# Formation of Structure in the Universe

Lent Term 2023

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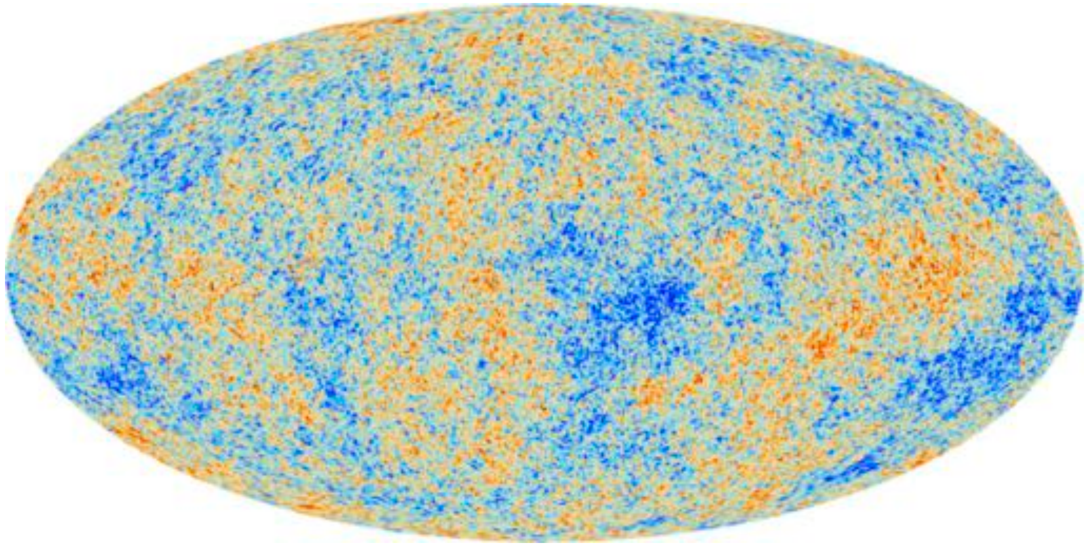
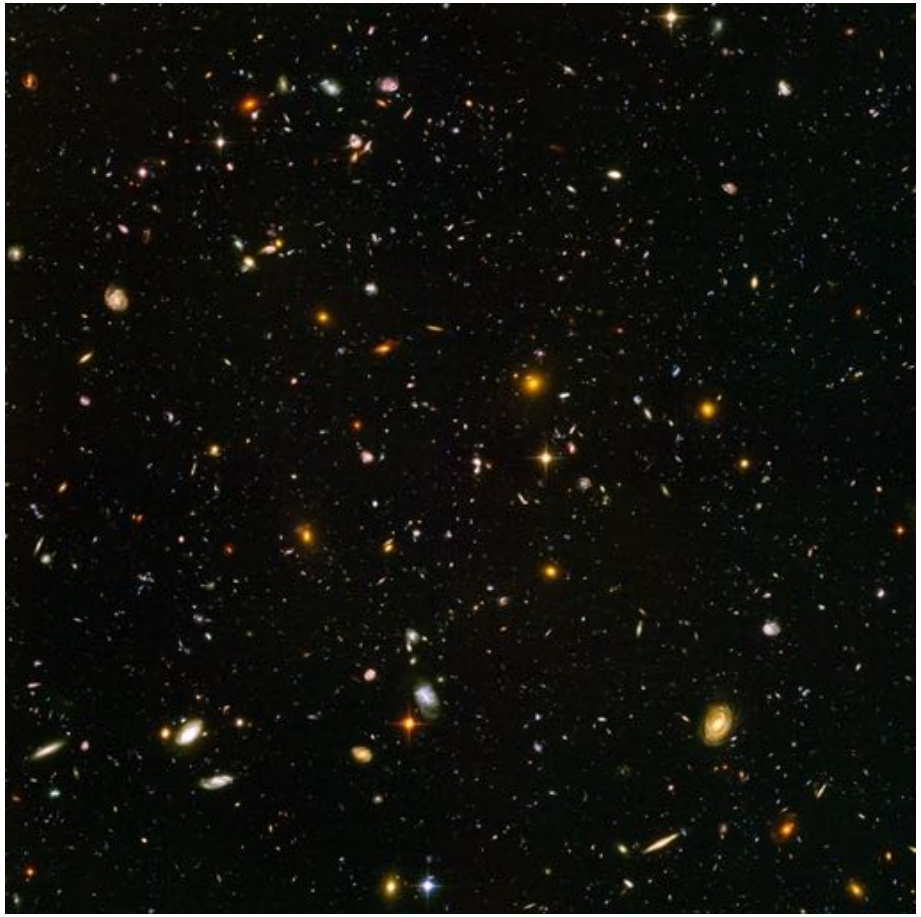
# Formation of Structure in the Universe in a nutshell





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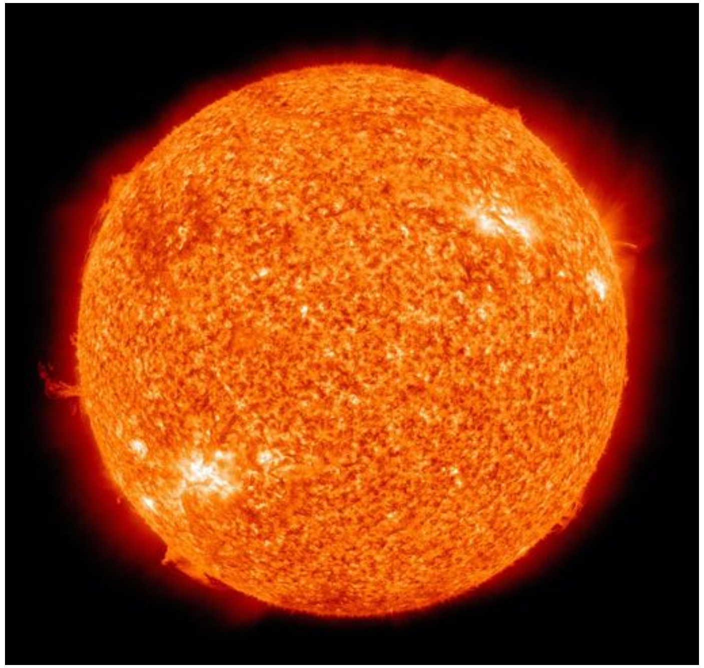


# Formation of Structure in the Universe in a nutshell

How can we heat and cool baryonic gas cloud ?

How stable is a gas cloud ?

How can we form a collapsed object from a gas cloud ?

What are the properties of galaxies ?

How stars in galaxies influence the on-going star-formation ?

How the first galaxies formed in the early Universe ?

# Formation of Structure in the Universe in a nutshell

| Date | Topic | Date | Topic | Date | Topic |
|---|---|---|---|---|---|
| 23/01/2023 | Introduction | 13/02/2023 | From gas cloud to collapsed object | 06/03/2023 | Gravitational instabilities in the cosmological context |
| 27/01/2023 | Physical process in baryonic gas (part 1) | 17/02/2023 | Galaxies and star-formation on galactic scales (part 1) | 10/03/2023 | Hierarchical structure formation (part 1) |
| 30/01/2023 | Physical process in baryonic gas (part 2) | 20/01/2023 | Galaxies and star-formation on galactic scales (part 2) | 13/03/2023 | Hierarchical structure formation (part 2) |
| 03/02/2023 | Gravitational stability and instability (part 1) | 24/02/2023 | Galaxies and star-formation on galactic scales (part 3) | 17/03/2023 | Galaxy formation and evolution |
| ~~06/02/2023~~ | Gravitational stability and instability (part 2) | 27/02/2023 | Feedback processes in star formation | April 2023 | Exam |
| 10/02/2023 | Gravitational collapse | 03/03/2023 | Galaxies interaction and triggering star-formation | | |

# Supervison

**3 groups** :

- 6-8 students/group
- please sign in (*email circulating soon*)
- 3 sessions

| Group ID | Session 1 | | Session 2 | | Session 3 | |
|---|---|---|---|---|---|---|
| | Date | Room | Date | Room | Date | Room |
| **1** | | | | | | |
| **2** | | | | | | |
| **3** | | | | | | |

# Introduction

## Chapter 1

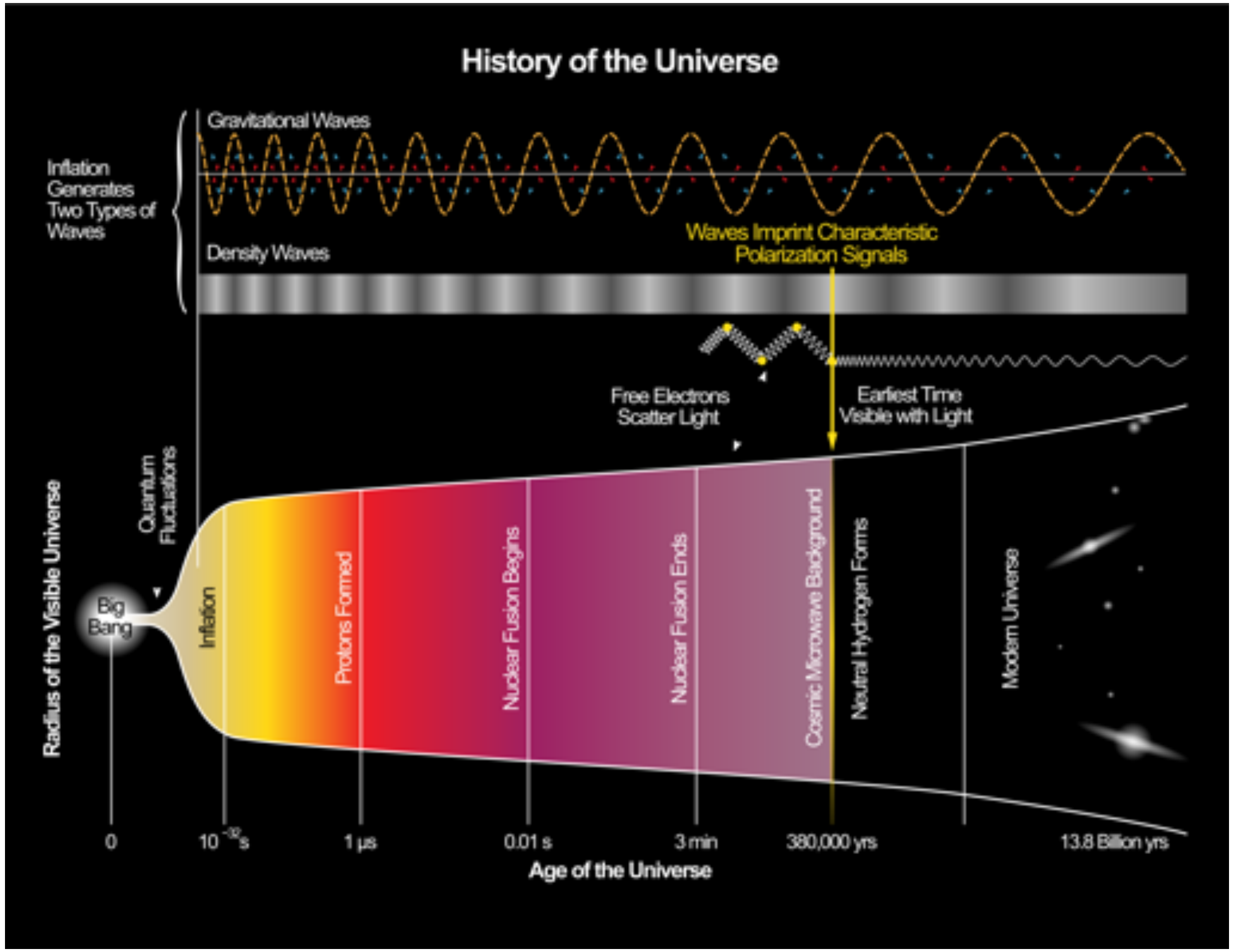
# The first 3 minutes of the Universe

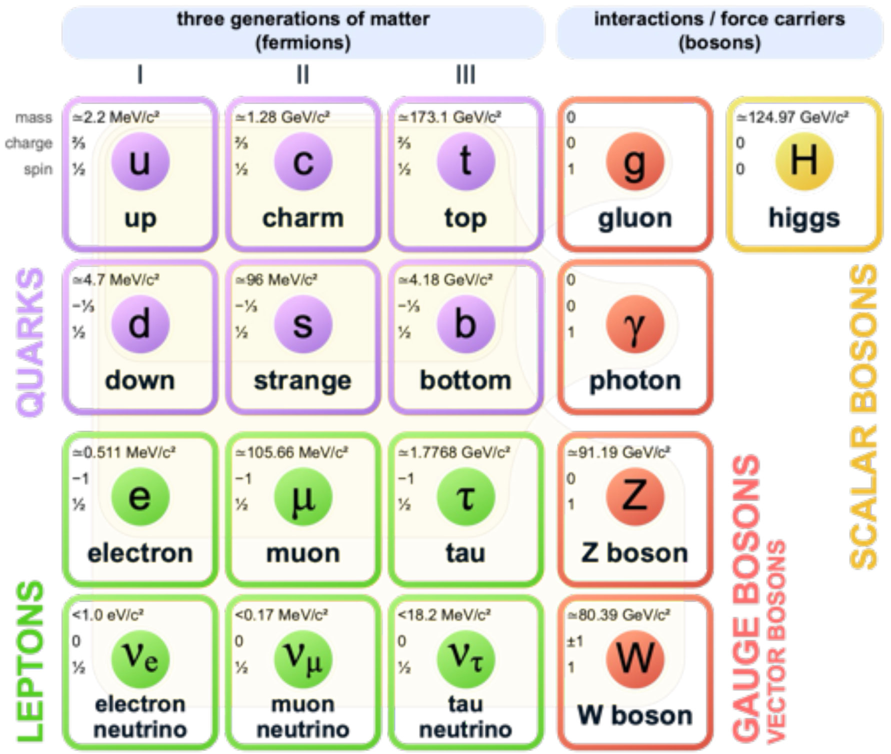
**t=0s** : *the Big-Bang*

**t=$10^{-36}$ to t=$10^{-32}$s** : *inflation*

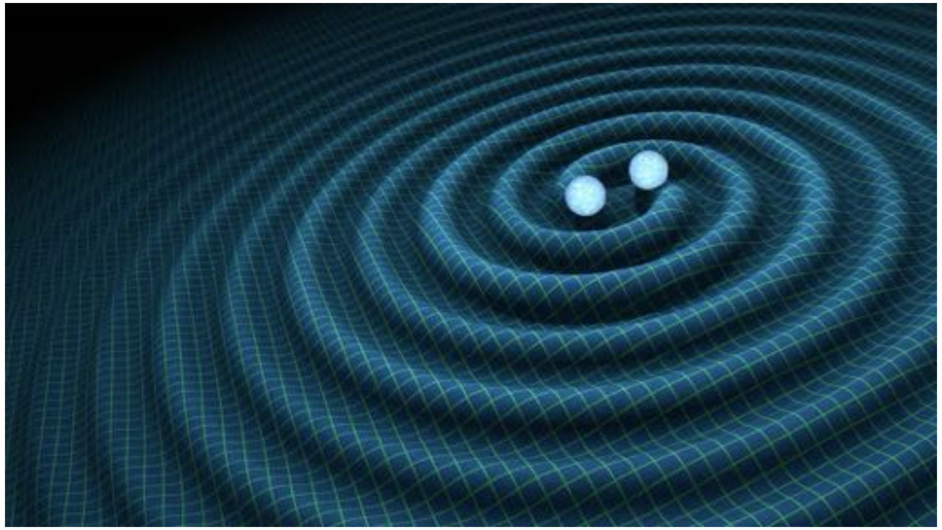
- *emission of gravitational waves*
- *emission of density waves*

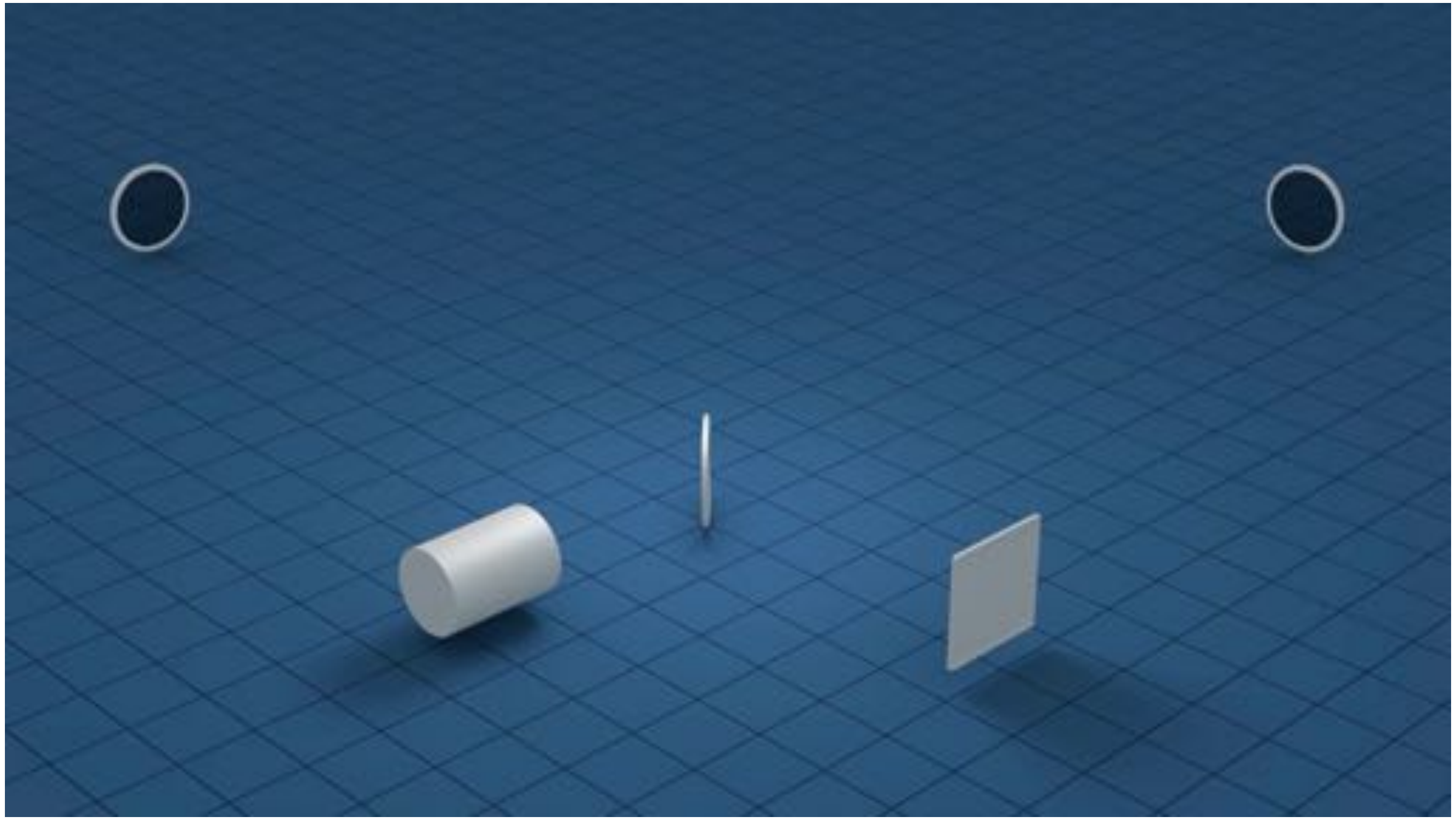
*Guzzetti et al. 2016, arXiv:* 1605.01615



# The first 3 minutes of the Universe





Resolution of current instrumentation : 1/1000 of the size of a proton.

# The first 3 minutes of the Universe

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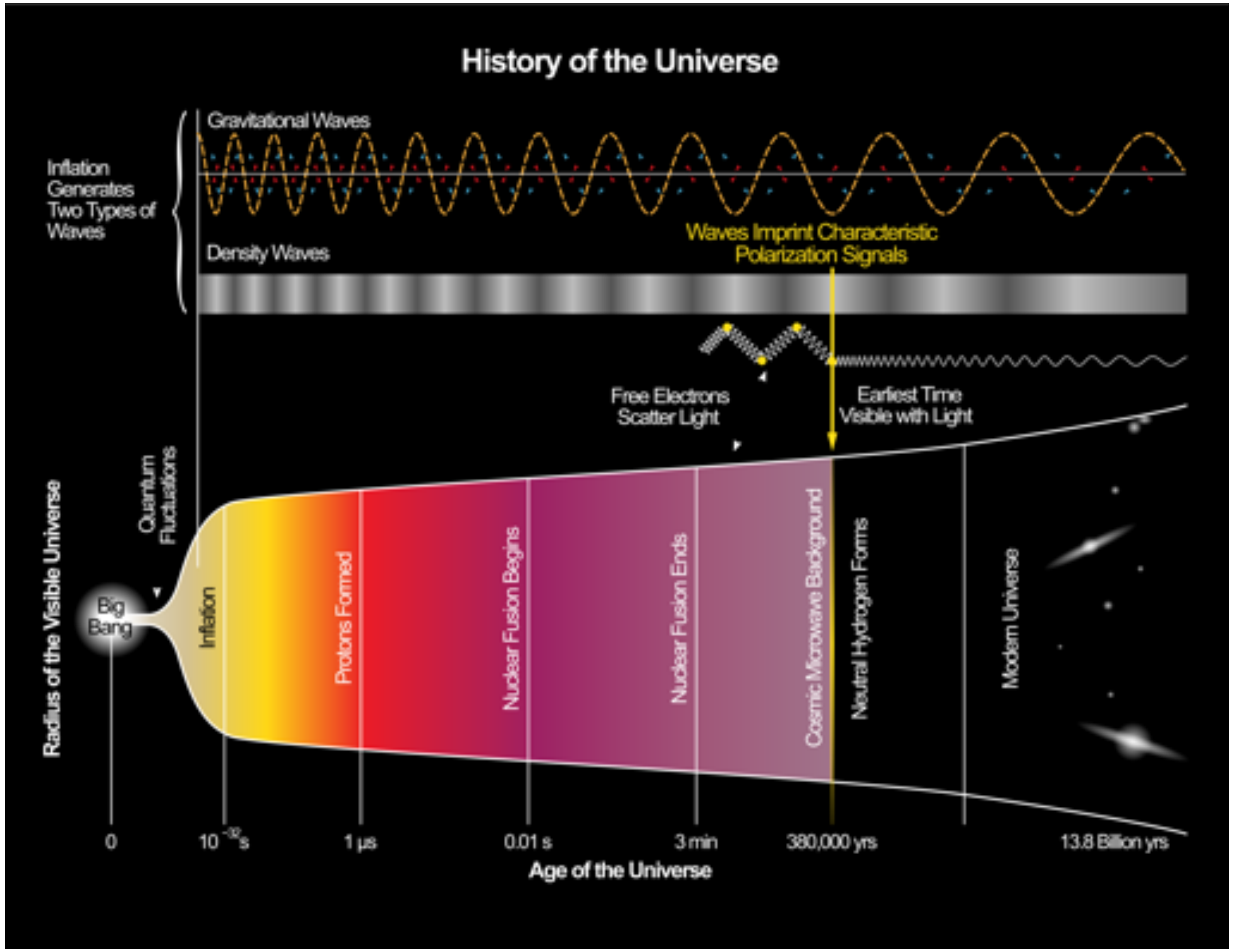


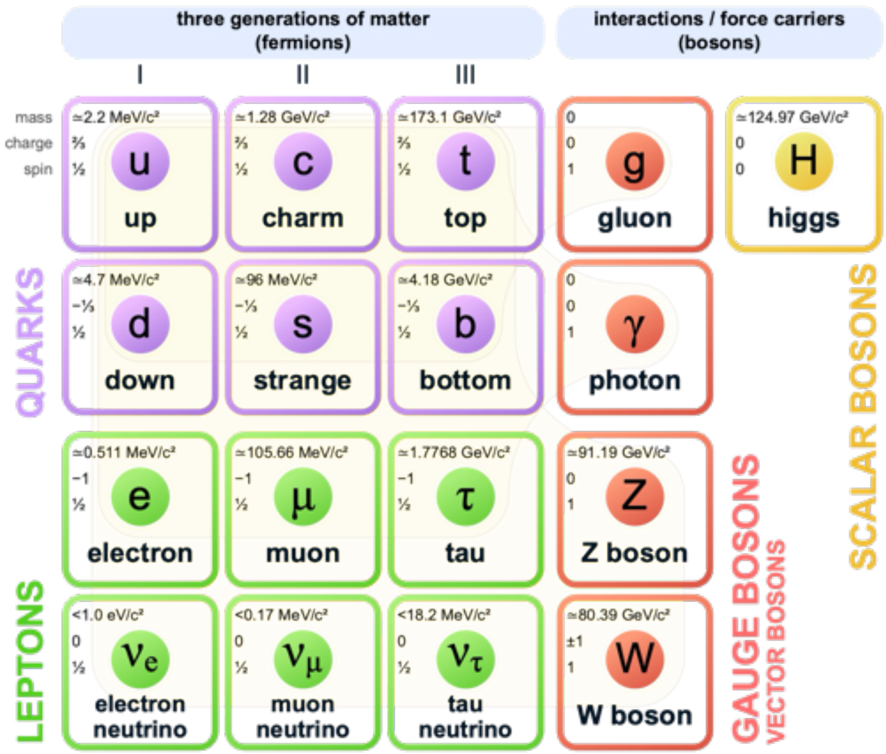

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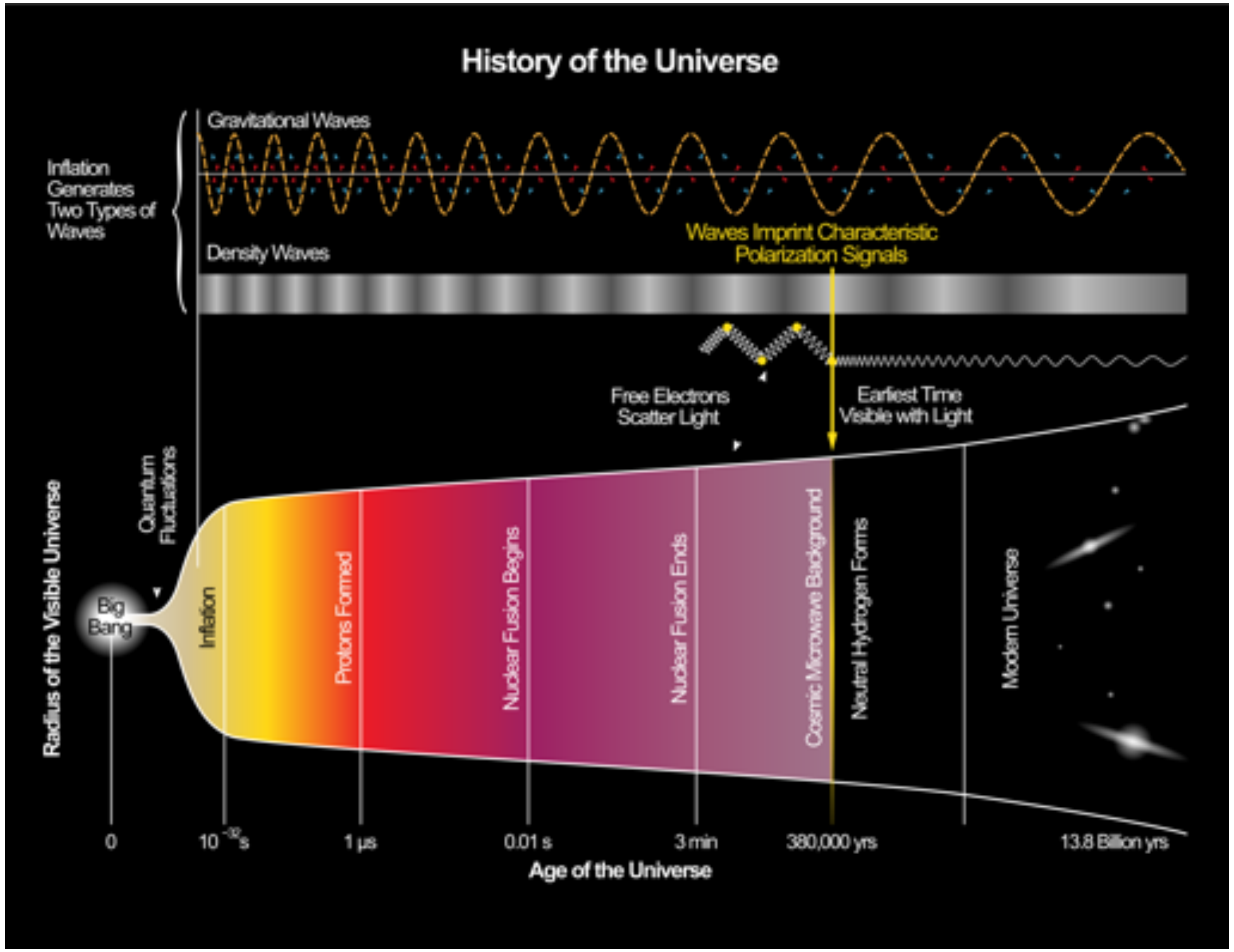


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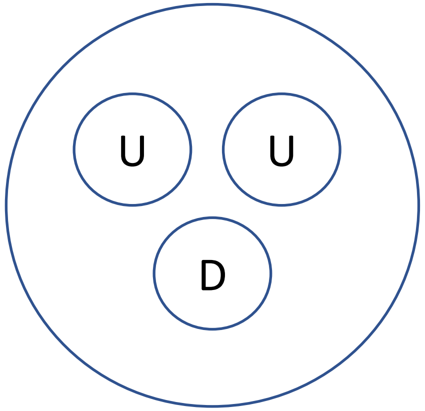
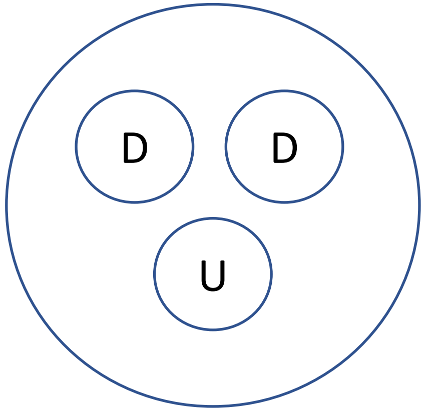
- *Universe mainly composed of quarks, leptons and photons*

**t=$10^{-6}$s :** *formation of protons and neutrons*

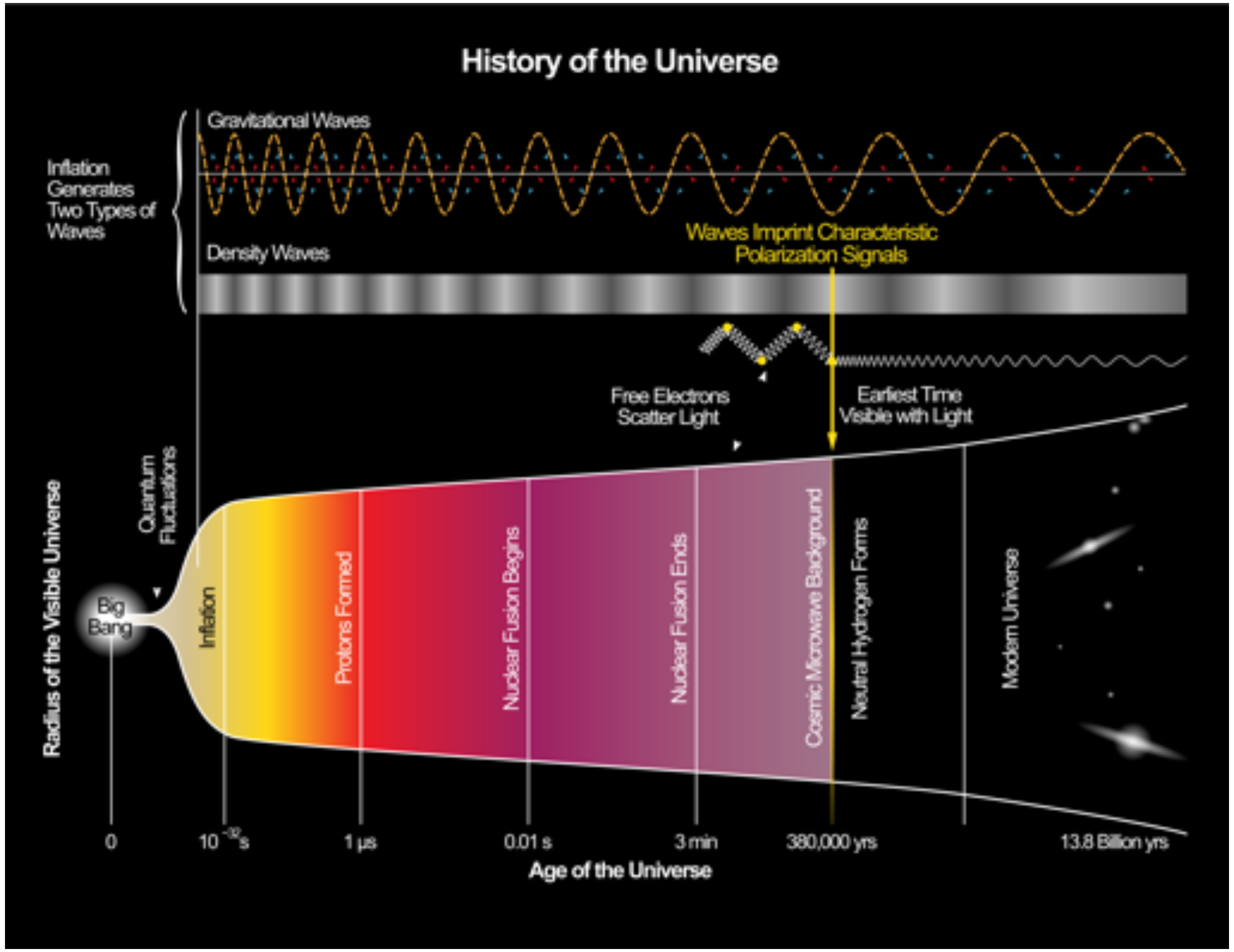



Proton




Neutron

# The first 3 minutes of the Universe

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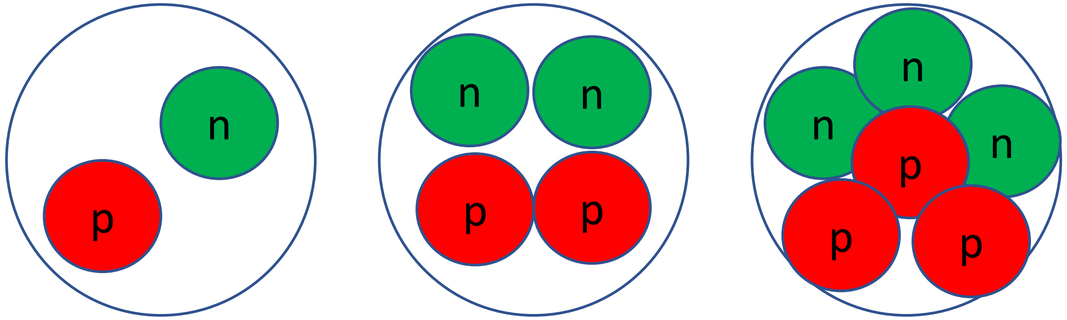
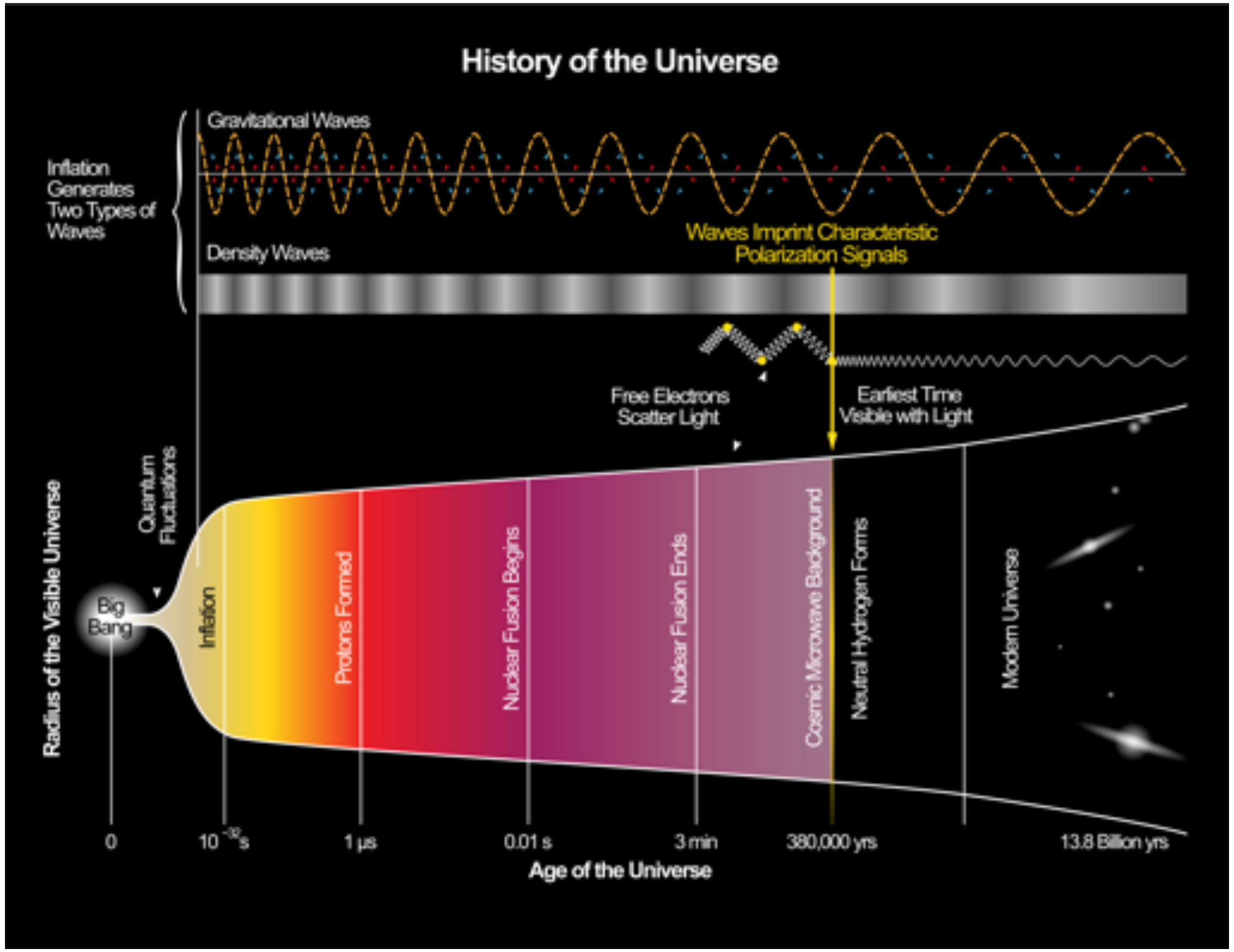
**t=$10^{-6}$s :** *formation of protons and neutrons, and then formation of nuclei of deuterium, helium and lithium*




Deuterium Helium Lithium

# The first 3 minutes of the Universe

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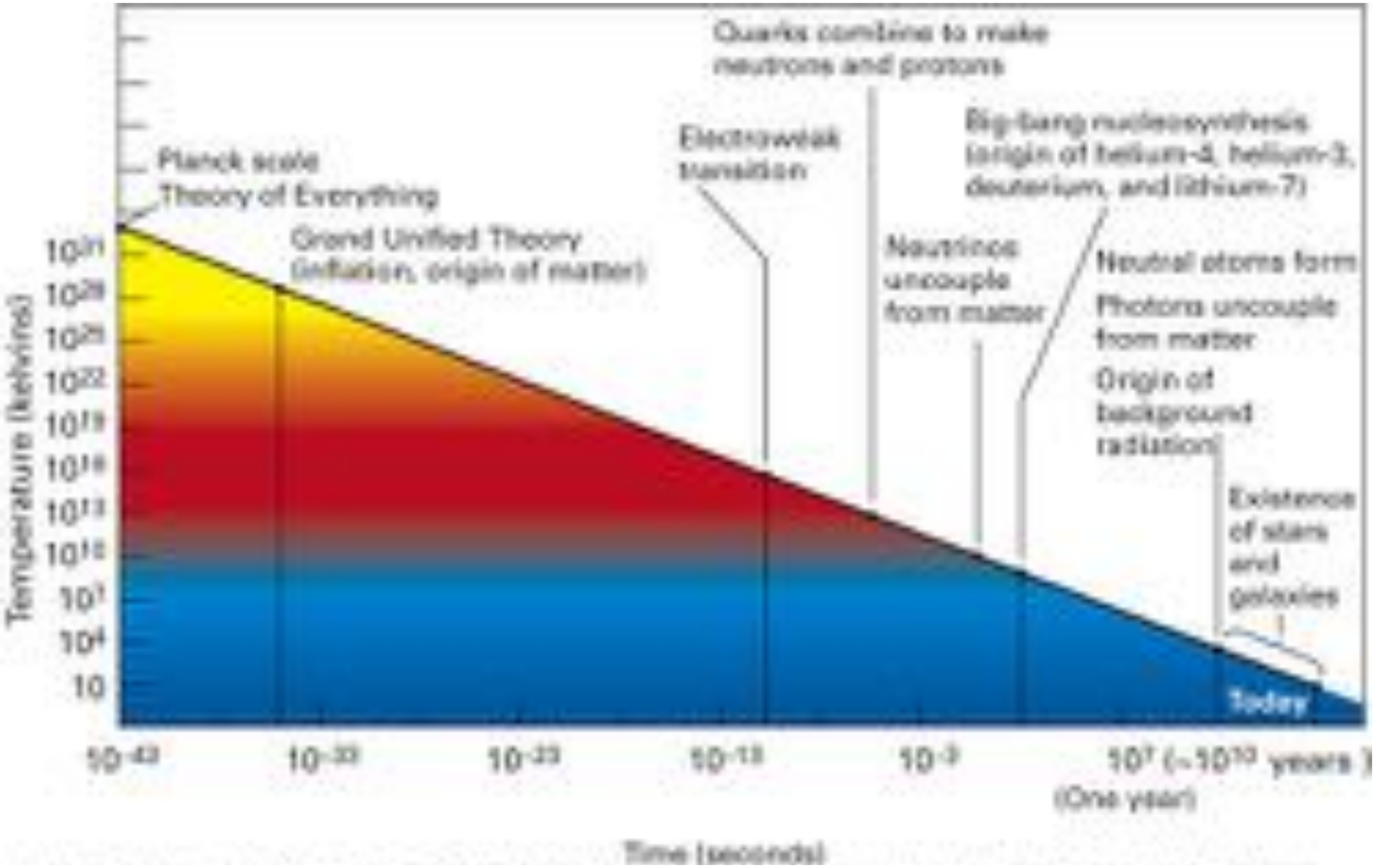
- *Universe mainly composed of quarks, leptons and photons*

**t=$10^{-6}$s :** *formation of protons and neutrons, and then formation of nuclei of deuterium, helium and lithium.*

**t=3mn :** *the Universe is mainly composed of radiation, baryonic matter, dark matter and dark energy.*

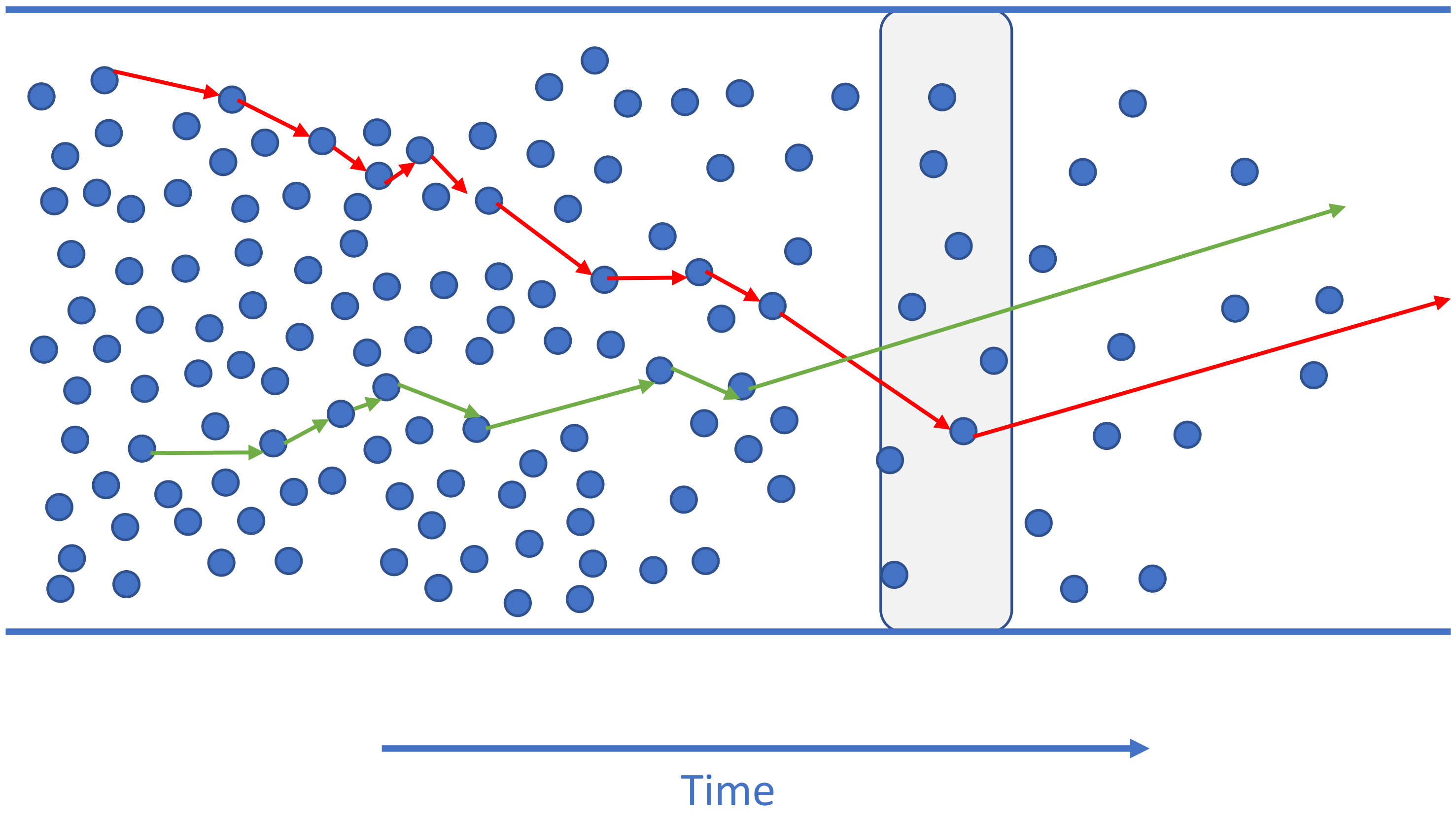
- *Electrons and nuclei are not bounded yet.*

# The formation of the first stars and galaxies




- Over-dense regions grow from the initial perturbation
- As the Universe is expanding, the Universe's temperature is decreasing.
- The Universe's density is also decreasing as the Universe is expanding

# The formation of the first stars and galaxies



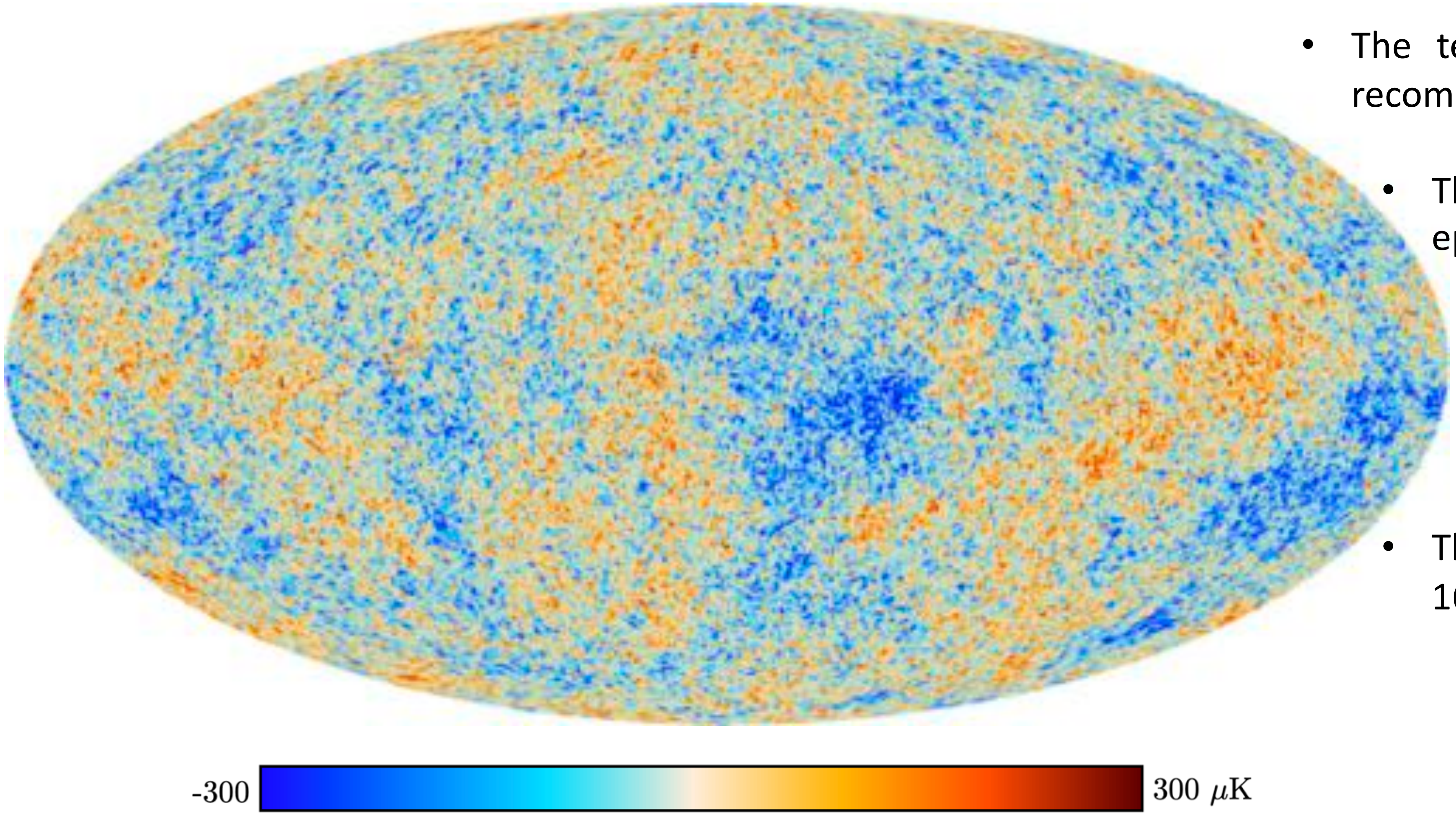

- Free electrons and protons start to be bounded but as soon as they are hit by a photon they become again unbounded :

$$e + p \Leftrightarrow H + \gamma$$

- When the Universe's density is sufficiently low to avoid interaction between photons and particles (*matter and radiation are decoupled*) :
    - The first atoms are formed (Hydrogen, Helium, Lithium)
    - Photons can escape, and form the first emitted radiation in the Universe : the **Cosmic Microwave Background**

This epoch is called **the recombination phase**

# The formation of the first stars and galaxies



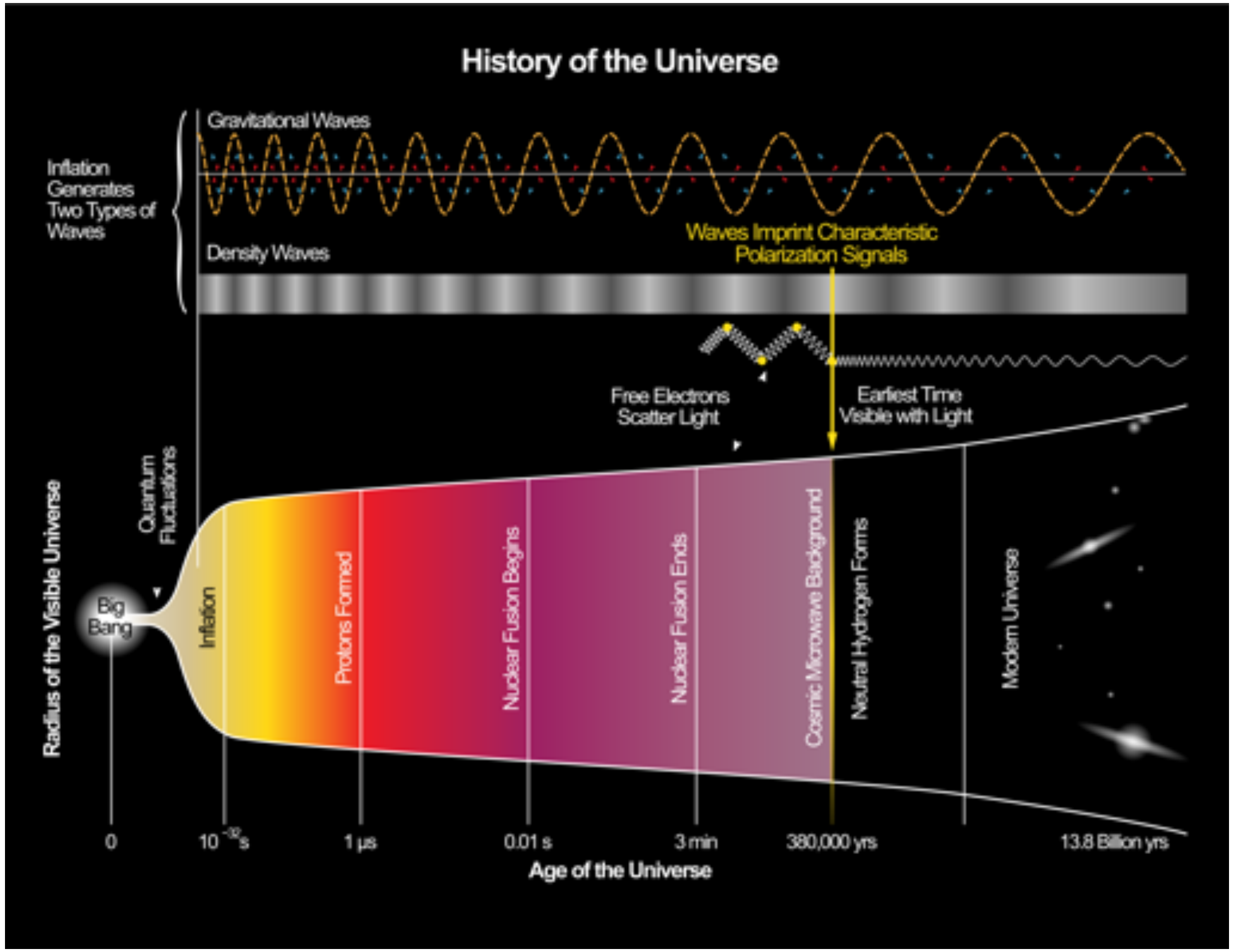

- The temperature of the Universe at the recombination epoch was ~3000K
- The redshift of the recombination epoch is $z\sim1100$
- Therefore, the observed temperature of the CMB is today :
$$T \approx \frac{3000}{1+z} = \frac{3000}{1101} \approx 2.7\ K$$
- The CMB is observed at a frequency of 160.23 GHz

# The formation of the first stars and galaxies

- European Space Agency (ESA) satellite
- Launched by an Ariane 5 rocket in May 2009
- Diameter :1.5m
- Objectives :
  - Map the Cosmic Microwave Background
  - Measure the cosmological parameters
  - Study galaxy clusters

# The formation of the first stars and galaxies

- After the recombination phase, the Universe enters the **Dark Ages**
- Overdense regions continue to grow
- Their density becomes sufficiently large that their gravitational field is dominated by their own mass
- Their evolution is now driven by their own gravity (self-gravitating objects) and not the evolution of the Universe (Hubble flow)
- At the center of these overdense regions, the gas cools and leads to the formation of the first stars : this epoch is called **Cosmic Dawn**



# The formation of the first stars and galaxies

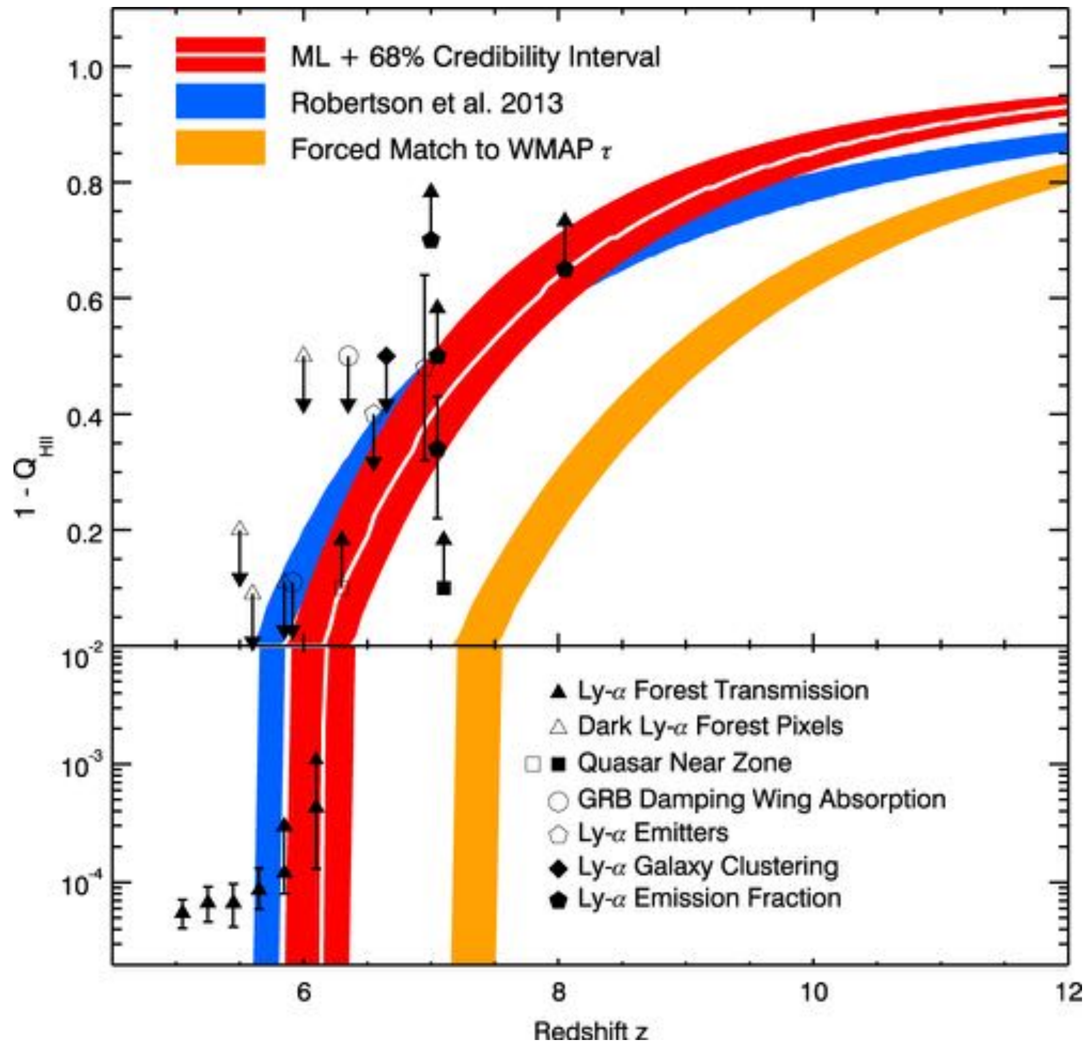
- The first stars are surrounded by neutral hydrogen

- The first stars emit UV photons* which will ionise the neutral hydrogen, creating bubbles of ionised hydrogen : this is the **epoch of reionisation**


* *This will be discussed later in this course*

# The formation of the first stars and galaxies

- The first stars are surrounded by neutral hydrogen
- The first stars emit UV photons* which will ionise the neutral hydrogen, creating bubbles of ionised hydrogen : this is the **epoch of reionisation**
- CMB observations by *Planck* reveals that the hydrogen is fully ionised 1 billion years after the Big-Bang (z=6)
- The study of the first generation of galaxies shows that they grow hierarchically by merging, leading to an evolving distribution of galaxies of different masses.




Robertson et al. (2015), ApJ, 802, 19

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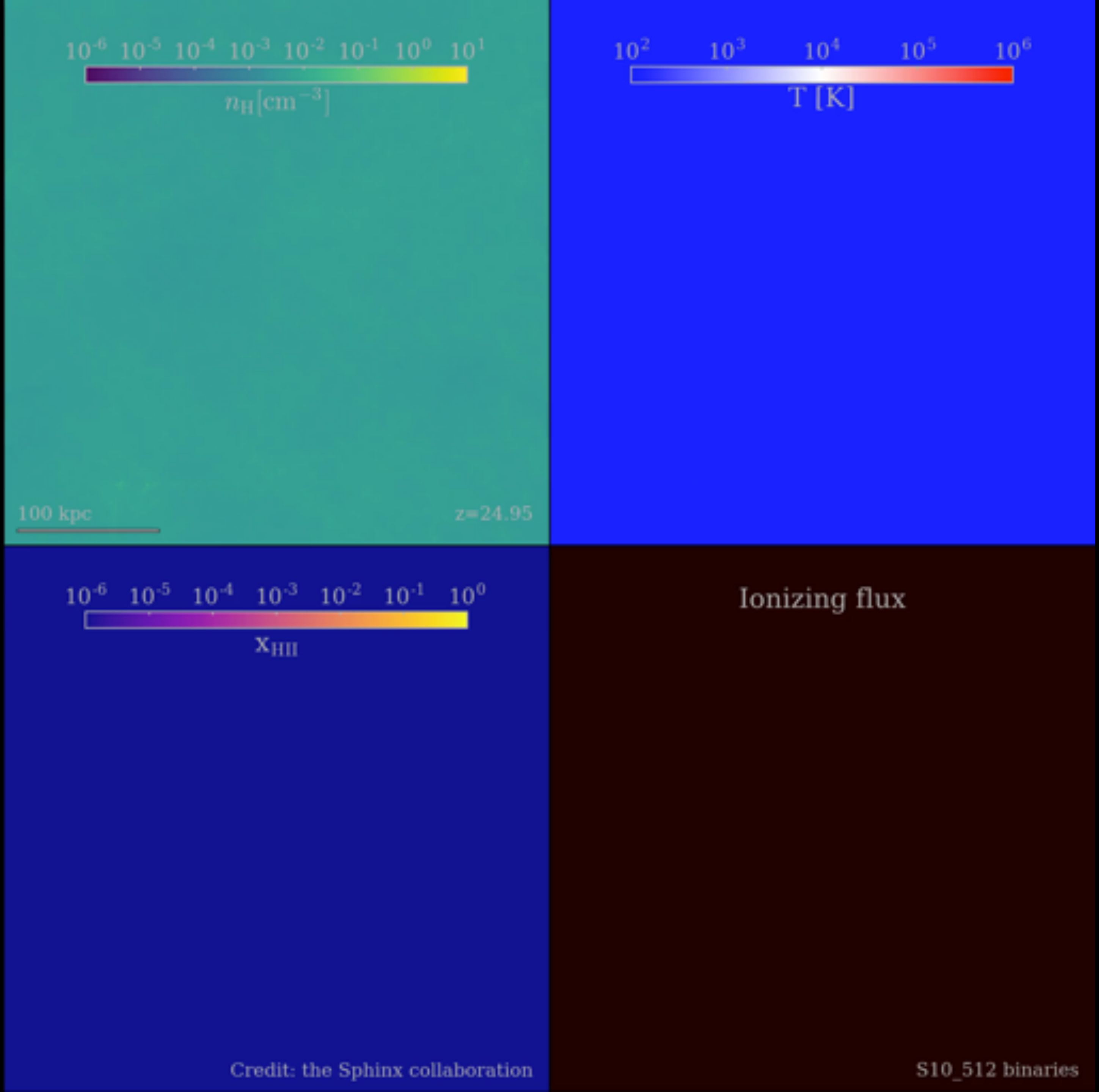
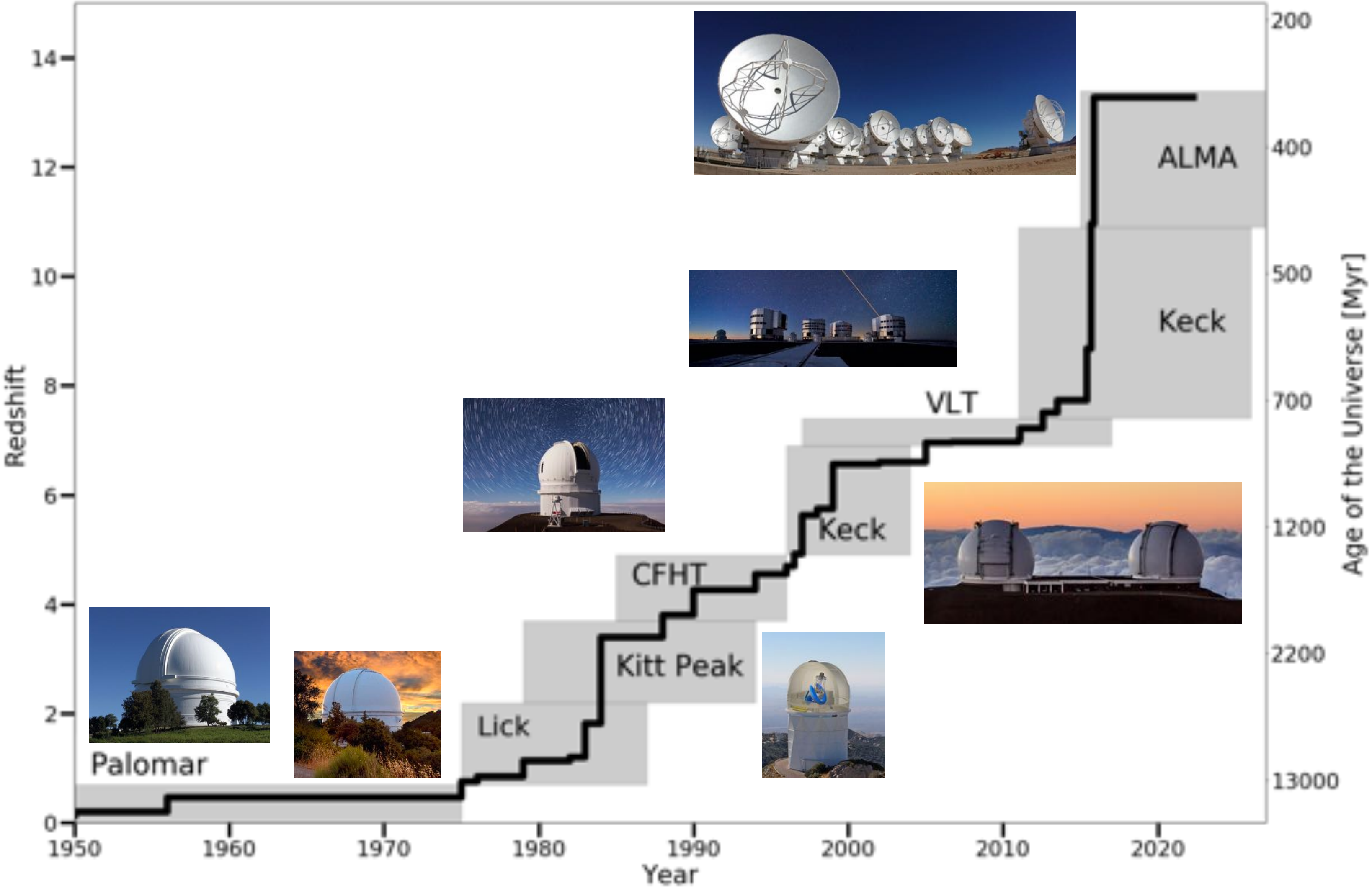
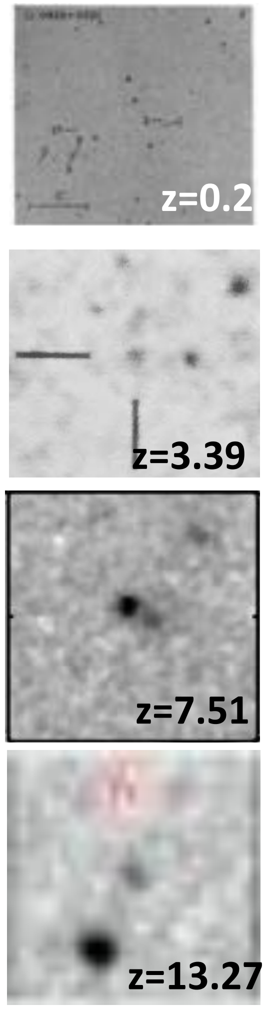



Rosdahl et al. 2018

# The “hunt” for the most distant galaxies
## *between 1950s and 2022*




Humason et al. 1956
Minkowski 1960
Spinrad et al. 1975
Spinrad & Smith 1976
Smith et al. 1979
Spinrad 1982
Spinrad & Djorgovsky 1984
Lilly 1988
Chambers et al. 1990
Lacy et al. 1994
Petitjean et al. 1996
Franz et al. 1997
Day et al. 1998
Hu et al. 1999, 2002
Pelló et al. 2004
Iye et al. 2006
Fontana et al. 2010
Vanzella et al. 2011
Ono et al. 2012
Shibuya et al. 2012
Finkelstein et al. 2013
Oesch et al 2014
Zitrin et al. 2015
Oesch et al. 2016
Harikane et al. 2022

# The arrival of the *James Webb* Space Telescope

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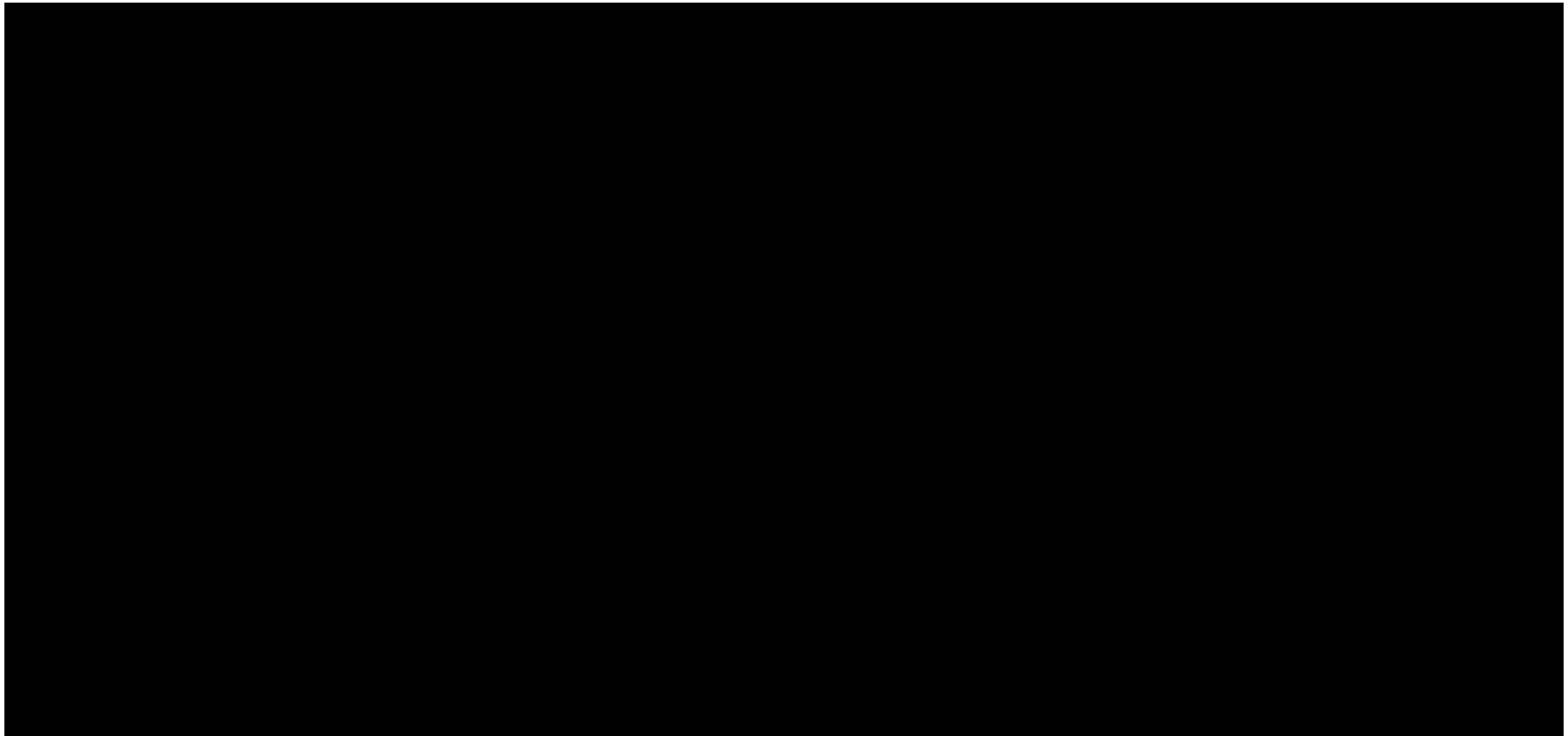

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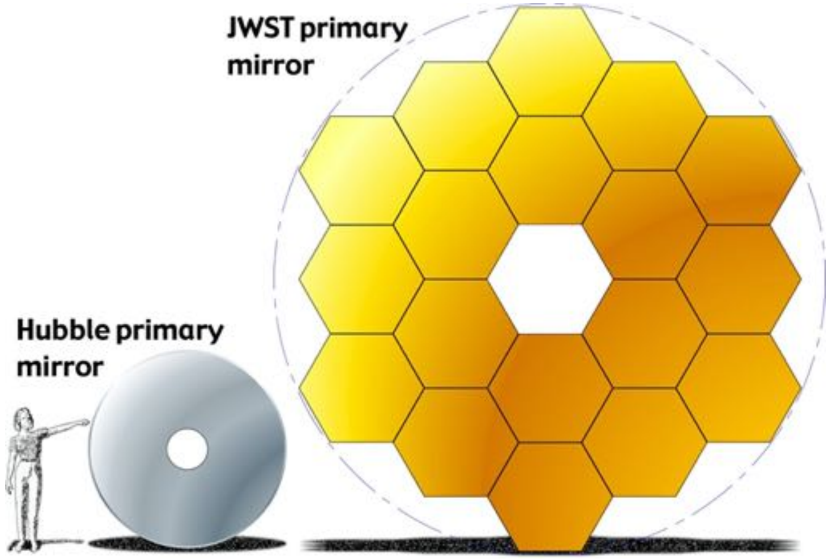

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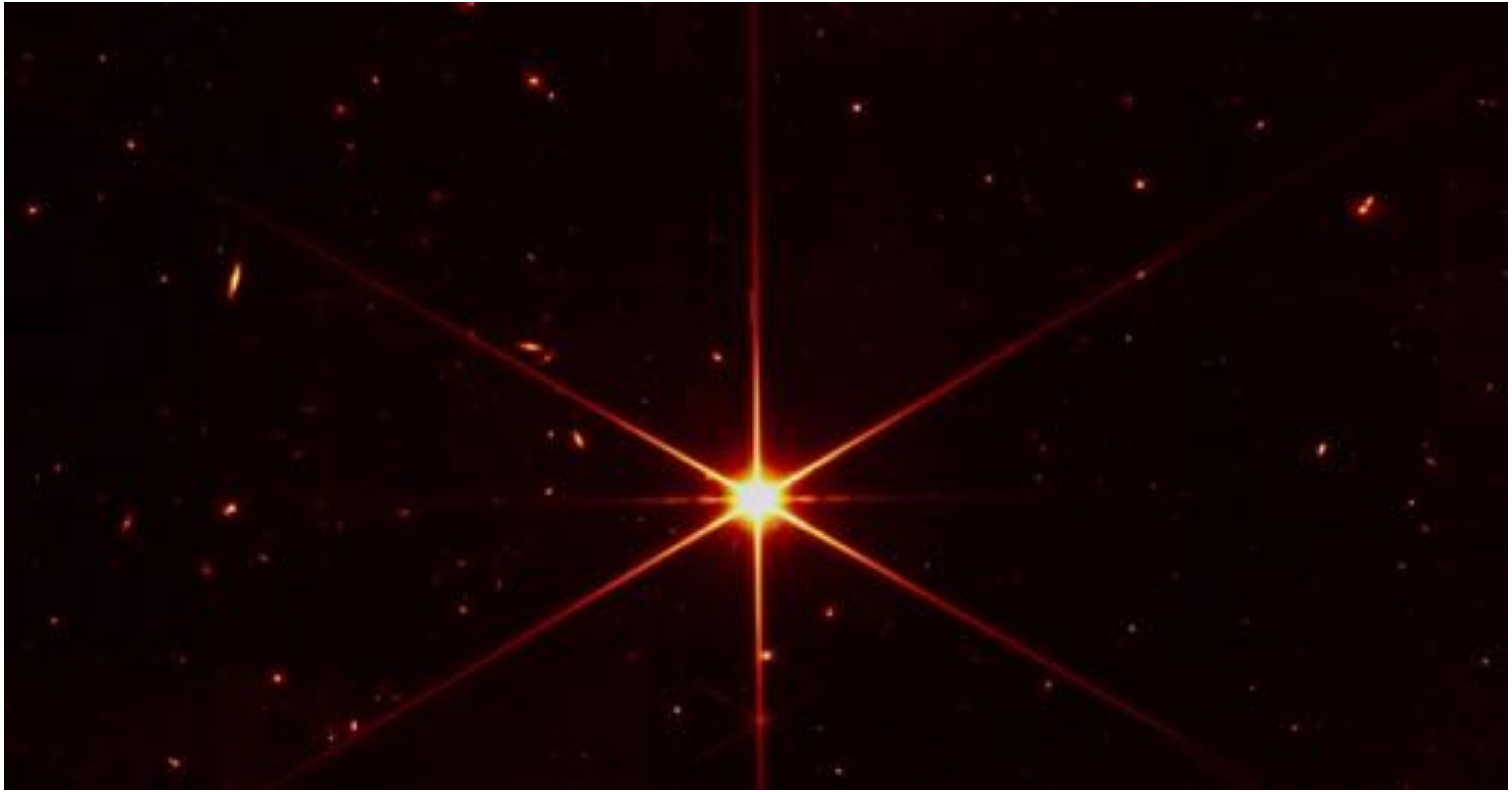

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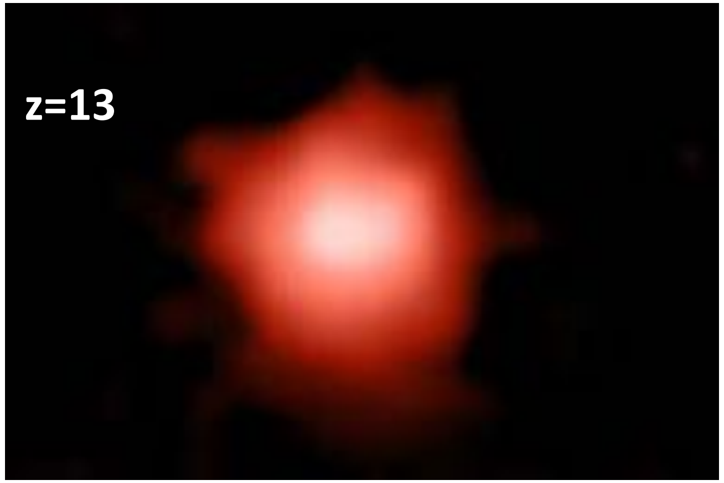
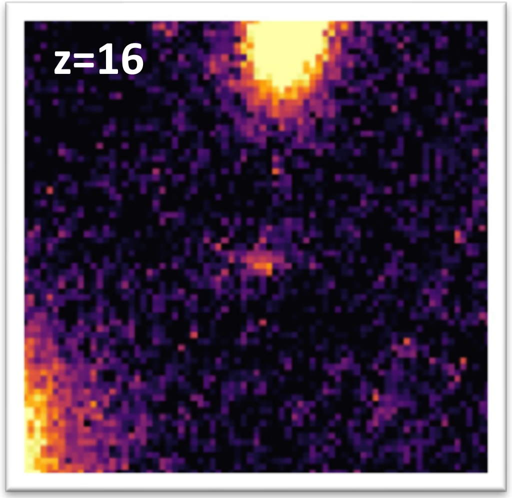
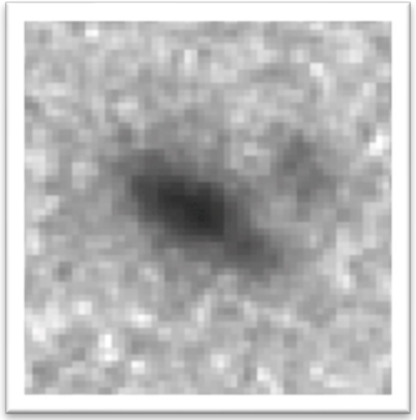



GLASS-z13 (Naidu et al. 2022)




SMACS-z16a (Atek et al. 2022)

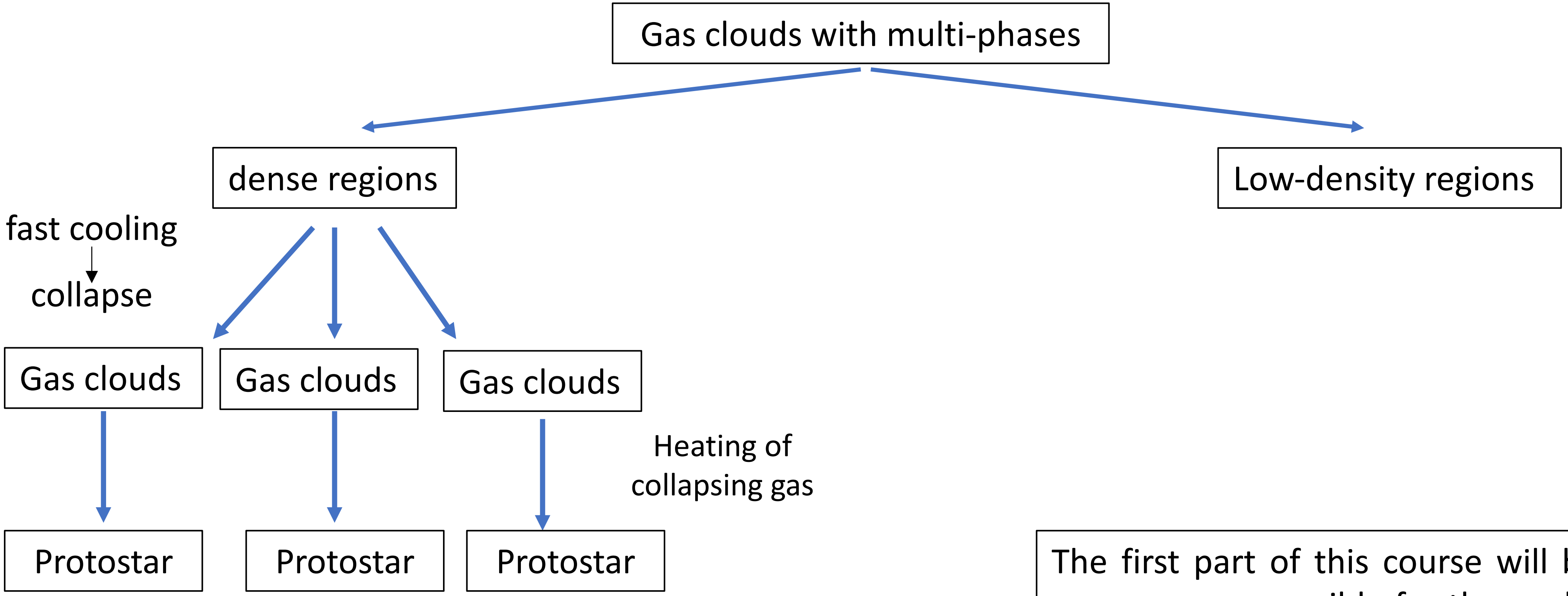
z=17



CEERS-DSFG-1 (Finkelstein et al. 2022)

Within a week, 11 papers have been submitted using the first dataset from the JWST to search for the first galaxies.

# Summary of the formation of structures in the Universe




The first part of this course will be to describe the processes responsible for the cooling of the gas, the heating of the gas and the formation of protostars.

# Toolbox for this course

## Euler's equation

*Adiabatic and inviscid flow*

pressure

$$\rho \frac{d\vec{v}}{dt} + \rho(\overrightarrow{v} \cdot \nabla)\vec{v} = -\nabla P - \rho \nabla \Phi_g$$

Flow velocity

Gravitational potential

## Equation of continuity

*Conservation of some quantities*

Density (mass/unit volume)

$$\frac{d\rho}{dt} + \nabla \cdot \vec{j} = 0$$

Flux=$\rho\vec{v}$

## Poisson's equation

*Differential equation*

$$\Delta\Phi = \nabla^2 \Phi = f$$

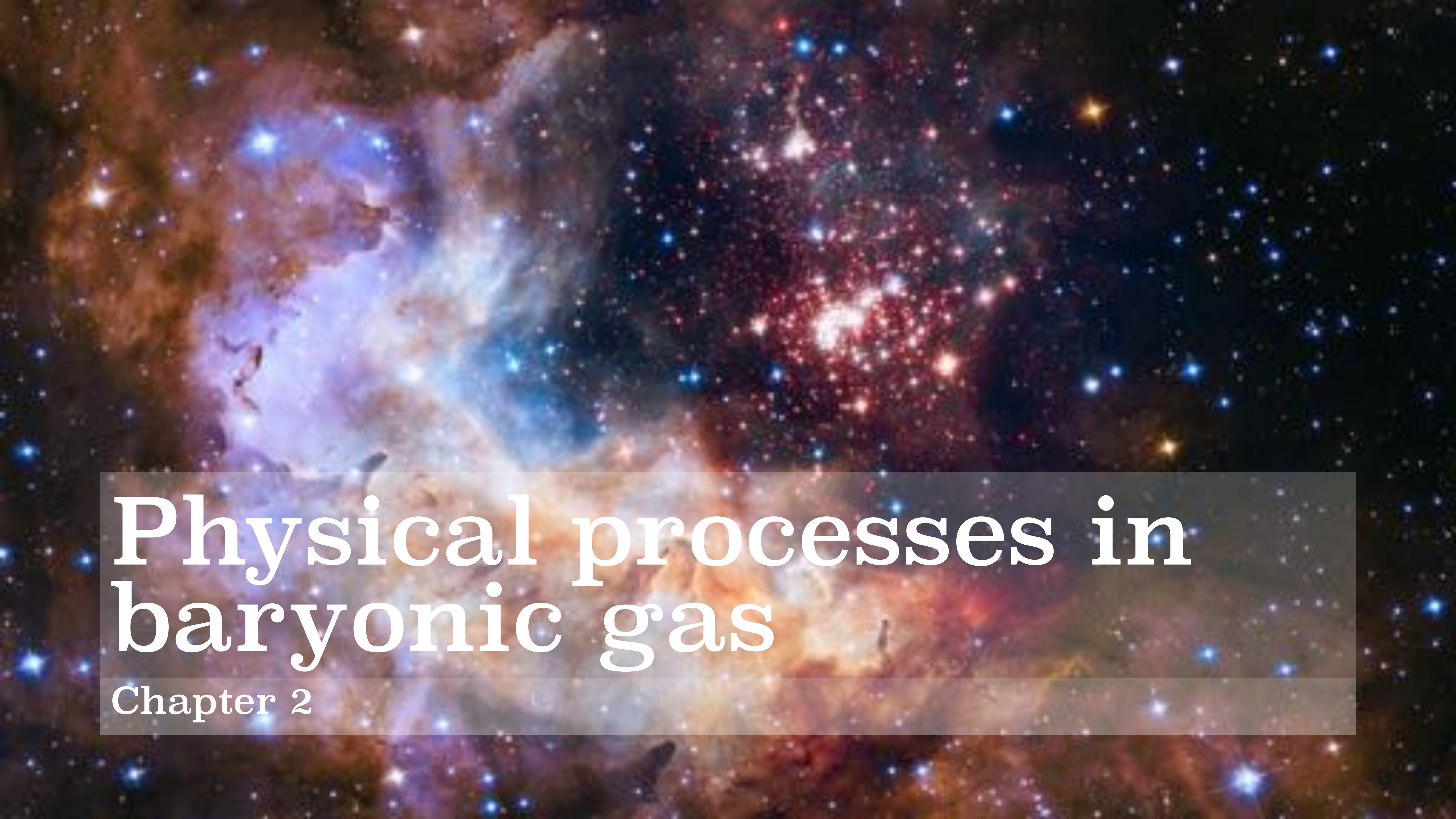
*For the gravitational potential we can write :*

$$\nabla^2 \Phi_g = 4\pi g \rho(\overrightarrow{r})$$

## Equation of state for an ideal gas

$$P = \frac{\rho k_B T}{\mu}$$

Mass particle



# Physical processes in baryonic gas

## Chapter 2

# Some definitions

The amount of energy ($dE$) passing through a surface should be proportional to the size of the surface ($dA$) and to the duration of the exposition ($dt$).

It is usually defined as :

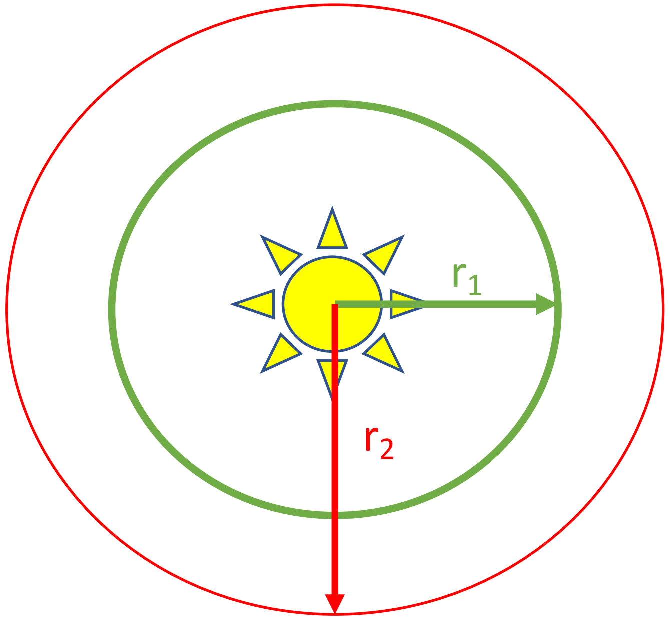
$$dE = dF \times dA \times dt$$

The energy flux $dF$ is measured in erg $s^{-1}$ $cm^{-2}$

A source of radiation is isotropic if it emits energy equally in all directions

By conservation of Energy, $dE(r_1) = dE(r_2)$ then :

$$F(r_1)4\pi r_1^2 = F(r_2)4\pi r_2^2$$



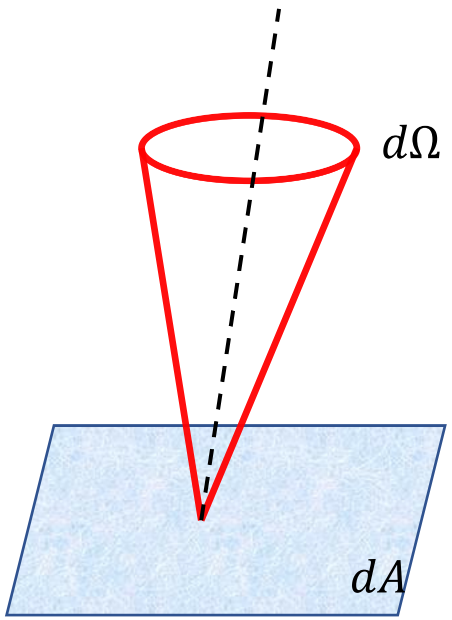

$$F(r) = \frac{F(r_1)r_1^2}{r^2}$$

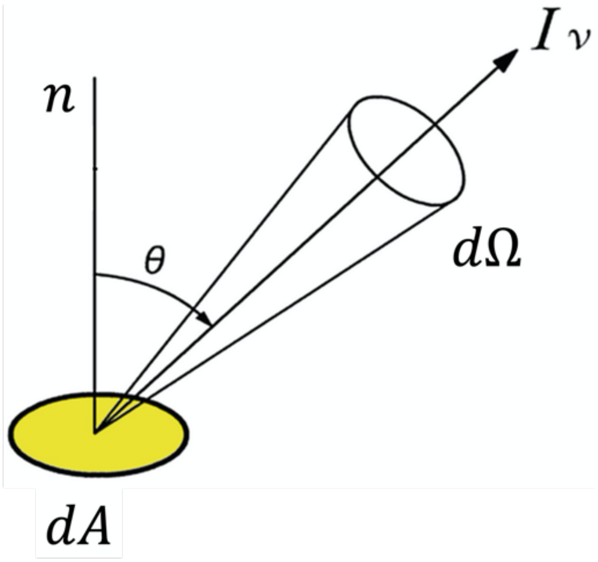
# Some definitions

Considering a surface normal to the direction of a given ray, and considering all the rays whose direction is within a solid angle $d\Omega$, then the energy passing through the element $dA$ is :

$$dE = I_\nu \, dA \, dt \, d\nu \, d\Omega$$

where $I_\nu$ is the <u>specific intensity</u> (or brightness)




If the surface is not perpendicular to the rays but has different orientation, then

$$\begin{aligned} dE &= I_\nu \, dA \, dt \, d\nu \cos\theta \; d\Omega \\ &= dF_\nu \, dA \, dt \, d\nu \end{aligned}$$

<u>N.B.</u> : If the radiation is isotropic, then $\int dF_\nu = 0$

# Some definitions

The <u>specific energy density</u> is the energy per unit volume per unit frequency range :

$$dE = u_\nu(\Omega)\, dV\, d\Omega\, d\nu$$

If we consider this cylinder, then $dV = dA\, c\, dT$ then

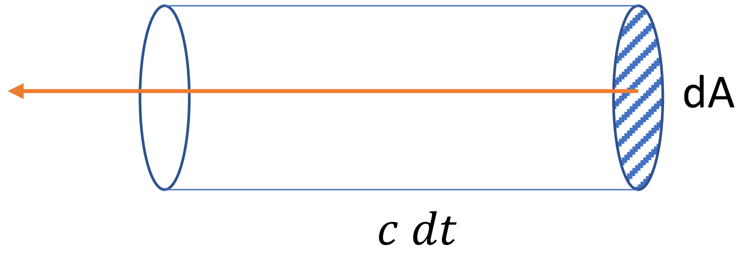
$$dE = u_\nu(\Omega)\, dA\, c\, dt\, d\Omega\, d\nu$$

but within $dt$ all radiation will pass out of the cylinder, then :

$$dE = I_\nu dA\, d\Omega\, dt\, d\nu$$

The specific energy density is defined as :

$$u_\nu(\Omega) = \frac{I_\nu}{c}$$




Integrating the specific energy density over solid angles :

$$u_\nu = \int u_\nu(\Omega) d\Omega = \frac{1}{c}\int I_\nu d\Omega$$

or $u_\nu = \frac{4\pi}{c} J_\nu$

The <u>mean density</u> is defined as :

$$J_\nu = \frac{1}{4\pi}\int I_\nu d\Omega$$

The <u>total radiation density</u> is given by

$$u = \int u_\nu d\nu = \frac{4\pi}{c}\int J_\nu d\nu$$

# Radiative transfer

We use radiative transfer each time a radiation is passing through a matter and add (absorption) or subtract (emission) energy.

## I- EMISSION

The spontaneous emission coefficient is defined as the energy emitted per unit time per unit solid angle per unit volume, such as :

$$dE = j\ dV\ d\Omega\ dt$$

If the emission is monochromatic (e.g., an emission line), we can define a monochromatic emission coefficient :

$$dE = \ j_\nu dV\ d\Omega\ dt\ d\nu$$

If the emission is isotropic, then :

$$j_\nu = \frac{1}{4\pi} P_\nu$$

where $P_\nu$ is the radiated power

We can also define the emissivity as the energy emitted per unit frequency per unit time per unit mass, and rewrite the transmitted energy in an isotropic emission as :

$$dE = \ \epsilon_\nu \rho\ dV\ dt\ d\nu\ \frac{d\Omega}{4\pi}$$

For an isotropic emission, the relation between the emission coefficient and the emissivity is given by :

$$j_\nu = \frac{\epsilon_\nu \rho}{4\ \pi}$$

Considering a beam of cross-section $dA$ traveling through a volume $dV = ds \times dA$ , then the energy added by spontaneous emission is :

$$dI_\nu = j_\nu\ ds$$

(Remember that $dE = I_\nu\ dA\ dt\ d\nu\ d\Omega$)

# Radiative transfer

We use radiative transfer each time a radiation is passing through a matter and add (absorption) or subtract (emission) energy.

## II- ABSORPTION

The absorption coefficient is defined as the loss of intensity in a beam as it travels a distance $ds$ :

$$dI_\nu = -\alpha_\nu \, I_\nu \, ds$$

The absorption depends on the density of absorbers along the travel of a beam.

If we assume a random distribution of absorbers, each of them with a cross section $\sigma_\nu$ and a density per unit volume $n$, then the effect of these absorbers on a radiation passing through $dA$ within a solid angle $d\Omega$ is :

$$dE = -dI_\nu \, dA \, dt \, d\Omega \, d\nu$$

or : $dE = I_\nu (n \, dA \, ds \, \sigma_\nu) d\Omega \, dt \, d\nu$

Therefore :

$$dI_\nu = -n \, \sigma_\nu \, I_\nu \, ds$$

We can rewrite the absorption coefficient such as :

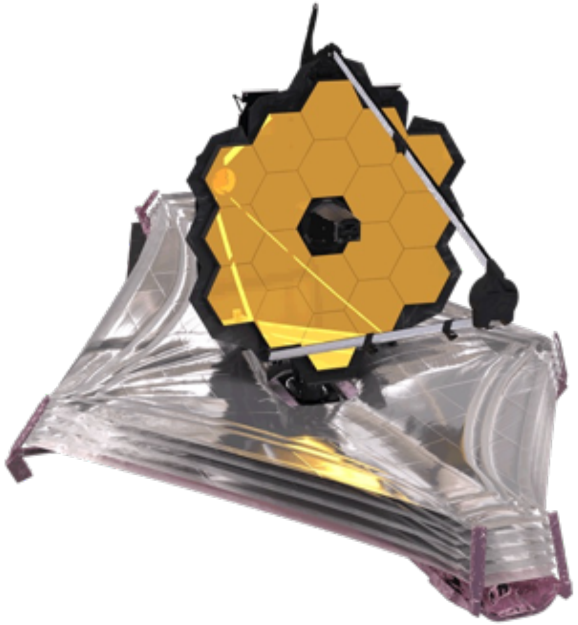
$$\alpha_\nu = n \, \sigma_\nu$$

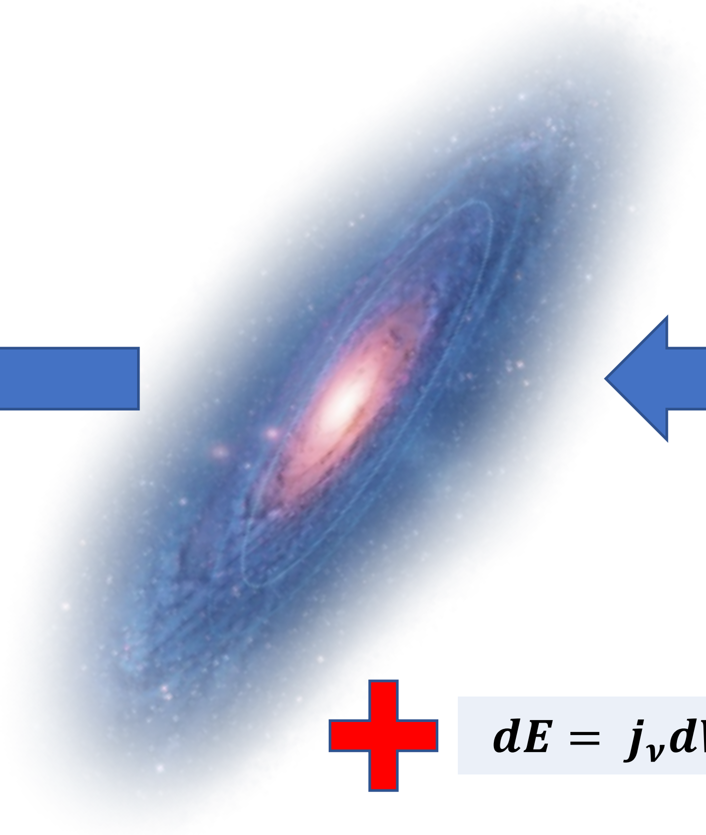
Usually, $\alpha_\nu$ is defined with the opacity (also known as the mass absorption coefficient) such as :

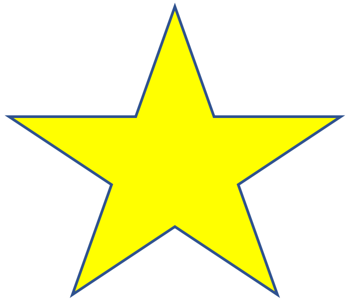
$$\alpha_\nu = \rho \, \kappa_\nu$$

# Summary of Monday's lecture

$$dE = I_\nu dA\, dt\, d\nu\, d\Omega$$







$+$ $$dE = j_\nu dV\, d\Omega\, dt\, d\nu$$

$-$ $$dE = -n\, \sigma_\nu\, dA\, dS\, I_\nu\, d\Omega\, dt\, d\nu$$

# Summary of Monday's lecture

## DEFINITIONS

The energy passing through a surface element $dA$ over a time $dt$ is given by

$$\boldsymbol{dE = dF \times dA \times dt = I_\nu dA\, dt\, d\nu\, d\Omega}$$

*Specific intensity*

We defined the <u>specific energy density</u> by :

$$\boldsymbol{dE = u_\nu(\Omega)\, dV\, d\nu\, d\Omega}$$

$$u_\nu = \frac{I_\nu}{c} \qquad dV = c\, dt\, dA$$

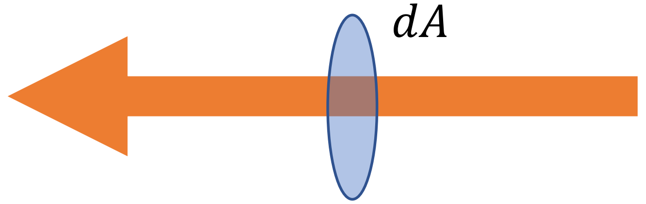
We also defined the <u>mean density</u> by :

$$\boldsymbol{J_\nu = \frac{1}{4\pi} \int I_\nu d\Omega}$$

The <u>total radiation density</u> is :

$$\boldsymbol{u = \int u_\nu d\nu = \frac{4\pi}{c} \int J_\nu d\nu}$$

## RADIATIVE TRANSFER




When a beam of light is passing through a surface $dA$ light can be added to the beam (emission by the material) or subtracted (absorption by the material)

**<u>Emission of light</u>** (adding energy to the beam) :

$$dE = j_\nu dV\, d\Omega\, dt\, d\nu$$

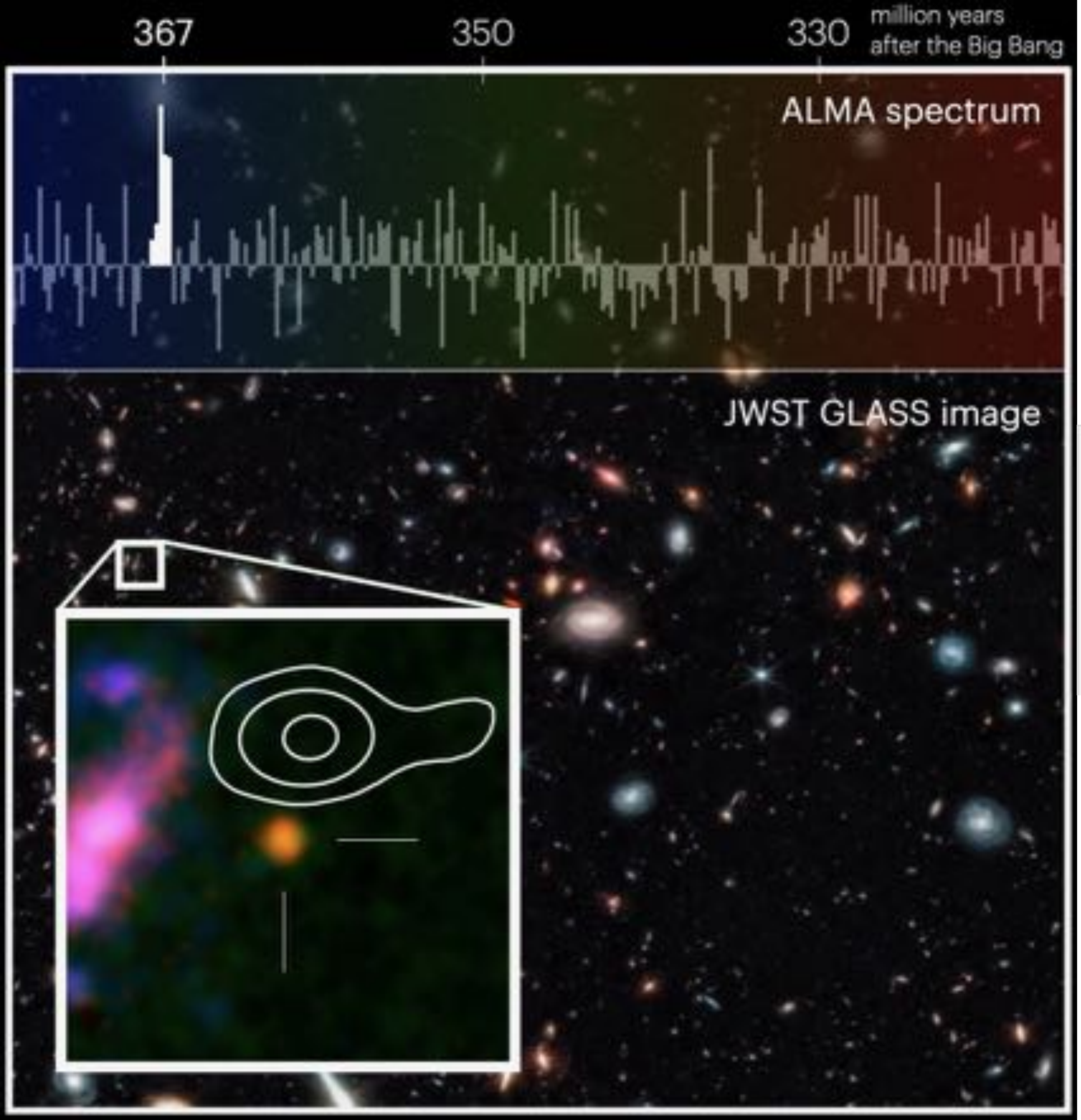
*Spontaneous emission coefficient*

**<u>Absorption of light</u>** (subtracting energy to the beam) :

$$dE = -\underbrace{n\, \sigma_\nu}_{\alpha_\nu} dA\, dS\, I_\nu\, d\Omega\, dt\, d\nu$$

*"-" absorption subtracts energy to the beam*

*Absorption coefficient*

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## Deep ALMA redshift search of a $z \sim 12$ GLASS-*JWST* galaxy candidate

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**$z=12.110 \pm 0.001$**

# Radiative transfer

We use radiative transfer each time a radiation is passing through a matter and add (absorption) or subtract (emission) energy.

## III- RADIATIVE TRANSFER EQUATION

The radiative transfer equation shows how the intensity of a beam evolves and includes contribution of spontaneous emission and absorption.

We showed that :

- Emission : $dI_\nu = j_\nu \, ds$
- Absorption : $dI_\nu = -n \, \sigma_\nu \, I_\nu \, ds$

Therefore :

$$\boxed{\boldsymbol{\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu}}$$

<u>Exact solutions of the radiative transfer equation :</u>

- **Emission only** : $\frac{dI_\nu}{ds} = j_\nu$

  ➔ $I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu \, (s')ds'$

- **Absorption only :** $\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$

  ➔ $I_\nu(s) = I_\nu(s_0) \exp[-\int_{s_0}^{s} \alpha_\nu(s') \, ds']$

Outside of these simple cases, the resolution of the radiative transfer equation requires numerical analysis

# Radiative transfer

## III- OPTICAL DEPTH AND SOURCE FUNCTION

The optical depth is a measure of the transparency of a medium, and is defined as :

$$d\tau_\nu = \alpha_\nu ds$$

*Absorption coefficient*

or by integrating :

$$\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

By definition, a medium is said optically thin (or transparent) if a photon can traverse it without being absorbed.

- Optically thin : $\tau_\nu \leq 1$
- Optically thick : $\tau_\nu \geq 1$

Filters used to observe the Sun are good examples of optically thick medium

# Radiative transfer

$$d\tau_\nu = \alpha_\nu ds$$

## IV- OPTICAL DEPTH AND SOURCE FUNCTION

$$\frac{dI_\nu}{dS} = j_\nu - \alpha_\nu I_\nu$$

The radiative transfer equation can now be rewritten to include the optical depth :

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

where $S_\nu$ is the source function defined as : $S_\nu = \frac{j_\nu}{\alpha_\nu}$

We can demonstrate that the formal solution of this equation can be written as :

$$I_\nu(\tau_\nu) = S_\nu + e^{-\tau_\nu}(I_\nu(0) - S_\nu)$$

# Radiative transfer

## V- MEAN FREE PATH

The mean distance a photon can travel through an absorbing material without being absorbed is the mean free path.

If we consider the photon mean free path as it tries to escape from an emitting region, and assuming $\tau_\nu = 1$, we have :

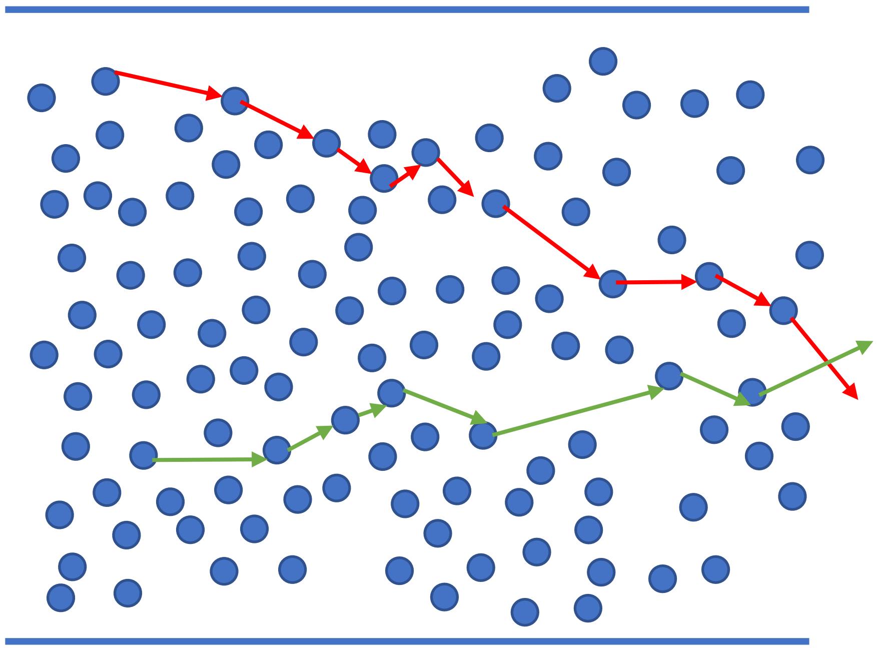
$$\tau_\nu = \alpha_\nu s = 1$$

then :

$$s = \frac{1}{\alpha_\nu} = \frac{1}{n\,\sigma_\nu} = l_\nu$$

A photon escaping from a region with an optical depth $\tau_\nu$ will undergo a random walk with N scatterings :

$$L = \sqrt{N l_\nu} \Rightarrow N \sim \frac{L^2}{l_\nu^2} \sim (\alpha_\nu L)^2 = \tau_\nu^2$$



# Thermal Radiation

<u>Thermal Radiation</u> is radiation emitted by matter in thermal equilibrium. The best example is the *black body radiation*.

The specific intensity depends only on the temperature of the radiation, such as :

$$I_\nu = B_\nu(T)$$

where $B_\nu(T)$ is the Planck function, defined as :

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

In the case of <u>thermal radiation</u>, the source function is defined by :

$$S_\nu = B_\nu(T)$$

then $j_\nu = \alpha_\nu B_\nu(T)$ *Kirchoff's law*

The radiative transfer equation can now be rewritten as :

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + \alpha_\nu B_\nu(T)$$

or $\frac{dI_\nu}{d\tau_\nu} = -I_\nu + B_\nu(T)$

DEFINITIONS

- Blackbody radiation : $I_\nu = B_\nu(T)$
- Thermal emission : $S_\nu = B_\nu(T)$

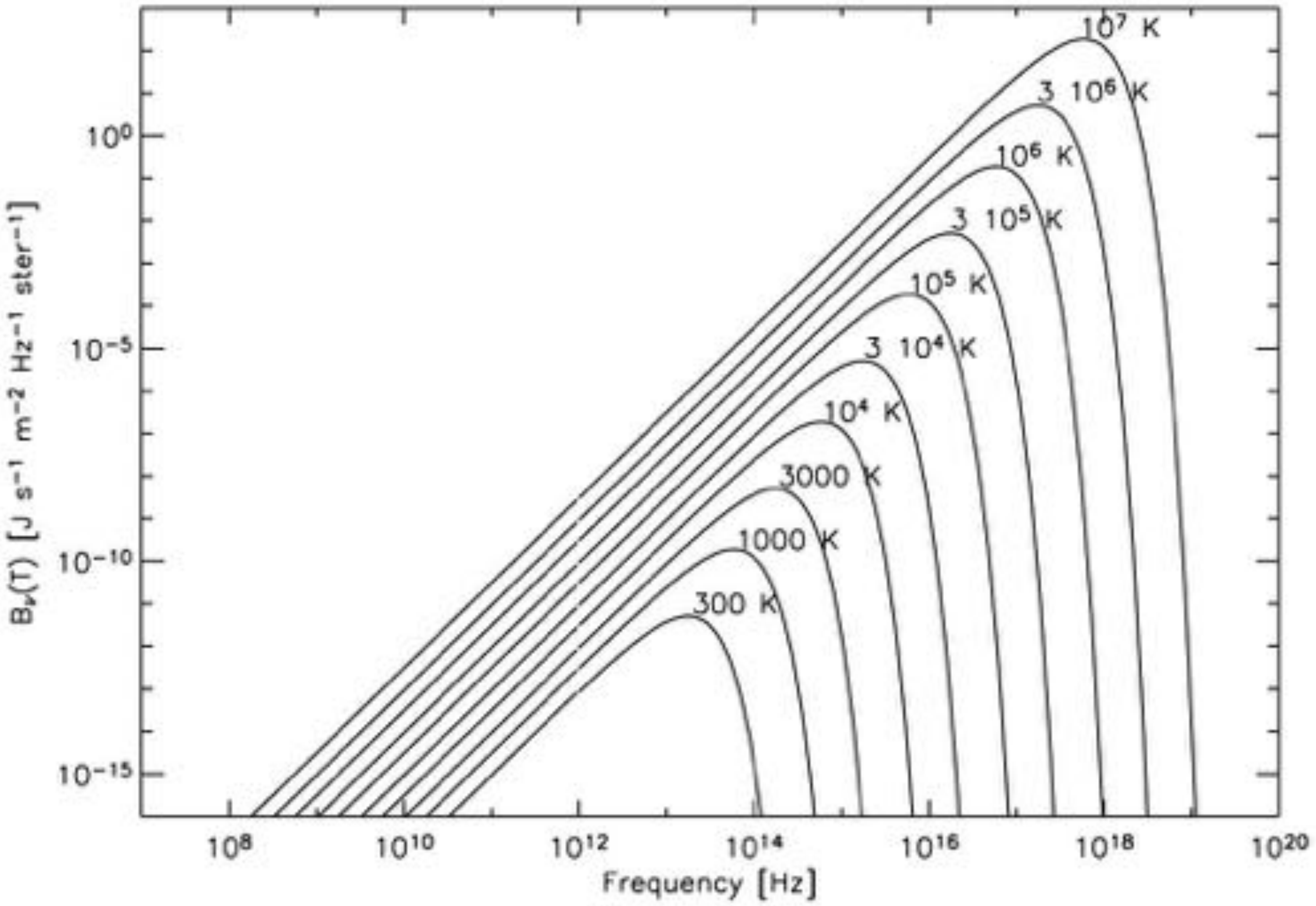
In thermal equilibrium

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + B_\nu(T) = 0$$

Hence

$$I_\nu = B_\nu \equiv S_\nu$$

# Thermal Radiation




Example of Blackbody radiation :

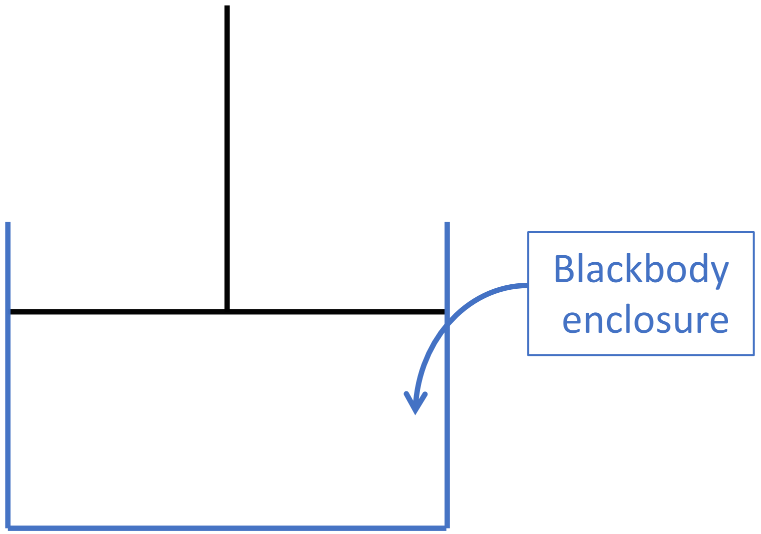
- *Cosmic Microwave Background*
- *Sun*
- *Lava*

The Wien's displacement law gives the wavelength of the peak :

$$\lambda = \frac{b}{T}$$

where $b$ is the Wien's displacement constant ($b = 2.898 \times 10^{-3}\ m\ K^{-1}$)

# Thermal Radiation




Considering a blackbody enclosure with a piston, so that work can be done or subtracted to the enclosure (the radiation).

The first law of thermodynamics gives :

$$dU = dQ \; - P\, dV$$

where $\mathrm{P}$ is the pressure, $Q$ the heat and $U$ is the total energy.

According to the second law of thermodynamics, we have :

$$dS = \frac{dQ}{T}$$

where $S$ is the entropy.

Radiation pressure

In thermodynamics :

- $U = u\, V$ (with $u$ the energy density)
- $P = \frac{u}{3}$

From radiative transfer : $u = \int u_\nu \, d\nu = \frac{4\pi}{c} \int j_\nu d\nu$

# Thermal Radiation

We can rewrite the first law of thermodynamics such as :

$U = u\,V$

$$dU = T\,dS - P\,dV$$

Then $dS = \frac{V}{T}\frac{du}{dT}dT + \frac{4u}{3T}dV$

Since $dS$ is a perfect differential, we can write :

- $\left(\frac{\partial S}{\partial T}\right)_V = \frac{V}{T}\frac{du}{dT}$
- $\left(\frac{\partial S}{\partial V}\right)_T = \frac{4u}{3T}$

And then :

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{1}{T}\frac{du}{dT} = -\frac{4u}{3T^2} + \frac{4}{3T}\frac{du}{dT}$$

$$\frac{du}{dT} = \frac{4u}{T} \Leftrightarrow \frac{du}{u} = 4\frac{dT}{T}$$

integrating gives :

$$\log u = 4 \log T + \log a$$

and therefore :

$$\boxed{u = aT^4}$$ *Stefan-Boltzmann law*

$J_\nu = \frac{1}{4\pi}\int I_\nu d\Omega$

For isotropic emission : $I_\nu = J_\nu$, and in the case of a blackbody radiation $I_\nu = B_\nu(T)$, therefore :

$$u = \int u_\nu\, d\nu = \frac{4\pi}{c}\int j_\nu d\nu = \frac{4\pi}{c}\int B_\nu(T) d\nu = \frac{4\pi}{c}B(T)$$

with $B(T) = \int B_\nu(T) d\nu = \frac{ac}{4\pi}T^4$ *[tabulated !]*

# Line emission

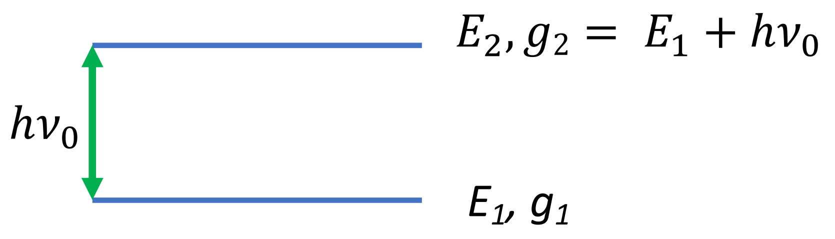
Statistical weight $g_i$: the total number of states possible with a given set of quantum numbers

Mean density

$$J_\nu = \frac{1}{4\pi}\int I_\nu d\Omega$$

*emission* *absorption*

$$j_\nu = \alpha_\nu B_\nu(T)$$ *Kirchoff's law*




The energy difference between two levels is not infinitely sharp, therefore photons with $E = h\nu_0 + \Delta E$ with $\Delta E \ll h\nu_0$ could also be absorbed/emitted.

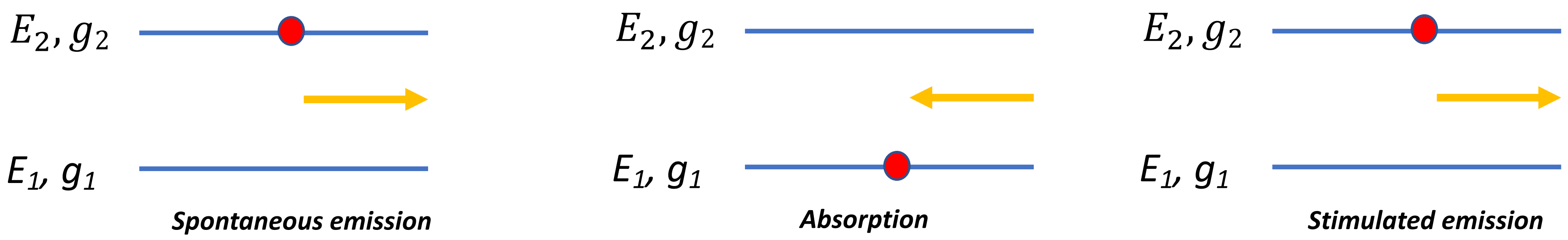
**EINSTEIN COEFFICIENTS**

$A_{21}$: transition probability per unit time for spontaneous emission

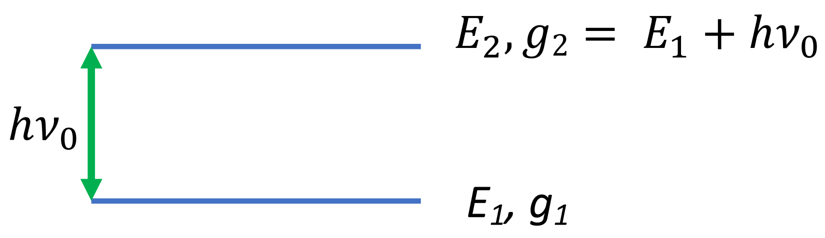
$B_{12}\bar{J}$ : transition probability per unit time for absorption

$$\bar{J} = \int_0^\infty J_\nu \Phi_\nu d\nu \text{ with } \int_0^\infty \Phi_\nu d\nu = 1$$

$B_{21}\bar{J}$ : transition probability per unit time for stimulated emission



# Line emission




In thermodynamics equilibrium :

$$absorption = emission$$

$$n_1 B_{12} \bar{J} = n_2 A_{21} + n_2 B_{21} \bar{J}$$

where $n_1$ and $n_2$ are the number density of atoms of level 1 and 2 respectively

Solving for $\bar{J}$ gives :

$$\bar{J} = \frac{A_{21}/B_{21}}{\left(\frac{n_1}{n_2}\right)\left(\frac{B_{12}}{B_{21}}\right) - 1}$$

In thermodynamics equilibrium :

$$\frac{n_1}{n_2} = \frac{g_1 \exp(-\frac{E}{k_b T})}{g_2 \exp(-\frac{E + h\nu_0}{k_b T})} = \frac{g_1}{g_2} \exp(\frac{h\nu_0}{k_b T})$$

Therefore :

$$\bar{J} = \frac{A_{21}/B_{21}}{\left(\frac{g_1 B_{12}}{g_2 B_{21}}\right) \exp\left(\frac{h\nu_0}{k_B T}\right) - 1}$$

In thermodynamics equilibrium : $J_\nu = B_\nu$ ,and $B_\nu$ is varying slowly with $d\nu$ then $\bar{J} = B_\nu$ :

$$\frac{A_{21}/B_{21}}{\left(\frac{g_1 B_{12}}{g_2 B_{21}}\right) \exp\left(\frac{h\nu}{k_B T}\right) - 1} = \frac{2h\nu^3/c^2}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

which gives, the following relations between Einstein coefficients :

$$g_1 B_{12} = g_2 B_{21} \qquad A_{21} = \frac{2h\nu^3}{c^2} B_{21}$$

# Summary of Friday's lecture

## RADIATIVE TRANSFER

The radiative transfer equation can be written as :

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

$$d\tau_\nu = \alpha_\nu ds$$

$$S_\nu = \frac{j_\nu}{\alpha_\nu}$$

Optically thin : $\tau_\nu \leq 1$
Optically thick : $\tau_\nu \geq 1$

We defined the mean free path as

$$s = \frac{1}{\alpha_\nu} = \frac{1}{n\,\sigma_\nu} = l_\nu$$

## THERMAL RADIATION

In thermal radiation, the specific intensity only depends on temperature and is given by the Planck function :

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

## LINE EMISSION

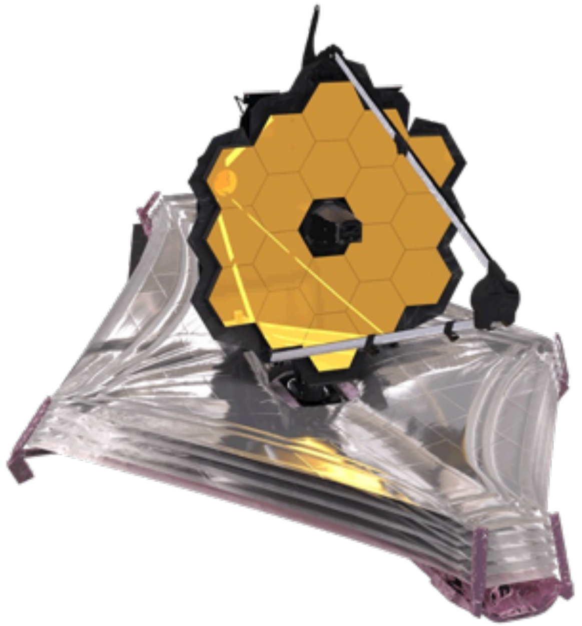
We defined 3 Einstein's coefficient :

- For spontaneous emission $A_{21}$
- For absorption $B_{12}\bar{J}$
- For stimulated emission $B_{21}\bar{J}$

# Summary of Friday's lecture

**<u>Question</u>** : Why the RTE uses $I_\nu$ and we computed $dI_\nu$ ?

$$\boldsymbol{dE = I_\nu dA\, dt\, d\nu\, d\Omega}$$



$$\boldsymbol{dI_\nu = j_\nu\, ds}$$

$$\boldsymbol{dI_\nu = -\,\alpha_\nu\, I_\nu\, ds}$$

$$\boldsymbol{\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu}$$

# Line emission

The line profile function describing an emission should be identical at the one describing absorption :

$$\int_0^{\infty} \phi_\nu^{em} d\nu = \int_0^{\infty} \phi_\nu^{abs} d\nu$$

We remind that the amount of energy emitted is given by :

$$dE = j_\nu dV\, d\Omega\, \mathrm{d}\nu\, dt$$

Each atom contributes to an energy $h\nu_0$ distributed over $4\pi$ solid angle, which can be expressed as :

$$dE = \left(\frac{h\nu_0}{4\pi}\right)\Phi(\nu) n_2 A_{21}\, dV\, d\Omega\, d\nu\, dt$$

$B_{12}\bar{J}$ with $\bar{J} = \int_0^{\infty} J_\nu \Phi_\nu d\nu$

We easily get the emission coefficient :

$$j_\nu = \frac{h\nu_0}{4\pi} n_2 A_{21} \phi(\nu)$$

The total energy absorbed in dt and dV is given by :

$$\frac{h\nu_0}{4\pi} n_1 B_{12} dV\, dt \int d\Omega \int_0^{\infty} J_\nu \Phi(\nu) d\nu$$

Then, the energy absorbed out of a beam is :

$$dE = \frac{h\nu_0}{4\pi} n_1 B_{12} dV\, dt\, d\Omega\, J_\nu \Phi(\nu) d\nu$$

Remember that : $dE = I_\nu(-\alpha_\nu\, dA\, ds\,) d\Omega\, dt\, d\nu$

We easily get the absorption coefficient :

$$\alpha_\nu = \frac{h\nu}{4\pi} n_1 B_{12} \phi(\nu)$$

# Line emission

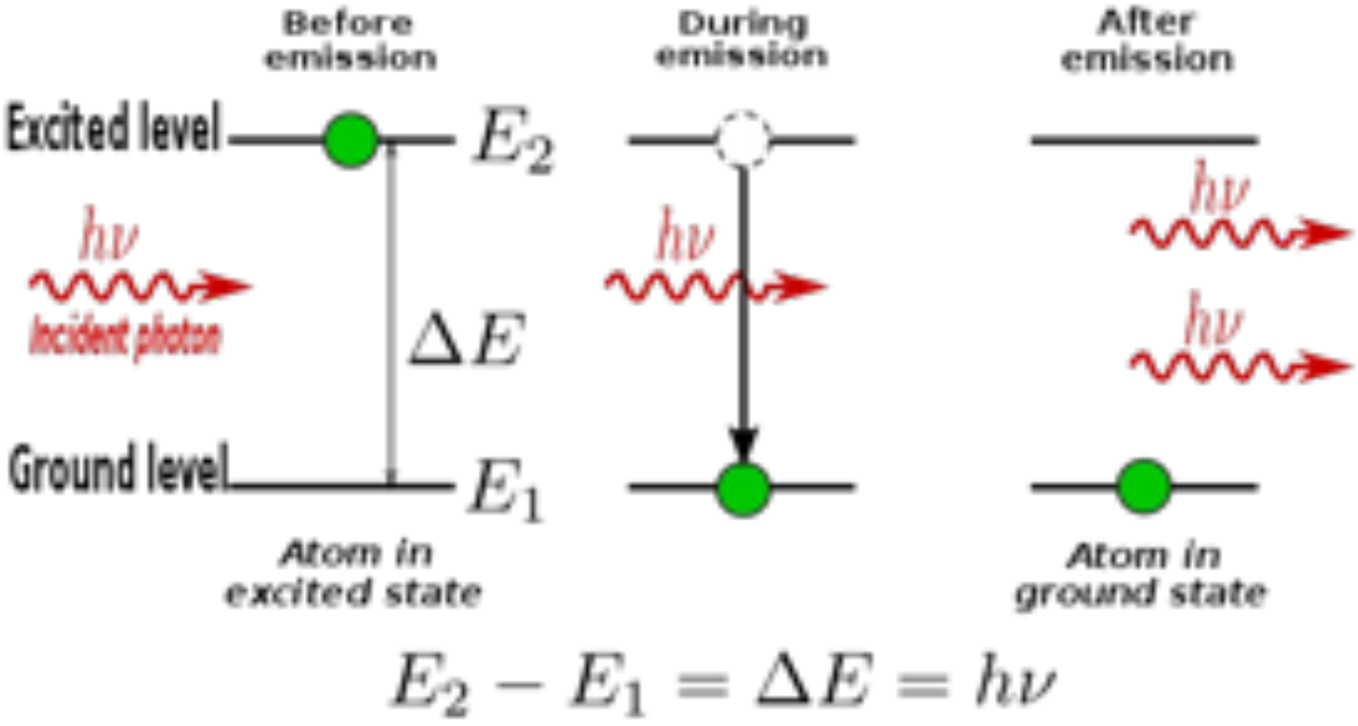
**Stimulated emission** is proportional to the intensity, and only affects photons along the given beam, similar to absorption.

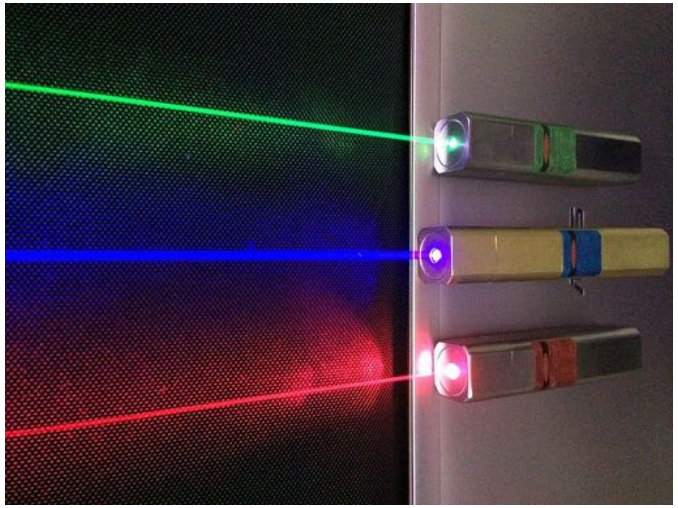
Then we can consider stimulated emission as negative absorption, such as :

$$\alpha_\nu^{stim} = -\frac{h\nu}{4\pi} n_2 B_{21} \phi(\nu)$$

Then the absorption coefficient, corrected for stimulated effect is :

$$\alpha_\nu = \frac{h\nu}{4\pi} \phi(\nu)(n_1 B_{12} - n_2 B_{21})$$



# Line emission

Collision between particles is also an important process producing emission lines.

Excitation/de-excitation of an atom can be expressed as : $n_0 C_{21}$ ; $n_0 C_{12}$ where $n_0$ is the density of colliding particles.

de-excitation

excitation

Usually collisions are dominated by electron-ions collisions, then $n_0 \sim n_e$

If the gas is in thermodynamic equilibrium :

$$n_e C_{21} g_2 \exp\left(\frac{E_2}{k_B T}\right) = n_e C_{12} g_1 \exp\left(\frac{E_1}{k_B T}\right)$$

then :

$$C_{12} = \frac{g_2}{g_1} C_{21} \exp(-[E_1 - E_2]/k_b T)$$

Excitation by collisions is one of the main process in astrophysical gas cooling (energy is dissipated by radiation).

In a thermodynamic equilibrium :

$$n_1(n_0 C_{12} + B_{12}\bar{J}) = n_2(n_0 C_{21} + B_{21}\bar{J} + A_{21})$$

If we assume that induced processes (absorptions) are much less important than spontaneous and collisions, then :

$$n_1 n_0 C_{12} = n_2(A_{21} + n_0 C_{21})$$

or

$$\frac{n_2}{n_1} = \frac{n_0 C_{12}}{A_{21}} \left[\frac{1}{1 + \frac{n_0 C_{21}}{A_{21}}}\right]$$

# Line emission

The line emissivity corresponds to the amount of energy emitted by the total number of atoms :

$$\epsilon = n_2 A_{21} h\nu_{21} = n_0 n_1 C_{12} h\nu_{21} \left[\frac{1}{1+\frac{n_0 C_{21}}{A_{21}}}\right]$$

$$\frac{n_2}{n_1} = \frac{n_0 C_{12}}{A_{21}} \left[\frac{1}{1+\frac{n_0 C_{21}}{A_{21}}}\right]$$

- **At low density** : $n_0 C_{21} \ll A_{21}$

$$\epsilon \approx n_0 n_1 C_{12} h\nu_{21}$$

*Every 1-> 2 transitions rise to a downwards 2-> 1 radiative transition*

- **At high density** : $n_0 C_{21} \gg A_{21}$

$$\epsilon \approx n_1 \frac{g_2}{g_1} \exp(-\frac{h\nu_{21}}{k_B T})\, A_{21} h\nu_{21}$$

*The emissivity depends on the conditions of the exited level; many downwards transitions are caused by collisions.*

$$C_{12} = \frac{g_2}{g_1} C_{21} \exp(-[E_1 - E_2]/k_b T)$$

The critical density above which the line is predominantly collisionaly de-excited is :

$$n_0^c \sim A_{21}/C_{21}$$

Generally :

$$n_0 \sim n_e \propto n_{atoms} \propto n$$

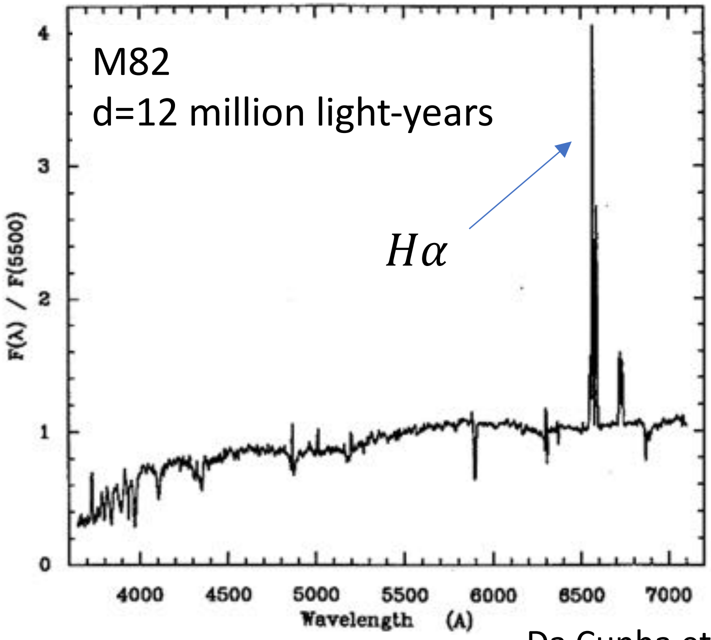
Therefore :

- At $n \ll n_0^c$ : $\epsilon_\nu \propto n^2$
- At $n \gg n_0^c$ : $\epsilon_\nu \propto n$

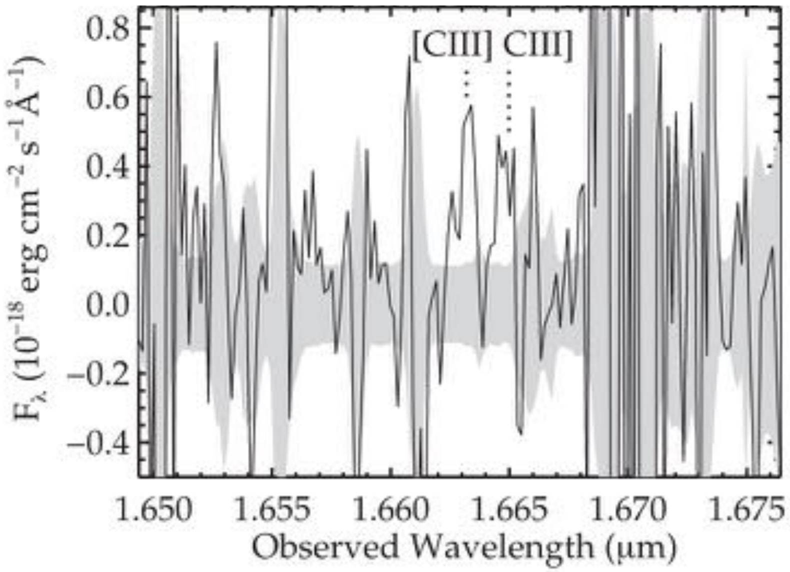
# Line emission

<u>Definition:</u> In Astrophysics some process timescales are so long that they can not be reproduced in laboratory. Some emission lines, with an extremely small probability may be detected in Space and not in laboratory : **forbidden lines**

| Type of atoms | Timescale | Nature of line | Example |
|---|---|---|---|
| Dipole | short | permitted | $H\alpha\ \lambda6563$ |
| Quadrupole | long | forbidden | $[OIII]\lambda5007$ |
| Intercombination | intermediate | Semi-forbidden | $CIII]\lambda1909$ |




Da Cunha et al. 2008

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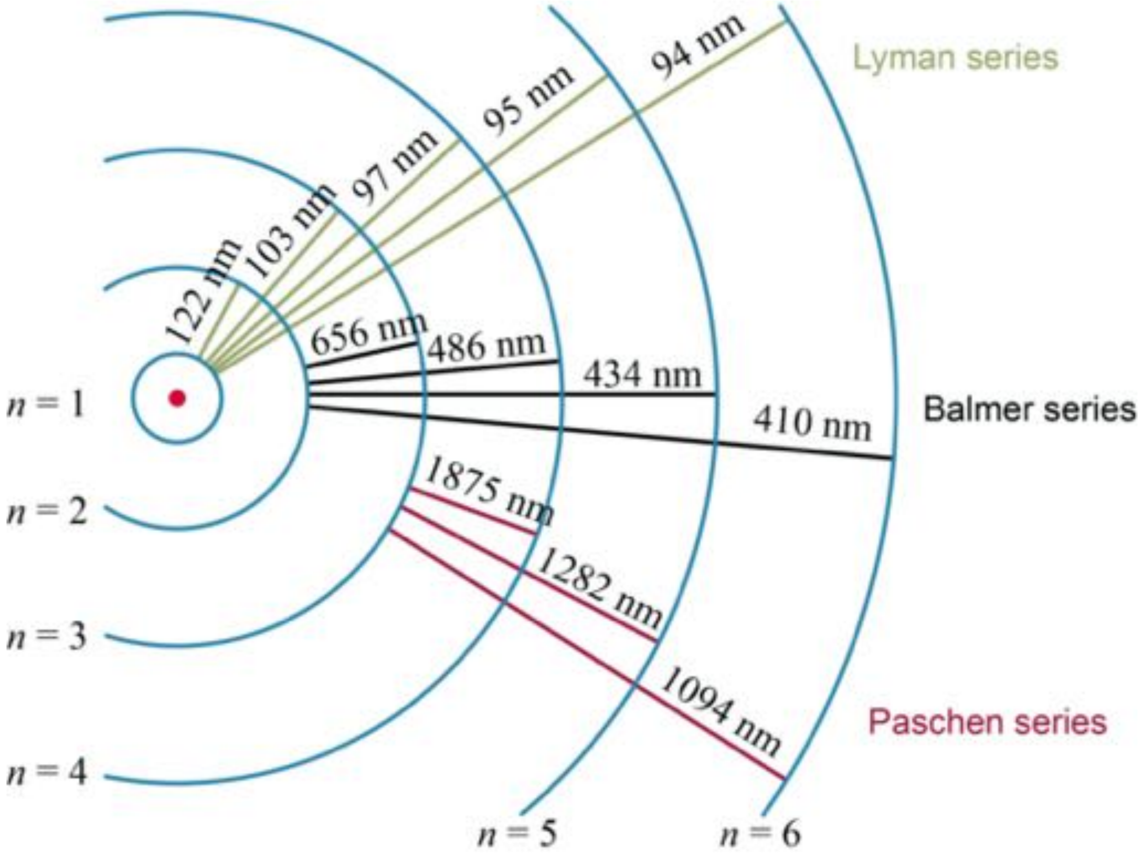
z=7.733

Stark et al. 2017

# Line emission

**THE HYDROGEN ATOM**




The electronic energy states are determined by :

$$\frac{1}{\lambda_{vac}^{m \to n}} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

with $R$ the Rydberg constant ($R = 1.09 \times 10^7 m^{-1}$)

Young stars are UV emitters and emit radiation at wavelength shorter than the Lyman edge ($\lambda = 91.1753\ nm$) ➔ the hydrogen around young stars is ionized.

In the ionised gas, electrons recombine in $H^+$from upper levels, and then decay to the fundamental by multiple transitions (and therefore multiple line emissions)

The recombination rate is given by :

$$\beta_{i \to j} = \alpha_{i \to j}(T) n_p n_e$$

where $\alpha_{i \to j}(T)$ is the effective recombination coefficient

The emissivity of a recombination line is given by :

$$\epsilon_{i \to j} = h\nu_{i \to j} \alpha_{i \to j} n_p n_e$$

# Line emission

## CASE OF BROAD LINE REGIONS

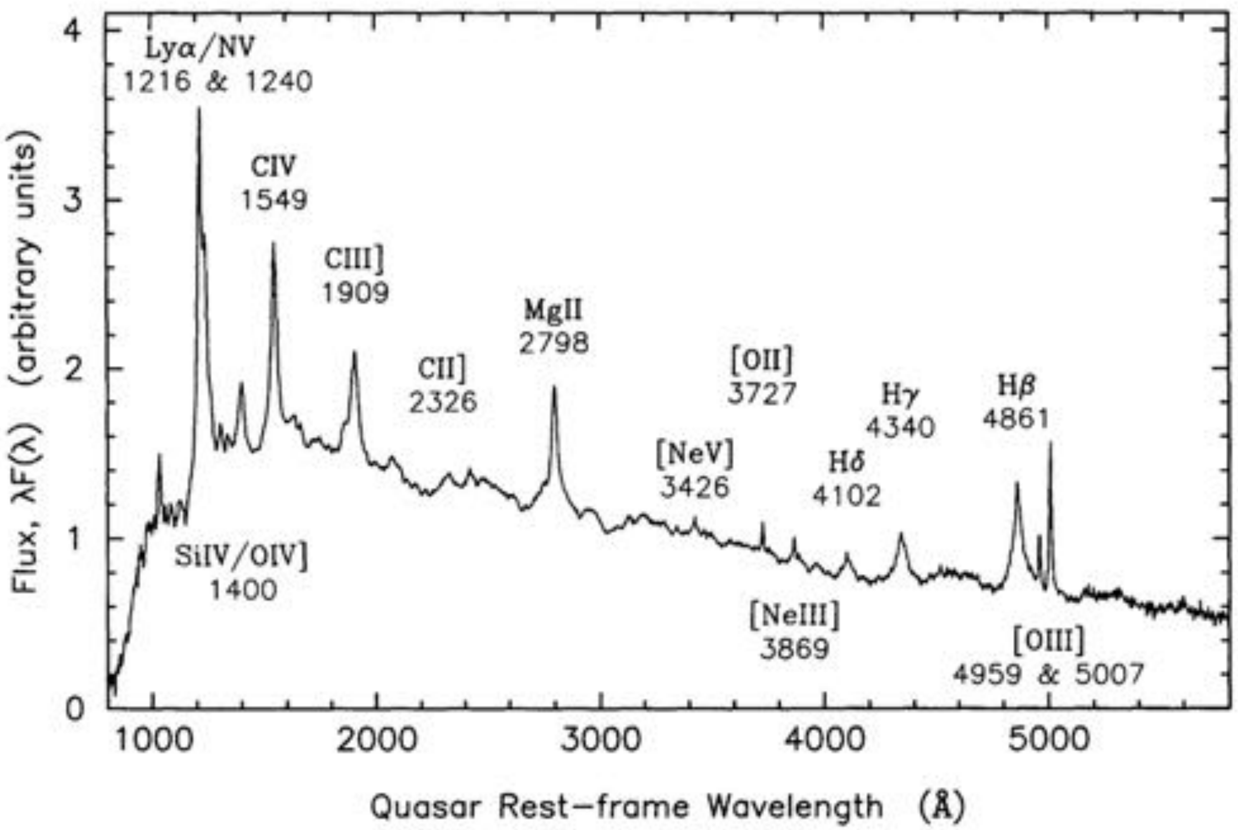
A broad line region is a very compact region (<1kpc) surrounding an accreting supermassive black holes.

In this region, clouds are photo-ionised by strong UV radiations emitted by the accreting black holes.

The mean density of these clouds is $\sim 10^{11} cm^{-1}$

Remember that at $\mathrm{n} \ll n_0^c : \epsilon_\nu \propto n^2$

Consequently, the permitted and recombination lines are much brighter than forbidden lines but also much brighter than any other lines coming from the host galaxy.



# Heating and Cooling

As we will see later in this course, processes of cooling and heating astrophysics gas are of central importance in the topic of structures formation.

The net heating rate ($Q$) is given by :

$$Q(n,T) = \underbrace{\Gamma(n,T)}_{\text{Total heating rate}} - \underbrace{\Lambda(n,T)}_{\text{Total cooling rate}}$$

The <u>equilibrium temperature</u> is the temperature at which cooling rate equal heating rate.

To test the stability of this equilibrium, we can slightly change the temperature from equilibrium temperature :

$$\Delta T = T - T_E$$

Then the enthalpy of the gas is :

$$\frac{d\Delta H}{dt} = Q(T) \approx Q_{|T_E} + \Delta T \left(\frac{\partial Q}{\partial T}\right)_{P|T_E}$$

But $Q_{|T_E} = 0$ (by definition), then :

$$\frac{d\Delta H}{dt} \approx \Delta T \left(\frac{\partial Q}{\partial T}\right)_{P|T_E}$$

The <u>cooling time</u> is defined as :

$$\tau_c = \frac{U}{\Lambda}$$

where $U$ is the thermal energy of the gas

# Heating and Cooling

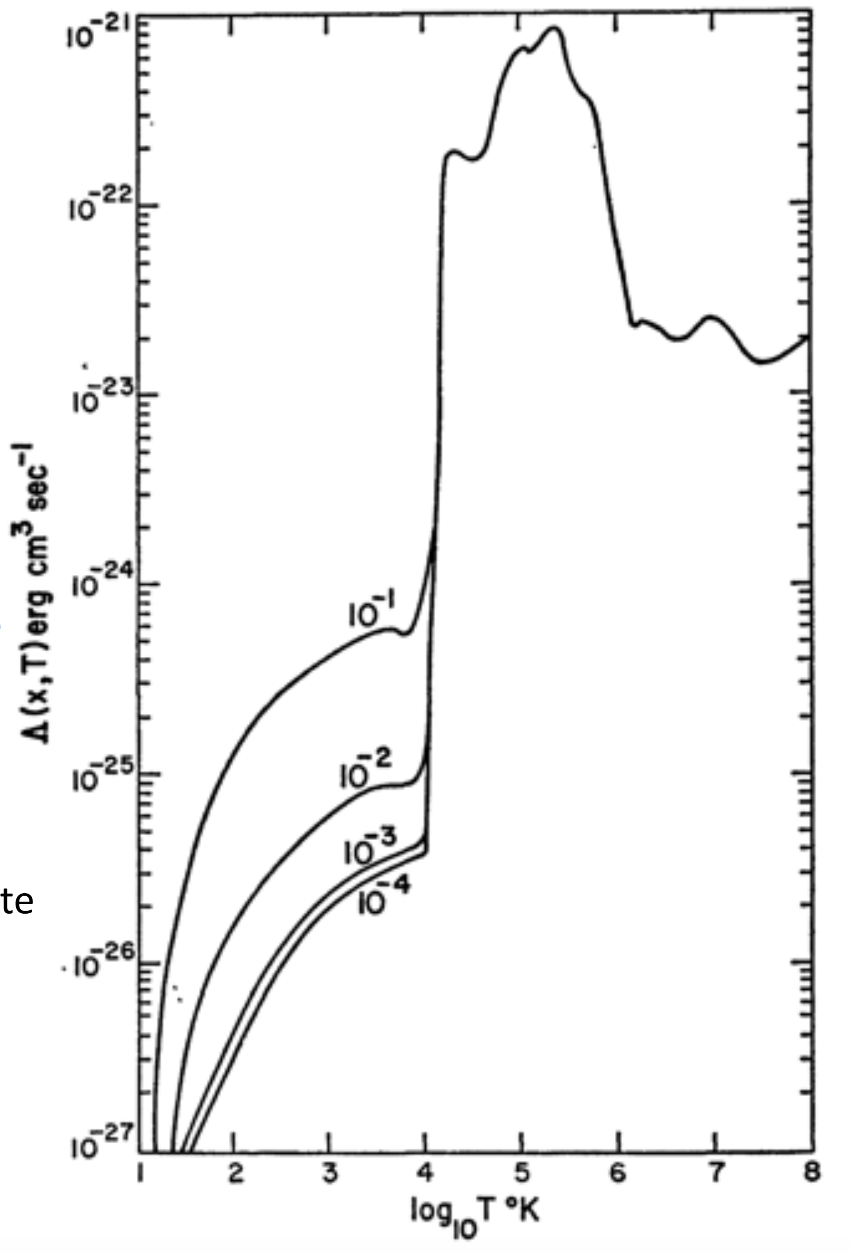
The cooling function $\Lambda(n, T)$ provides a description of the way the gas will cool considering all cooling processes, and is then defined as :

$$\Lambda(T) = \sum_i \Lambda_i$$

The main cooling processes are :

- Cooling by lines emission
- Cooling by free-free emission in ionized gas
- Cooling by dust
- Cooling by recombination

Net cooling rate



# Heating and Cooling

## COOLING BY LINE EMISSION

Remember that at high-density :

$$\epsilon \approx n_1 \frac{g_2}{g_1} \exp(-\frac{h\nu_{21}}{k_B T})\ A_{21} h\nu_{21}$$

and at low-density :

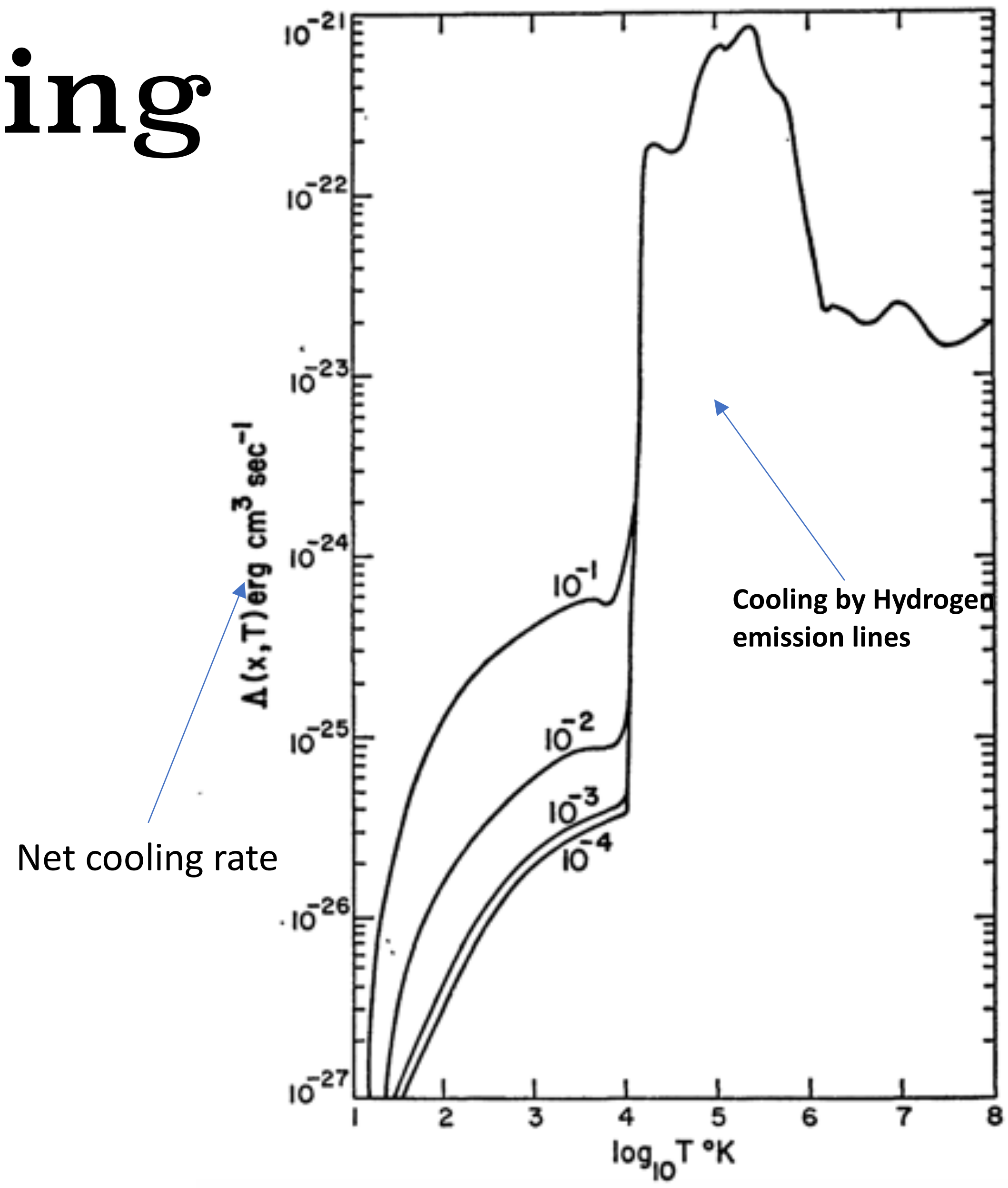
$$\epsilon \approx n_0 n_1 C_{12} h\nu_{21}$$

We also demonstrated that (*relation between upwards and downwards transition*):

$$C_{12} = \frac{g_2}{g_1} C_{21} \exp(-[E_1 - E_2]/k_B T)$$

To be an effective cooling process (i.e, to get a high value of $C_{12}$) : $\Delta E \sim k_B T$

For hydrogen, typical $\Delta E < 10\ eV$, which means a temperature of $\mathrm{T} \sim 10^4 K$



# Heating and Cooling

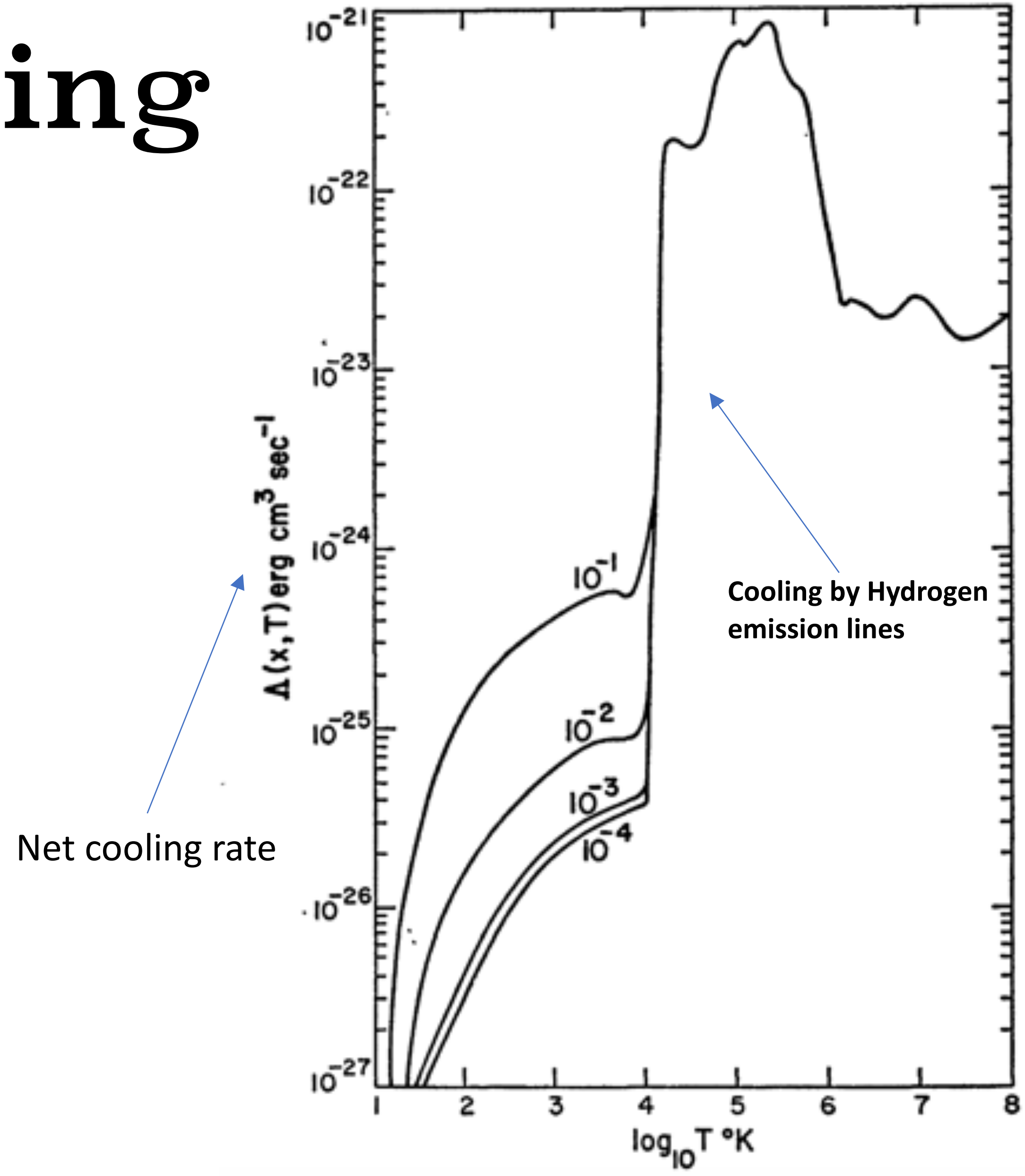
## COOLING BY LINE EMISSION

At lower temperature, several other atoms can be used to cool the gas :

- $C^+$ ($^2P_{1/2} \rightarrow {}^2P_{3/2}$) : $\frac{\Delta E}{k}$ =92 K
- $Si^+$ ($^2P_{1/2} \rightarrow {}^2P_{3/2}$) : $\frac{\Delta E}{k}$ = 413 K
- O ($^3P_2 \rightarrow {}^2P_1$) : $\frac{\Delta E}{k}$ = 228 K
- O ($^3P_2 \rightarrow {}^3P_0$) : $\frac{\Delta E}{k}$ = 326 K

Focussing on $C^+$, collisional excitation can occur via collisions with electrons or hydrogen atoms. For collisions with electron, the cooling rate is given by :

$$\Lambda_{C+} = n_e n_{c+} 8\times 10^{-33} T^{-1/2} \exp(-\frac{92}{T})\, Jm^{-3} s^{-1}$$



# Heating and Cooling

**COOLING BY FREE-FREE EMISSION**

In hot ($T \gg 10^5$K) fully ionised gas, radiation is produced via *Bremsstrahlung*

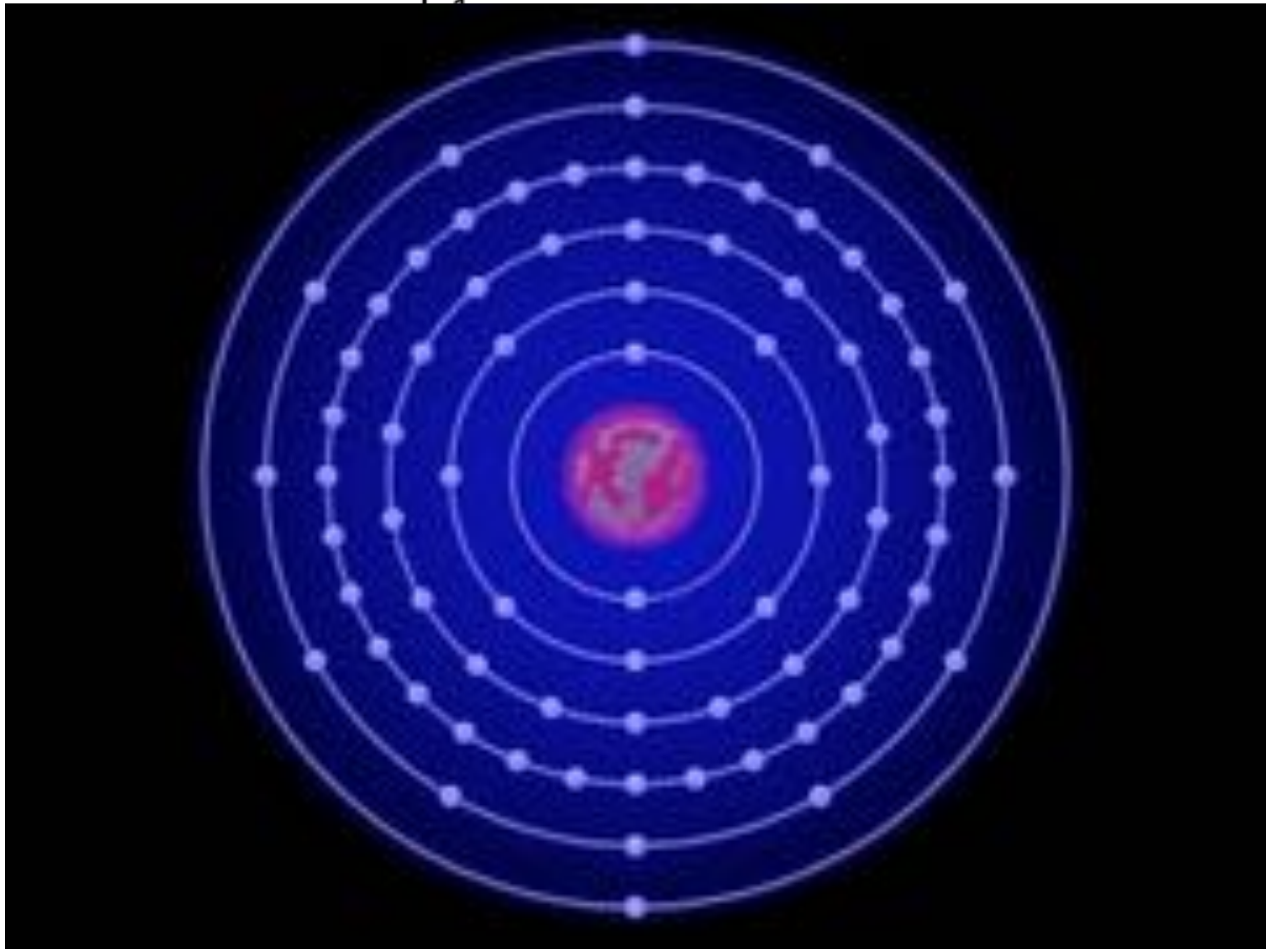
Bremsstrahlung radiation is produced when a charged particle is decelerated when deflected by another charged particle. The moving particle loses kinetic energy, which is converted into radiation.

The Bremsstrahlung emissivity is given by :

$$\epsilon_\nu^{ff} = \frac{\mu_0 Z^2 e^6}{3\pi^2 c \epsilon_0^2 m^2} \left(\frac{\pi m}{6 k_B}\right)^{\frac{1}{2}} g_{ff} n_e n_i T^{-\frac{1}{2}} e^{-h\nu/k_B T}$$

$$\epsilon_\nu^{ff} = a_1 g_{ff} n_e n_i Z^2 T^{-1/2} e^{-h\nu/k_B T}$$

where $g_{ff}$ is the *Gaunt factor* and is tabulated.



# Heating and Cooling

## COOLING BY FREE-FREE EMISSION

We can determine the absorption coefficient linked to the emission coefficient for free-free emission. We remember that :

$$j_\nu = \frac{\epsilon_\nu}{4\pi}$$
$$j_\nu = \alpha_\nu B_\nu(T)$$

And $B_\nu(T) = \frac{2h\nu^3/c^2}{\exp\left(\frac{h\nu}{k_B T}\right)-1}$

Then $\alpha_\nu^{ff} = \frac{\epsilon_\nu^{ff}}{4\pi B_\nu(T)}$

$$\epsilon_\nu^{ff} = \frac{\mu_0 Z^2 e^6}{3\pi^2 c \epsilon_0^2 m^2}\left(\frac{\pi m}{6k_B}\right)^{\frac{1}{2}} g_{ff} n_e n_i T^{-\frac{1}{2}} e^{-h\nu/k_B T}$$

or

$$\alpha_\nu^{ff} = \frac{\mu_0 Z^2 e^6 c}{24\pi^3 \epsilon_0^2 m^2 h}\left(\frac{\pi m}{3k_B}\right)^{\frac{1}{2}} g_{ff} n_e n_i \nu^3 T^{-\frac{1}{2}}\left(1 - e^{-\frac{h\nu}{k_B T}}\right)$$

In the Rayleigh-Jeans limit (low energy) : $h\nu \ll k_b T$, therefore :

$$1 - e^{-\frac{h\nu}{k_B T}} \approx \frac{h\nu}{k_B T}$$

Then the absorption coefficient can be simplified as :

$$\alpha_\nu^{ff} = a_2 g_{ff} n_e n_i Z^2 \nu^{-2} T^{-3/2}$$

# Heating and Cooling

**COOLING BY FREE-FREE EMISSION**

**Example of Hydrogen atom**

Considering a cloud of fully ionised hydrogen at a temperature T, and assuming that we are in a Rayleigh-Jeans limit (i.e. $h\nu \ll k_B T$).

The coefficient absorption is given by :

$$\alpha_\nu^{ff} = a_2 g_{ff} n_e n_i \nu^{-2} T^{-3/2}$$

Remember that the optical depth is given by :

$$\tau_\nu = \alpha_\nu s$$

Then if L is the distance through the region, then :

$$\tau_\nu^{ff} = a_2 g_{ff} n_e n_i \nu^{-2} T^{-3/2} L$$

And in the case of an hydrogen cloud, then $n_i = n_e$ :

$$\tau_\nu^{ff} = a_2 g_{ff} n_e^2 \nu^{-2} T^{-3/2} L$$

Numerical calculations show that :

$$g_{ff} \propto T^{0.15} \nu^{-0.1}$$

We have demonstrated earlier than :

$$I_\nu = B_\nu(T)\left(1 - e^{-\tau_\nu}\right)$$

Which could be simplify in the two limits :

- Optically thin : $\tau_\nu \ll 1$ $\quad I_\nu = \tau_\nu B_\nu$
- Optically thick : $\tau_\nu \gg 1$ $\quad I_\nu = B_\nu$

Therefore :

- Optically thin : $\tau_\nu \ll 1$ $\quad I_\nu \propto \nu^{-0.1}$
- Optically thick : $\tau_\nu \gg 1$ $\quad I_\nu \propto \nu^2$

# Heating and Cooling

Optically thin : $\tau_\nu \ll 1$ $\quad I_\nu \propto \nu^{-0.1}$
Optically thick : $\tau_\nu \gg 1$ $\quad I_\nu \propto \nu^{2}$

**COOLING BY FREE-FREE EMISSION**

**Example of Hydrogen atom**

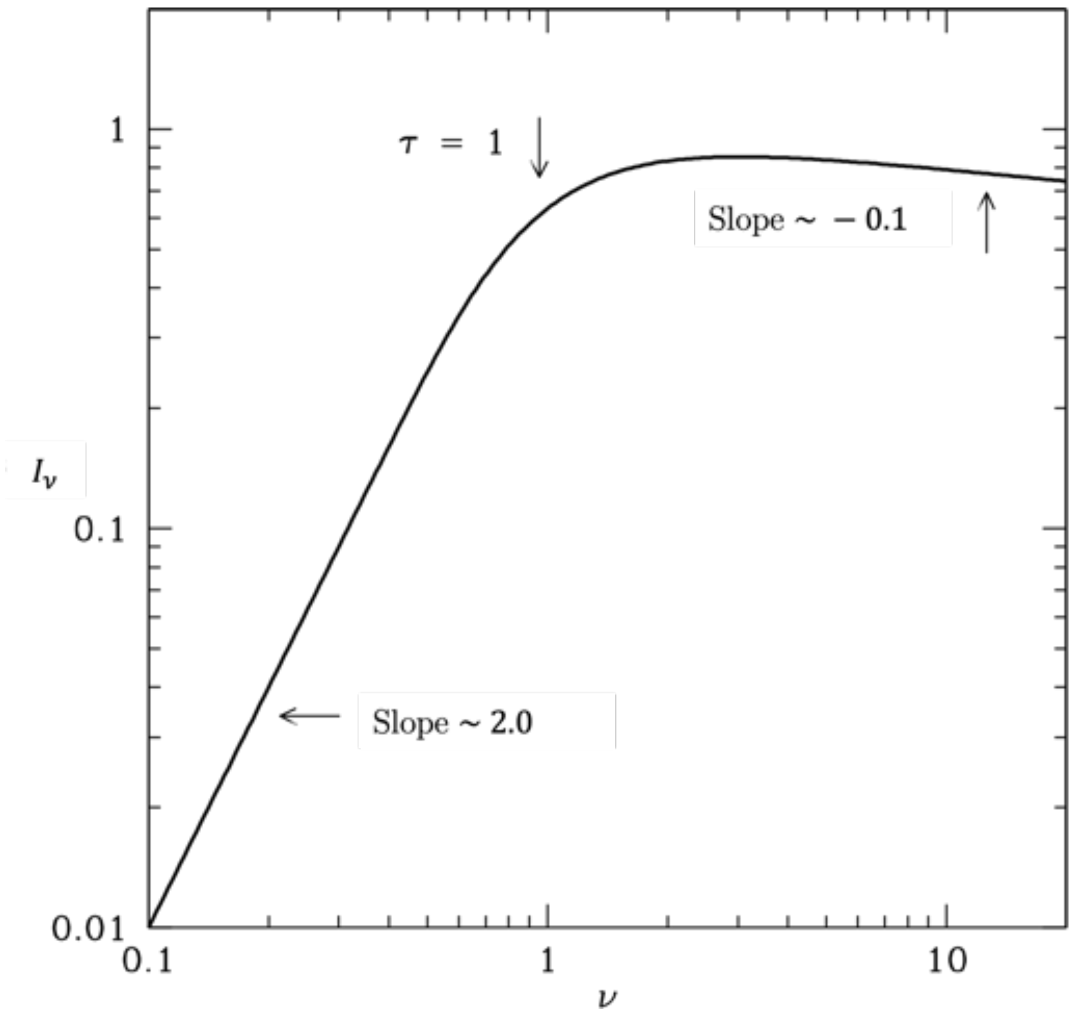
The cooling rate is found by integrating the quantity :

$$\Lambda_{ff} = \int_{\nu_0}^{\nu_{cut}} \epsilon_{ff}^{\nu} d\nu$$

Therefore :

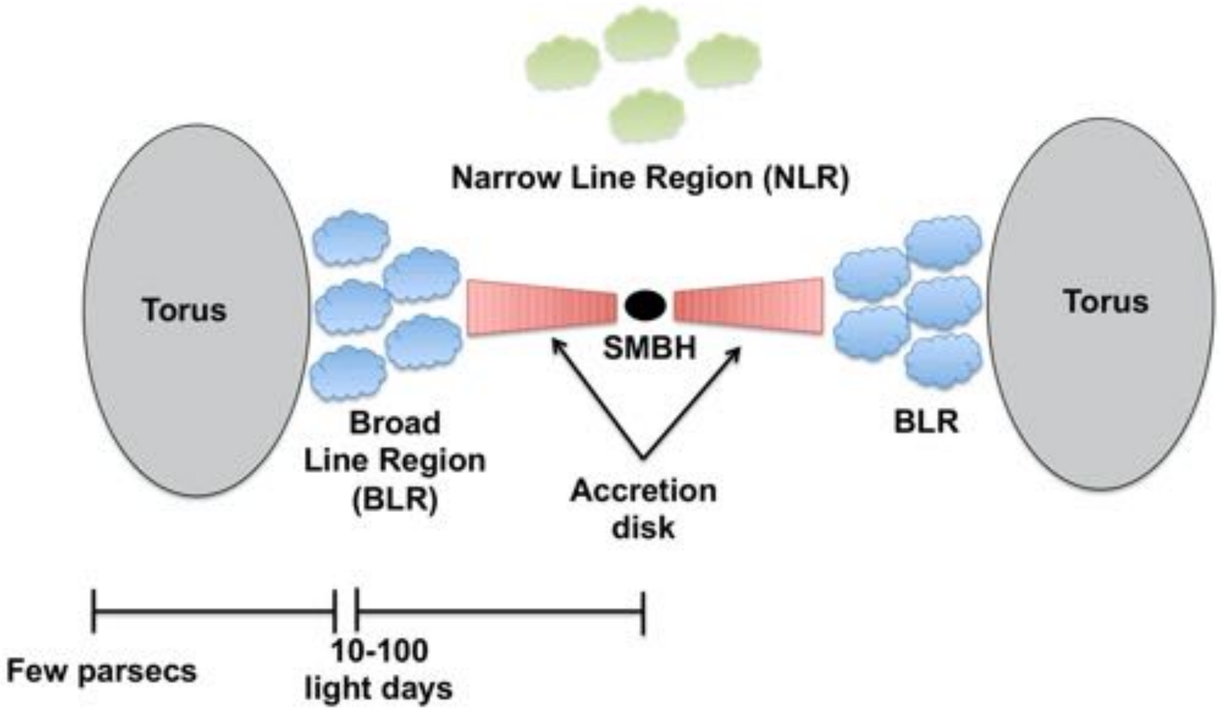
$$\Lambda_{ff} \propto n_e n_i Z^2 T_e^{-1/2}$$

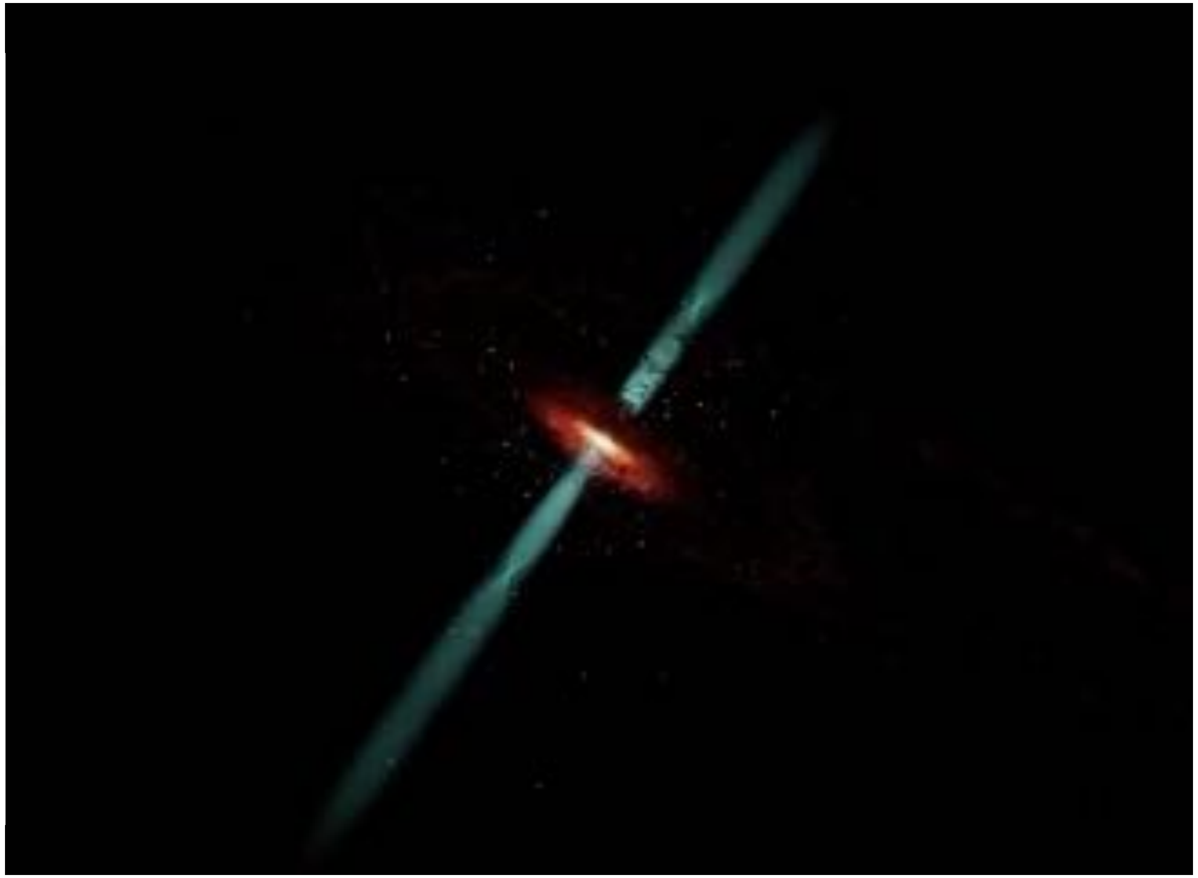
Given that $\frac{d\Lambda}{dT} > 0$, if the heating is constant, this results into a stable cooling process.



# Summary of Monday’s lecture

**EMISSION LINES IN THE CASE OF A BROAD LINE REGIONS**



# Summary of Monday's lecture

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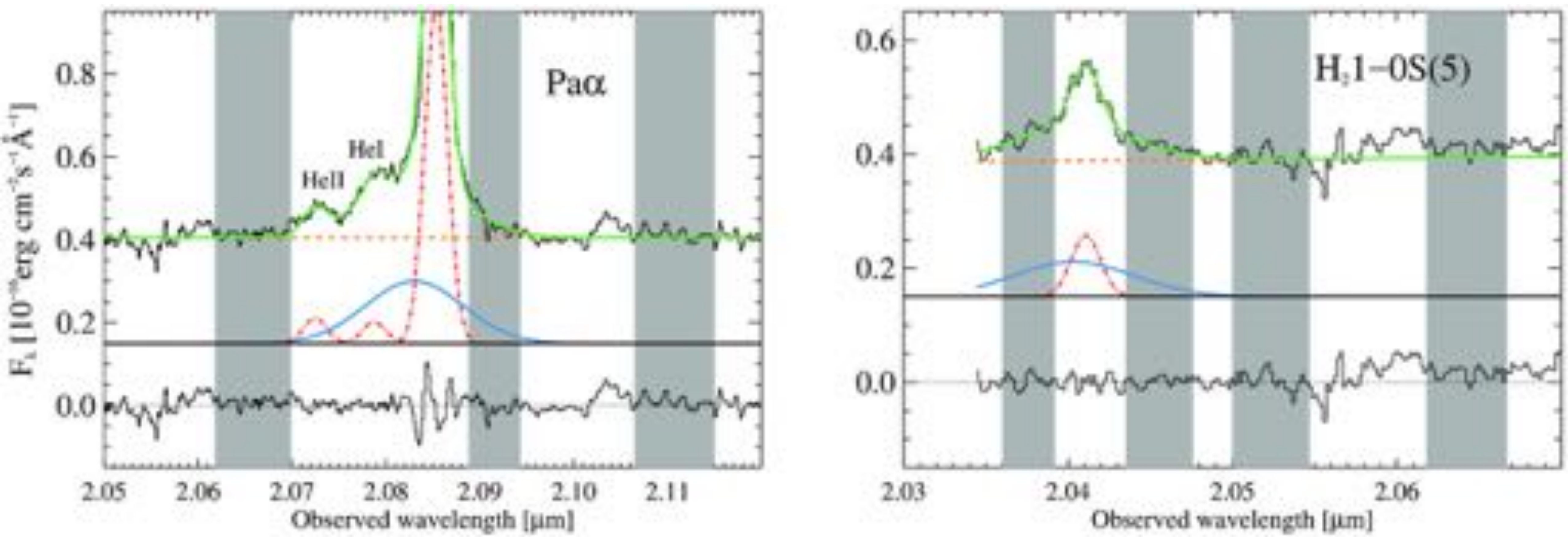



Ramos-Almeda et al. (2019)

# Summary of Monday's lecture

**COOLING AND HEATING**

We defined the cooling function $\Lambda(n, T)$ as :

$$\Lambda(T) = \sum_i \Lambda_i$$

where $\Lambda_i$ is the cooling function for individual process

*By line emission*

Hydrogen is efficient only at $T > 10^4 K$, at lower temperature C and O are more efficient.

*By free-free emission*

For hot and fully ionised H-gas ($> 10^5\ K$), if the heating is constant, the cooling is a stable process.

# Heating and Cooling

**COOLING OF MOLECULAR GAS**

In the cool molecular phase of the ISM, the excitation conditions for rotational transitions of molecules are matched to the typical temperature of molecular clouds.

From your Quantum mechanics course, you know that :

$$E_j^{rot} = J(J+1)\frac{\hbar^2}{2I} = J(J+1)B$$

The Einstein A coefficient for a rotational transition can be written as :

$$A_{nm} = \frac{8\pi^2}{3\,\hbar c^2} Z_0 \nu^3 |< n|\hat{d}|m >^2$$

The molecule must have a permanent dipole (to justify the rotational transition) $\hat{d} = \mu$, then for J+1→J transition, we can quote the result of QM :

$$A_{J+1,j} = \frac{8\pi^2}{3\hbar c^2} Z_0 \nu^3 \mu^2 \frac{J+1}{2J+1}$$

The energy spacing between the level is :

$$h\nu_{J+1,J} = 2B(J+1)$$

$H_2$ is the most abundant molecule in the Universe, but it has no permanent dipole, hence it cannot have transition $\Delta J = \pm 1$, it can only undergo quadrupole transition $\Delta J = \pm 2$, which corresponds to excitation temperature T>500K ($\lambda \rightarrow$ mid-IR)

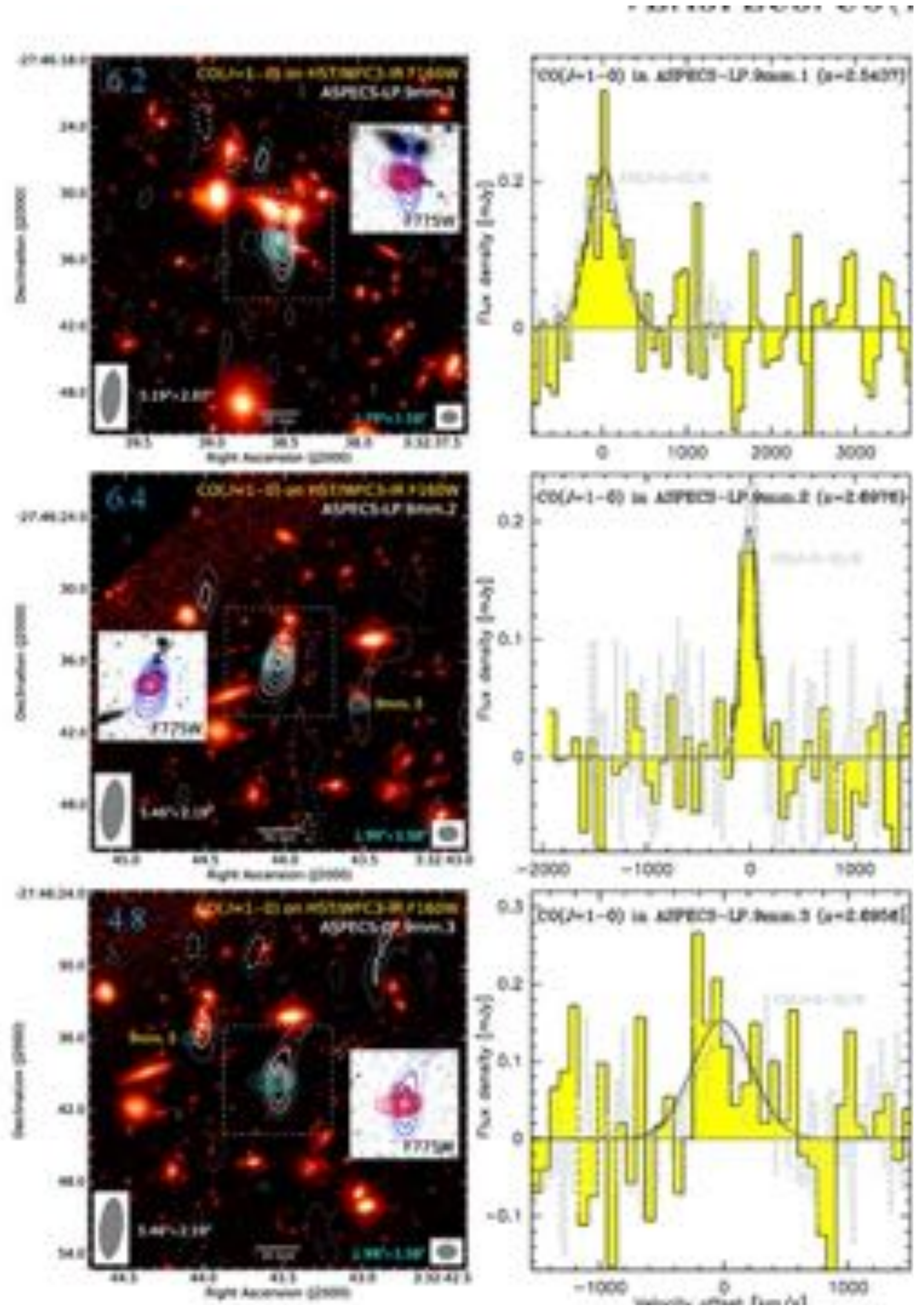
# Heating and Cooling

**COOLING OF MOLECULAR GAS**

However, molecules with dipole transitions exist in the Universe :

| Molecules | Transition | Frequency |
|---|---|---|
| $^{12}CO$ | $J = 1 \rightarrow 0$ | 115.27 GHz |
| $^{12}CO$ | $J = 2 \rightarrow 1$ | 230.54 GHz |
| CS | $J = 1 \rightarrow 0$ | 48.99 GHz |
| NCN | $J = 1 \rightarrow 0$ | 86.63 GHz |

All these transitions are excellent coolant of the cold molecular phase of the ISM



Riechers et al. 2020

# Heating and Cooling

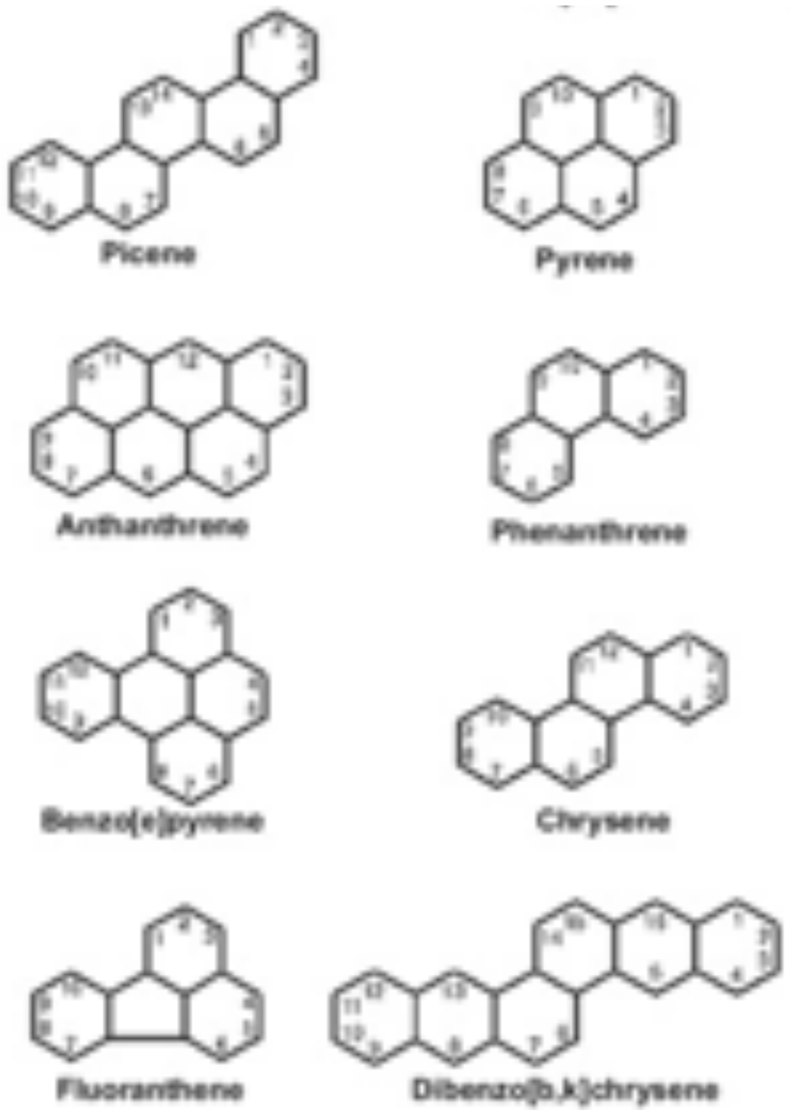
**COOLING BY DUST**

Dust is an important component of the ISM. It accounts for

- 50% of the heavy elements
- 1% of the baryonic mass of a galaxy
- 40% of the luminosity of a galaxy

Dust grains are produced by stars at the end of their life (e.g., after SNe). Their size is ranging from 1nm to $1\mu m$ with a mean size of about 0.1 $\mu m$. This explains why dust grain absorb mainly at optical wavelength.

The smallest dust grains are just large molecules such as PAHs, the larger are amorphous grain of silicates and carbon, but with an icy surface layer



# Heating and Cooling

**COOLING BY DUST**

The main consequences of dust in the ISM are :

- Absorption at optical wavelength (*extinction*)
- Radiates thermally in the mid- and far-infrared
- Acts as catalysts for chemical reaction in the ISM and the formation of large molecule
- Can also scatter photons elastically

Consider a dust grain of radius $a$ and assume that dust scattering does not contribute to dust heating, then only absorption is considered. The absorption coefficient of a spherical grain is then given by :

$$\alpha_{ext}(\nu) = n_g \sigma_{ext}(\nu, a) = n_g Q_{ext}(\nu, a)\sigma_a = n_g Q_{ext}(\nu, a)\pi a^2$$

where $Q_{ext}(\nu, a)$ is the efficiency of extinction

The extinction coefficient can be expressed as :

$$Q_{ext} = Q_{abs} + Q_{sca}$$

The power absorbed by a single grain from an incident radiation field $F_\nu$ is given by :

$$P_{abs} = \int_0^{+\infty} F_\nu \sigma_a Q_{abs}(\nu, a) d\nu$$

At equilibrium, the emissivity is given by :

$$\epsilon = 4\pi j_\nu = 4\pi \alpha_\nu B_\nu(T_g)$$

Hence, for a single grain the power radiated is :

$$P_{rad} = 4\pi \int_0^{+\infty} \sigma_a Q_{abs}(\nu, a) B_\nu(T_g) d\nu$$

And in equilibrium : $P_{abs} = P_{rad}$

# Heating and Cooling

**COOLING BY DUST**

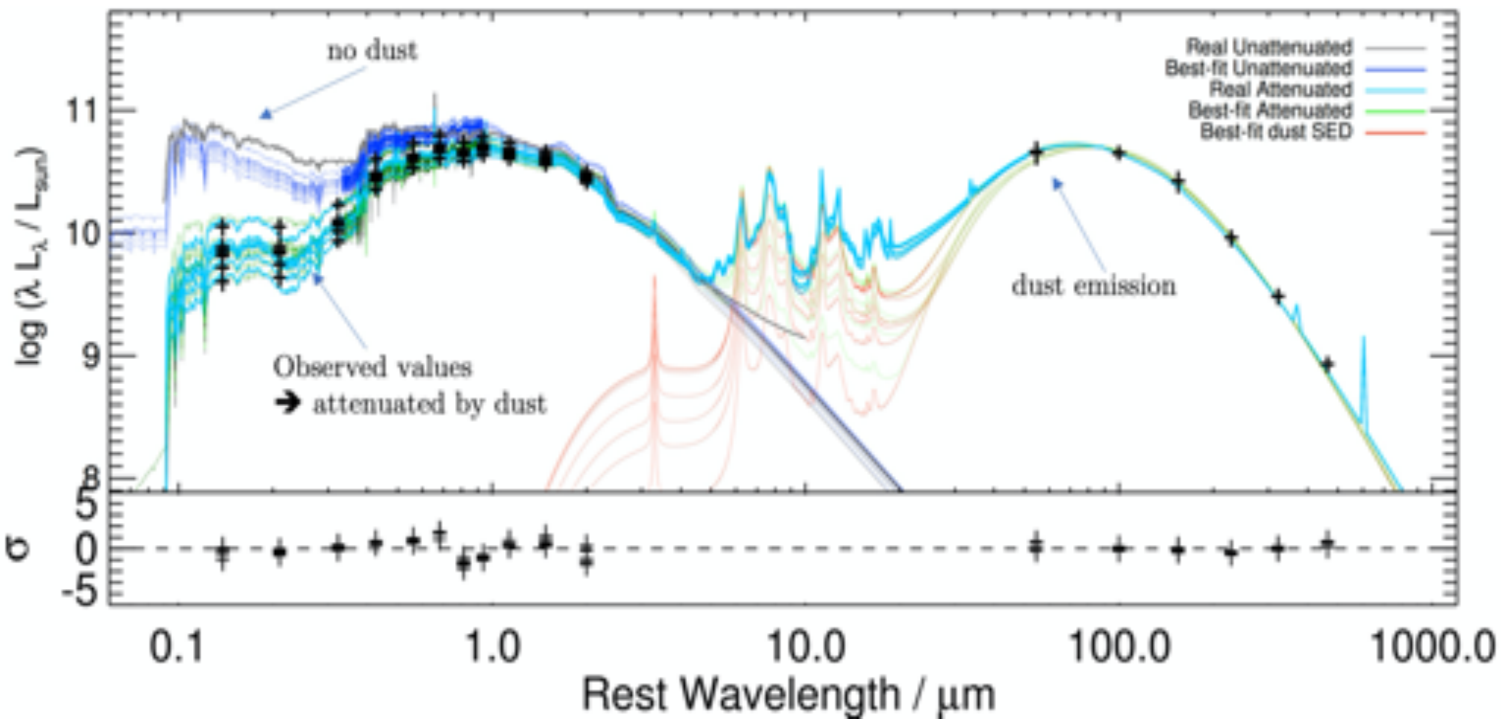
For typical dust : $T_g^{eq} \propto F_{UV}^{1/5}$

Since $F = \frac{L}{4\pi R^2}$, then :

$$T_g^{eq}[K] \approx 40 L_{39}^{1/5} R_{pc}^{-2/5}$$

Typically, dust heated by UV emission reaches temperature of 100K in star-forming regions.

If there is a significant amount of dust, then the ISM is optically thick and leads in efficient cooling.



# Heating and Cooling

## RADIATIVE HEATING AND COOLING BY RECOMBINATION

Photons with energies greater than the ionisation potential of a species lead to the ejection of an electron with energy $h\nu - I_i$

This electron can then heat the gas via collision processes.

Some of the electron kinetic energy will lead to excitation of electronic levels, and subsequently re-radiation and hence no heating of the gas.

Considering a cloud of pure hydrogen.

The ionisation rate is given by :

$$n_H S_* \sigma_i$$

Ionisation cross-section

Ionising photons flux

In equilibrium, ionisation rate must equal the recombination rate :

$$n_H S_* \sigma_i = n_e^2 \alpha_B$$

where $\alpha_B$ is the net recombination coefficient and $n_e = n_i$

The heating rate is given approximately by :

$$\mathrm{T} = n_H S_* \sigma_i (h\bar{\nu} - I_H)$$

where $I_H$ is the ionisation energy of hydrogen and $h\bar{\nu} - I_H$ is the mean energy of ejected electrons.

The mean kinetic energy per electron is $\frac{3}{2} k_B T_E$, and is lost on recombination, hence the cooling rate is :

$$\Lambda = n_e^2 \alpha_B \frac{3}{2} k_B T_E$$

# Heating and Cooling

## RADIATIVE HEATING AND COOLING BY RECOMBINATION

Equating heating and cooling rate gives :

$$n_H S_* \sigma_i (h\bar{\nu} - I_H) = n_e^2 \alpha_B \frac{3}{2} k_B T_e$$

therefore :

$$T_e = \frac{2}{3} \frac{h\bar{\nu} - I_H}{k_B}$$

If the ionising flux is coming from a central star, then the emission can be approximated by a thermal emitter of temperature $T_*$ , and a reasonable approximation is that $h\bar{\nu} - I_H \sim k_B T_*$.

Then :

$$T_e \sim \frac{2}{3} T_*$$

Typically $T_* \sim 3\times10^4 - 6\times10^4 K$ giving $T_e \sim 4\times10^4 - 8\times10^4$K

- In a ionised gas : heat via collisions
- In a neutral gas : heat via inelastic collisions (hydrogen emission lines escaping the cloud and not contributing to the heating of the gas)

Similar arguments can also apply to the heating by X-Rays and Cosmic Rays which ionise principally hydrogen, and the emission of photo-electrons from dust particles.

## MECHANICAL HEATING

Main processes :

Heating in shocks

Heating in viscous accretion discs

In a strong shock all of the kinetic energy is converted into internal energy.

# Summary of cooling processes

**COOLING AND HEATING**

We defined the cooling function $\Lambda(n, T)$ as :

$$\Lambda(T) = \sum_i \Lambda_i$$

where $\Lambda_i$ is the cooling function for individual process

*By line emission*

Hydrogen is efficient only at $T > 10^4 K$, at lower temperature C and O are more efficient.

*By recombination*

In ionised gas, the energy of free electrons will heat the gas by collisions. In a neutral gas, we have inelastic collisions leading to photons emission (and no heating)

*By free-free emission*

For hot and fully ionised H-gas ($> 10^5\ K$), if the heating is constant, the cooling is a stable process.

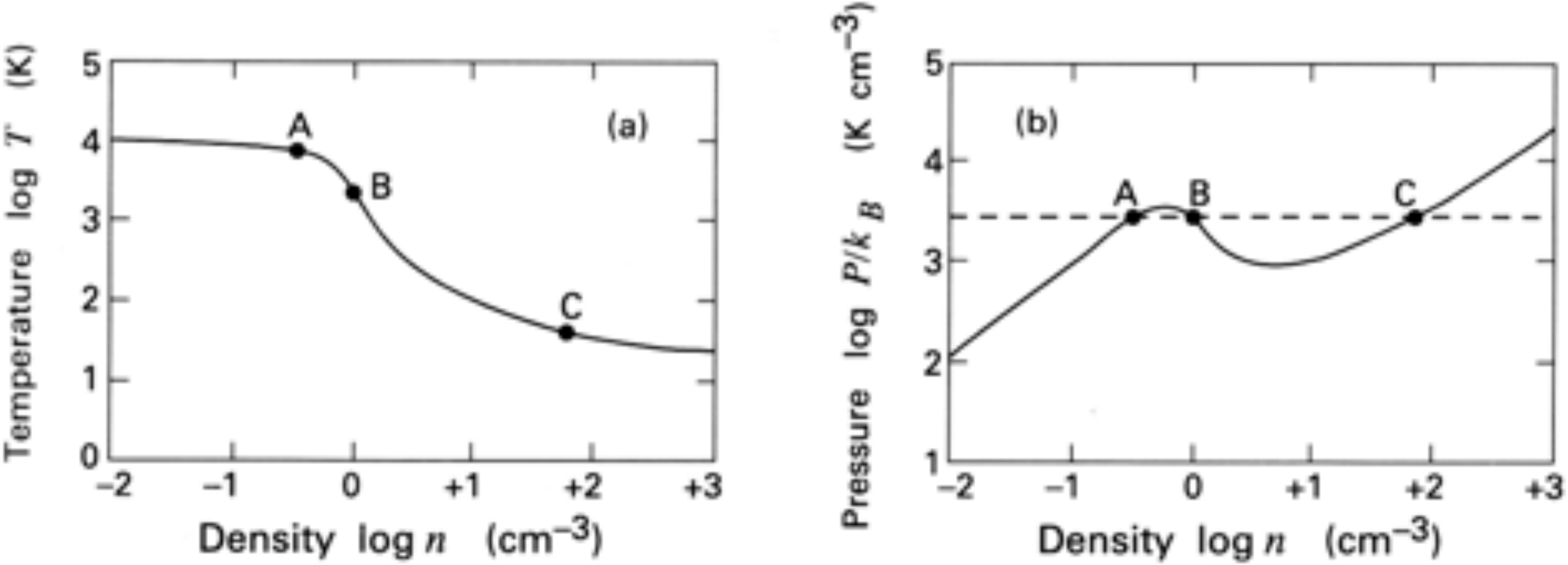
*Molecular gas*

Cooling of molecular gas is due to rotational transitions, which are more efficient for molecules with a permanent dipole. Then $H_2$ is not the main responsible for molecular gas cooling. Other molecules with permanent dipole (e.g., CO)

*By dust*

Dust grains absorb UV light and re-radiate in FIR. If the dust content is high, then cooling by dust is very efficient

# The multi-phase ISM




- 3 stable points in pressure ($\frac{dT}{dn} = 0$), but only 2 “real” stable points

Then we conclude that clouds of hydrogen are composed of warm and cold phases (multi-phase medium).

# The multi-phase ISM

**CAN WE EXPLAIN MOLECULAR CLOUDS IN THIS WAY ?**

Properties of Giant Molecular Clouds :

- Typical masses : $M \sim 10^5 M_{\odot}$
- Radius : $R \sim 50\text{pc}$
- Gravitational Energy $E_g \sim \frac{GM^2}{r}$
- Thermal Energy $E_k \sim \frac{3}{2}\left(\frac{M}{m_H}\right) k_B T$

Therefore :

$$\frac{E_g}{E_k} \sim \frac{GMm_H}{rk_BT} \approx 100$$

➔ The gravitational potential energy is much higher than the thermal energy meaning that such clouds are self-gravitating.

For the hot ionised phase, it is useful to calculate the cooling time :

$$\tau_c = \frac{E_k}{\Lambda}$$

Cooling at $\sim 5\times10^5$K is dominated by line emission from collisionnaly-excited ions and :

$$\Lambda \approx 1.6\times10^{-35} n_e n_i \left(\frac{T}{10^6}\right)^{-0.6} \text{W m}^{-3}$$

Then the cooling time for the gas with $n_e \sim 3\times10^3$m$^{-3}$ :

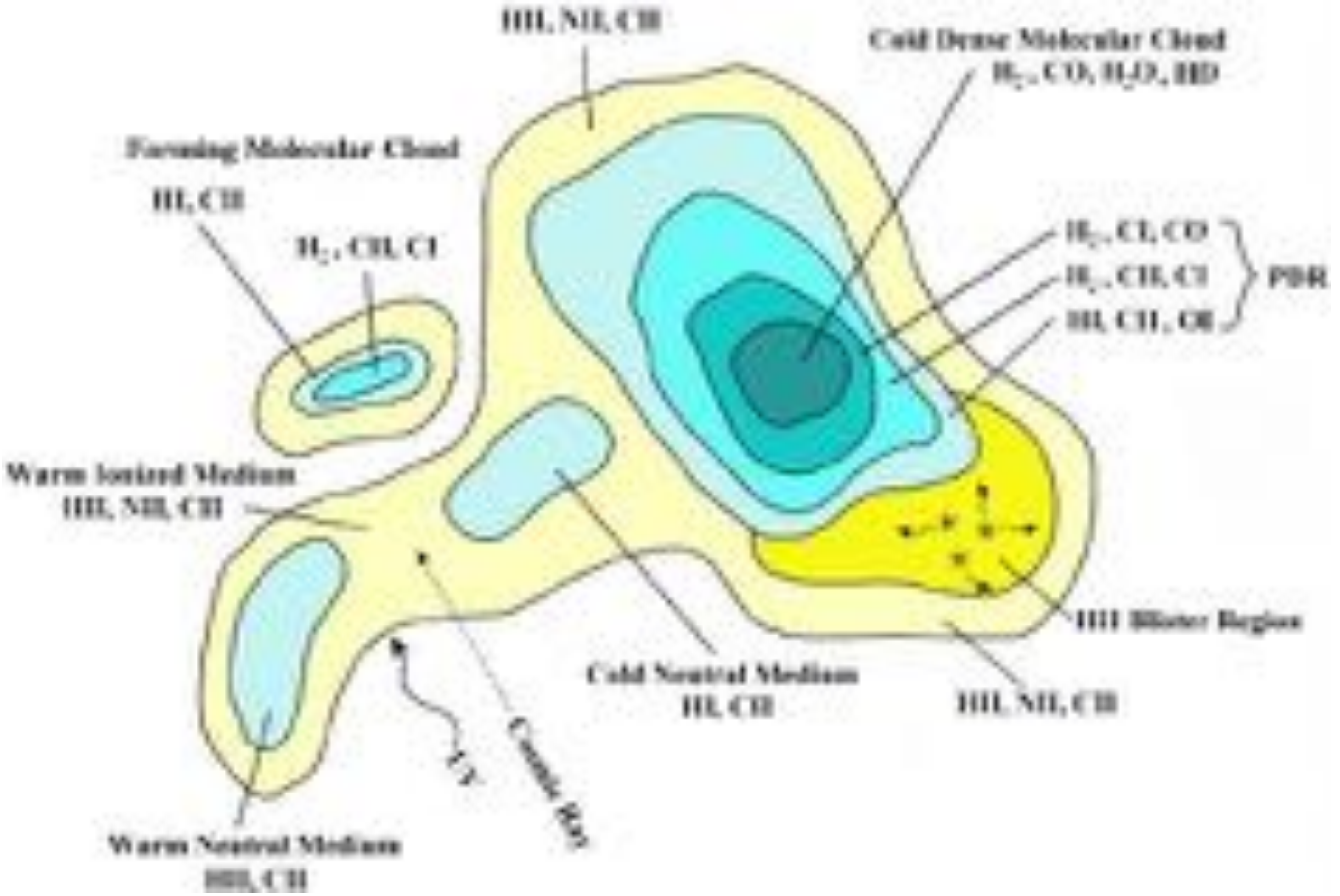
$$\tau_c = \frac{\frac{3}{2} n_e k_B T}{\Lambda} \sim 4\times10^6 \left(\frac{T}{5\times10^5}\right)^{1.6} \left(\frac{n_e}{3\times10^3}\right)^{-1} \text{yr}$$

The gas does not need a constant heating source, but still cools quickly on timescale (1 million years) much shorter than those over which a galaxy evolves (~billion years).

The gas can cool rapidly especially in denser regions, and then condense to one of the denser phases.

# The multi-phase ISM

| Phase | $n_{tot}$ ($10^6$ m$^{-3}$) | T(K) | M/$10^9$ $M_o$ | *f* |
|---|---|---|---|---|
| Molecular | >300 | 10 | 4.0 | 0.01 |
| Cold neutral | 50 | 80 | 3.0 | 0.04 |
| Warm neutral | 0.5 | ~5000 | 2.0 | 0.3 |
| Warm ionised | 0.3 | 10 000 | ~0.2 | 0.15 |
| Hot ionised | 3x$10^3$ | 3x$10^5$ | <0.02 | 0.5 |



# Gravitational stability and instability

Chapter 3

# Equations of hydrodynamics and hydrostatic equilibrium

## TOOLBOX FOR THIS CHAPTER

Euler's equation governs adiabatic and inviscid flow :

$$\rho \frac{d\vec{v}}{dt} + \rho(\overrightarrow{v} \cdot \nabla)\vec{v} = -\nabla P - \rho \nabla \Phi_g$$

Density of the fluid

Flow velocity

Pressure

Gravitational Potential

In equilibrium $v = 0$ , then :

$$-\nabla P - \rho \nabla \Phi_g = 0$$

Poisson's equation gives the gravitational potential :

$$\nabla^2 \Phi_g = 4\pi G \rho$$

The equation of state for an ideal gas gives the pressure :

$$P = \frac{\rho k_B T}{\mu}$$

The equation of continuity

$$\frac{d\rho}{dt} + \nabla.(\rho v) = 0$$

We also need an equation describing the energy flux but if we use the general form, we will not be able to solve the equations analytically. Approximations are needed !

# The isothermal sphere

Hydrostatic equilibrium
$(v = 0)$

0 0

$$\rho\frac{d\vec{v}}{dt} + \rho(\overrightarrow{v}\cdot\nabla)\vec{v} = -\nabla P - \rho\nabla\Phi_g$$

The simplest model in which pressure and gravity allow stable configuration is the isothermal sphere.

In this model, we assume :
- a spherical symmetry
- a gas at a uniform temperature T
- the equation of state given by : $P = \rho\frac{k_B T}{\mu} = a_T^2\rho$

where $a_T^2$ is the isothermal sound speed.

The gravitational potential and the pressure are only function of the radius $r$, then the Euler's equation can be written as :

$$\boxed{-\frac{1}{\rho}\frac{dP}{dr} - \frac{d\Phi_g}{dr} = 0}$$

and the Poisson's equation :

$$\boxed{\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\Phi_g}{dr} = 4\pi G\rho}$$

In spherical coordinates $(r, \theta, \varphi)$, the gradient is given by :

$$\nabla f = \frac{\partial f}{dr}\vec{r} + \frac{1}{r}\frac{\partial f}{d\theta}\vec{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{d\varphi}\overrightarrow{\varphi}$$

$$\nabla^2\Phi_g = 4\pi G\rho$$

In spherical coordinates $(r, \theta, \varphi)$, the Laplace operator is given by :

$$\Delta f = \nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{d\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$

# The isothermal sphere

## SINGULAR ISOTHERMAL SPHERE

In the following we assume an isolated single isothermal sphere, and we will try to get the solution (pressure, mass, radius)

The gravitational force is given by :

$$-\rho \nabla \Phi_g = -\frac{GM\rho}{r^2}$$

$$-\frac{1}{\rho}\frac{dP}{dr} - \frac{d\Phi_g}{dr} = 0$$

Then, from the Euler's equation we get :

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}$$

where the mass $M$, is the mass within a radius $r$ :

$$M = M(r) = \int_0^r \rho(r') 4\pi r'^2 dr'$$

The equation of state ($P = \rho \frac{k_B T}{\mu} = a_T^2 \rho$) can be written as :

$$\frac{d\rho}{dr} = -\frac{GM}{a_T^2}\frac{\rho}{r^2}$$

Then :

$$\frac{d \ln \rho}{dr}$$

$$r^2 \frac{1}{\rho}\frac{d\rho}{dr} = -\frac{GM}{a_T^2}$$

Taking the radius derivative gives :

$$\frac{d}{dr}\left(r^2 \frac{d \ln \rho}{dr}\right) = -\frac{G}{a_T^2}\frac{dM}{dr}$$

with $\frac{dM}{dr} = 4\pi r^2 \rho$, then

$$\frac{d}{dr}\left(r^2 \frac{d \ln \rho}{dr}\right) = -\frac{G}{a_T^2} 4\pi r^2 \rho$$

The exact solution of this equation is :

$$\boxed{\rho(r) = \frac{a_T^2}{2\pi G r^2}}$$

# The isothermal sphere

## SINGULAR ISOTHERMAL SPHERE

From the previous equation, we can determine :

- **The total mass of the cloud**

$$\mathrm{M}(\mathrm{r}_0) = \int_0^{r_0} \frac{a_T^2}{2\pi G r^2} 4\pi r^2 dr = \frac{2a_T^2 r_0}{G}$$

- <u>In equilibrium</u>, there must be an **external pressure** equaling the pressure at the surface of the cloud :

$$P_0 = a_T^2 \rho(r_0) = \frac{a_T^4}{2\pi G r_0^2}$$

- The **cloud radius** and **isothermal sound speed** can be estimated from the Mass and external pressure
- Although the density and pressure diverge at $r \to 0$, the total mass, internal energy, etc.. are bounded

$$\rho(r) = \frac{a_T^2}{2\pi G r^2}$$

$$M(r) = \int_0^r \rho(r') 4\pi r'^2 dr'$$

*Singular sphere* because the density and pressure diverge at r=0

# The isothermal sphere

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$$\mathrm{M(r_0)} = \int_0^{r_0} \frac{a_T^2}{2\pi G r^2} 4\pi r^2 dr = \frac{2a_T^2 r_0}{G}$$

- <u>In equilibrium</u>, there must be an **external pressure** equaling the pressure at the surface of the cloud :

$$P_0 = a_T^2 \rho(r_0) = \frac{a_T^4}{2\pi G r_0^2}$$

- The **cloud radius** and **isothermal sound speed** can be estimated from the Mass and external pressure
- Although the density and pressure diverge at $r \to 0$, the total mass, internal energy, etc.. are bounded

$$\rho(r) = \frac{a_T^2}{2\pi G r^2}$$

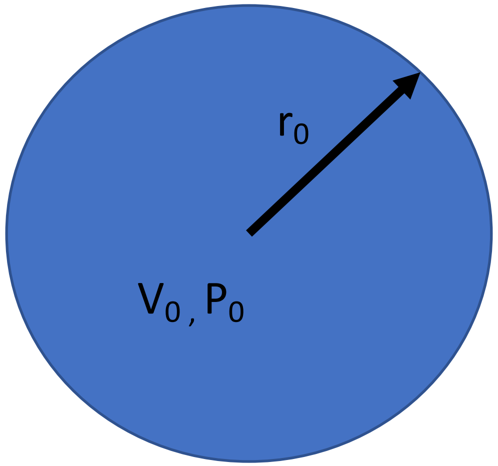
$$M(r) = \int_0^r \rho(r') 4\pi r'^2 dr'$$

*Singular sphere* because the density and pressure diverge at r=0

# Summary of the isothermal sphere

Singular Isothermal sphere

*T=cst*




$$\rho\frac{d\vec{v}}{dt} + \rho(\overrightarrow{v}\cdot\nabla)\vec{v} = -\nabla P - \rho\nabla\Phi_g$$

*Hydrostatic equilibrium*
*Spherical coordinates*

$$-\frac{1}{\rho}\frac{dP}{dr} - \frac{d\Phi_g}{dr} = 0$$

$$\nabla^2\Phi_g = 4\pi G\rho$$

*Hydrostatic equilibrium*
*Spherical coordinates*

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\Phi_g}{dr} = 4\pi G\rho$$

$$\rho(r) = \frac{a_T^2}{2\pi G r^2}$$

$$P_0 = a_T^2\rho(r_0) = \frac{a_T^4}{2\pi G r_0^2}$$

These solutions ($\rho(r)$ and $\mathrm{P}(r)$) diverge at $r \to 0$
Good description of the problem when $r \gg 0$

# The isothermal sphere

## GENERAL SOLUTION

The Euler's equation gives :

$$-\frac{1}{\rho}\frac{dP}{dr} - \frac{d\Phi_g}{dr} = 0$$

with $P = a_T^2 \rho$, then :

$$-\frac{a_T^2}{\rho}\frac{d\rho}{dr} = \frac{d\Phi_g}{dr}$$

Integrating gives :

$$-\ln\rho = \frac{\Phi_g}{a_T^2} + cste$$

Or :

$$\rho(r) = \rho_c \exp\left(-\frac{\Phi_g(r)}{a_T^2}\right)$$

$\rho_c = \rho(r = 0) \neq 0$

$$\nabla^2\Phi_g = 4\pi G\rho$$

To simplify the analysis, we need to introduce dimensionless variables :

$$\psi = \frac{\Phi_g}{a_T^2} \quad and \ \ \xi = \left(\frac{4\pi G\rho_c}{a_T^2}\right)^{1/2} r$$

The the Poisson's equation becomes :

$$\frac{1}{\xi^2}\frac{d}{d\xi}\xi^2\frac{d\psi}{d\xi} = e^{-\psi}$$

The solution of previous equation is :

$$\psi = \ln\left(\frac{\xi^2}{2}\right)$$

# The isothermal sphere

## GENERAL SOLUTION

The boundary solutions we can assume are :

- No gravitational force at the center of the cloud :

$$\left(\frac{d\psi}{d\xi}\right)_{\xi=0} = 0$$

- The density at the center of the cloud must be $\rho_c$, then $\psi(\xi = 0) = 0$

No analytical solution, equation must be integrated numerically.

However, we can estimate the total mass of the cloud :

$$M(r_0) = \int_0^{r_0} \rho 4\pi r^2 dr$$

Introducing the dimensionless variables gives :

$$M(r_0) = 4\pi\rho_c \left(\frac{a_T^2}{4\pi G\rho_c}\right)^{3/2} \int_0^{\xi_0} e^{-\psi}\xi^2 d\xi$$

Then :

$$\boxed{M(r_0) = 4\pi\rho_c \left(\frac{a_T^2}{4\pi G\rho_c}\right)^{\frac{3}{2}} \left(\xi^2 \frac{d\psi}{d\xi}\right)_{\xi=\xi_0}}$$

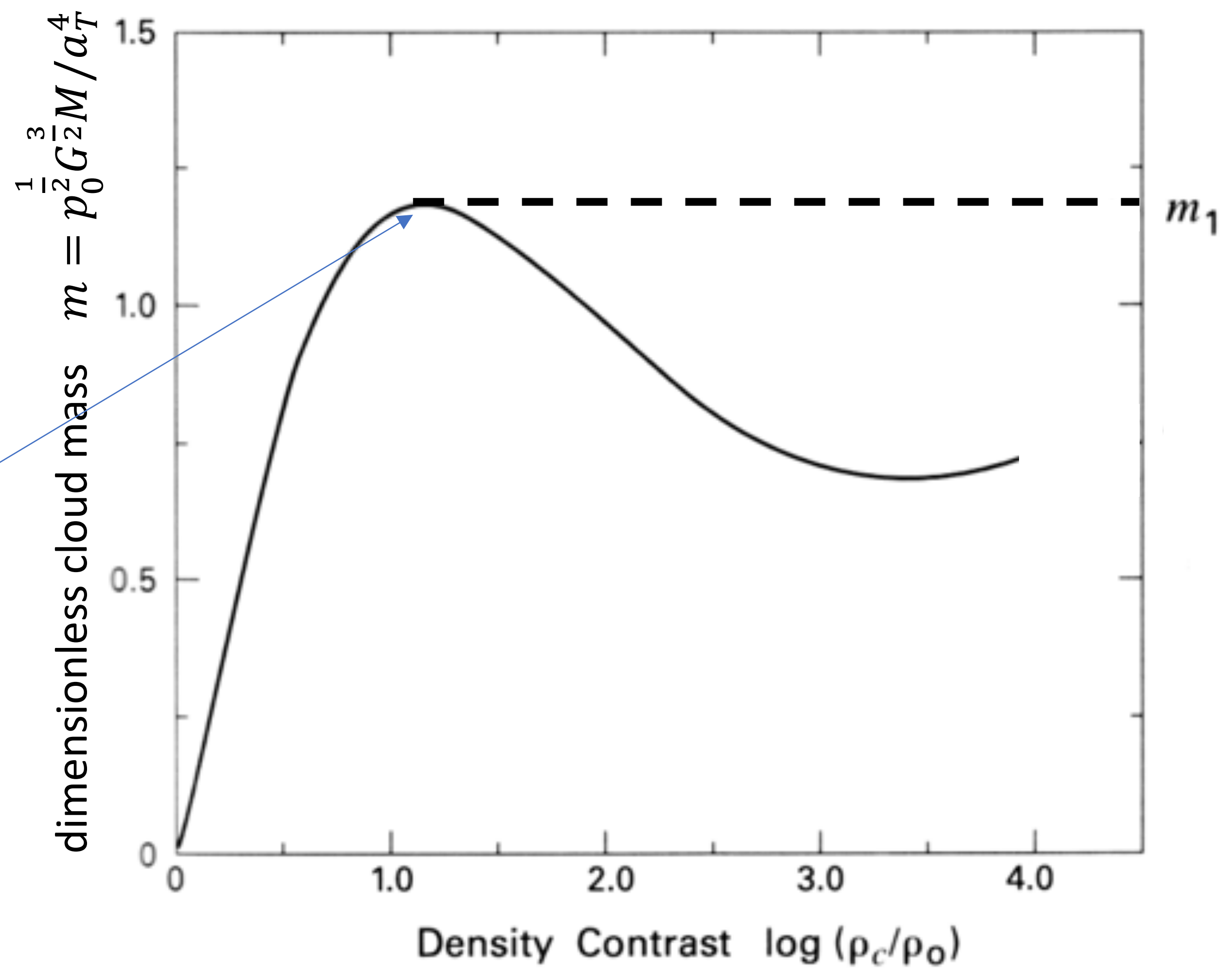
There is one more parameter compared to the singular solution : $\rho_c$ , and we need to also specify $a_T^2$ , $r_0$

# The isothermal sphere

**GENERAL SOLUTION**

We can also study the mass of the cloud as a function of the density contrast defined as $\rho_c/\rho_0$ , where $\rho_0 = \rho(r_0)$

There is a maximum cloud mass ($m_1$) for which equilibrium can be reached.



# The polytropic sphere

The equation state of the polytropic sphere is :

$$P = \mathrm{K}\rho^{1+\frac{1}{n}} = \mathrm{K}\rho^{\Gamma}$$

where $n$ is the polytropic index :

- n=0 for rocky planets
- n=1.5 for star cores

For the general polytropic case, we will demonstrate in problem sheet that the temperature always follows the gravitational potential :

$$k_B T = \frac{1-\Gamma}{\Gamma}\mu\Phi_g$$

# Virial equilibrium for the self-gravitating sphere

In the following, we will test if the solutions we find for the isothermal sphere are stable.

The equations of hydrostatic equilibrium are :

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}$$

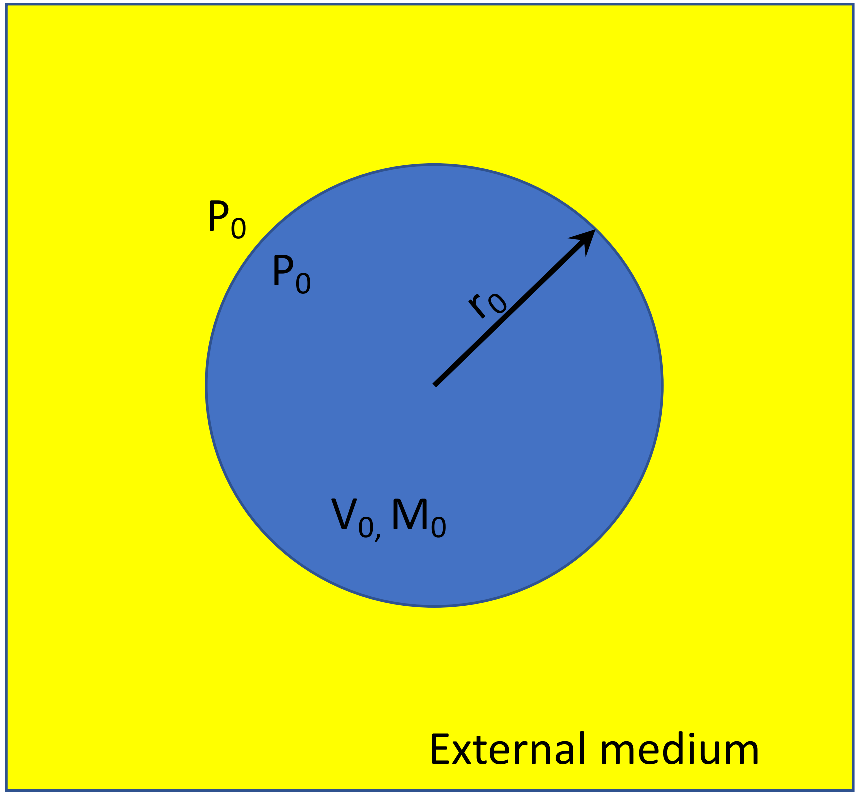
and

$$\frac{dM}{dr} = 4\pi r^2 \rho$$

To simplify, we will consider the mass as an independent variable, such as : $dr = \frac{dM}{4\pi r^2 \rho}$, then:

$$4\pi r^3 dP = -4\pi r\, GM\rho dr = -\frac{GM}{r} dM$$

$$\Rightarrow \int_{V=0,p=p_c}^{V=V_0,p=p_0} 3V\, dP = -\int_0^{M_0} \frac{GM}{r} dM$$



# Virial equilibrium for the self-gravitating sphere

$$\int_{V=0,p=p_c}^{V=V_0,p=p_0} 3V\,dP = -\int_0^{M_0} \frac{GM}{r} dM$$

To solve the previous equation, we need to integrate by part :

$$\int_a^b u(x)v'(x)dx = [u(x)v(x)]_a^b - \int_a^b u'(x)v(x)$$

Therefore, we obtain :

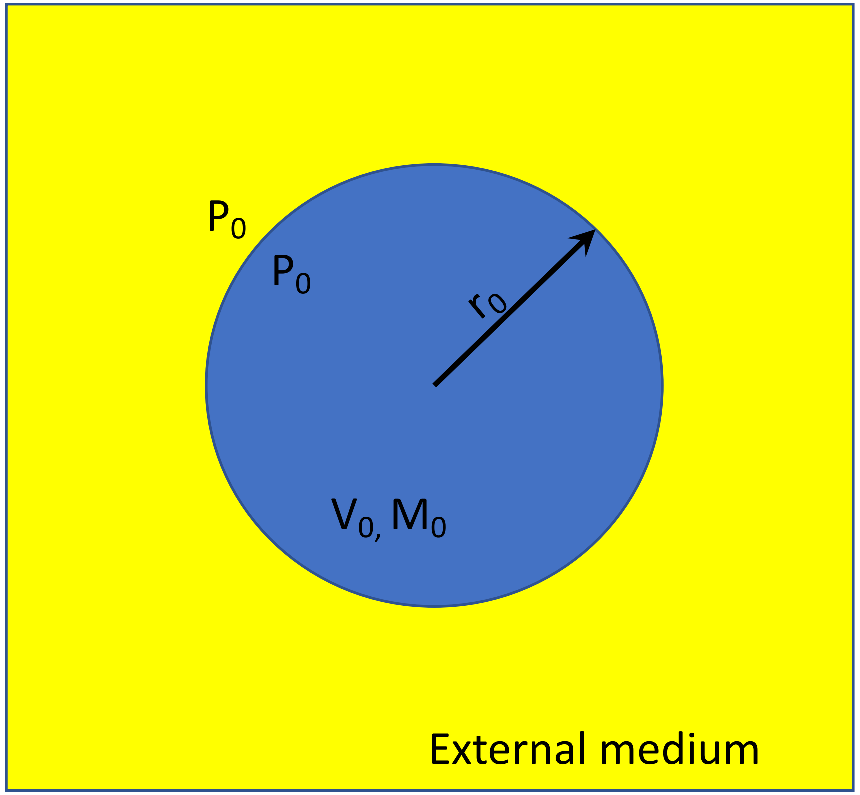
$$3[PV]_{0,p_c}^{V_0,p_0} - 3\int_{0,p_c}^{V_0,p_0} PdV = -\int_0^{M_0} \frac{GM}{r} dM$$

$$3P_0V_0 = 4\pi r_0^3 P_0$$

Gravitational potential ($\Omega$)

$$\boxed{3\int_0^{M_0} \frac{P}{\rho} dM + \Omega = 4\pi r_0^3 P_0}$$

**Equation of virial equilibrium**



# Stability of an isothermal cloud

$$\boxed{3\int_0^{M_0} \frac{P}{\rho} dM + \Omega = 4\pi r_0^3 P_0}$$

For an isothermal sphere, we know that : $P = a_T^2 \rho$, hence :

$$3\int_0^{M_0} \frac{P}{\rho} dM = 3a_T^2 M_0 = 3\frac{k_B T}{\mu} M_0$$

We also know that :

$$\Omega = -\frac{3}{5}\frac{GM_0^2}{r_0}$$

Then the virial equilibrium equation for an isothermal cloud becomes :

$$3\frac{k_B T}{\mu} M_0 - \frac{3}{5}\frac{GM_0^2}{r_0} - 4\pi r_0^3 P_0 = 0$$

Associated with thermal pressure

Associated with gravity

Associated with external pressure

**Stability conditions**

- If =0 ➔ equilibrium
- If <0 ➔ external pressure and gravity are "stronger" than thermal pressure : **the cloud is collapsing**
- If >0 ➔ the thermal pressure is larger than external pressure and gravity : **the cloud is expanding**.

# Stability of an isothermal cloud

The external pressure $P_0$ is given by :

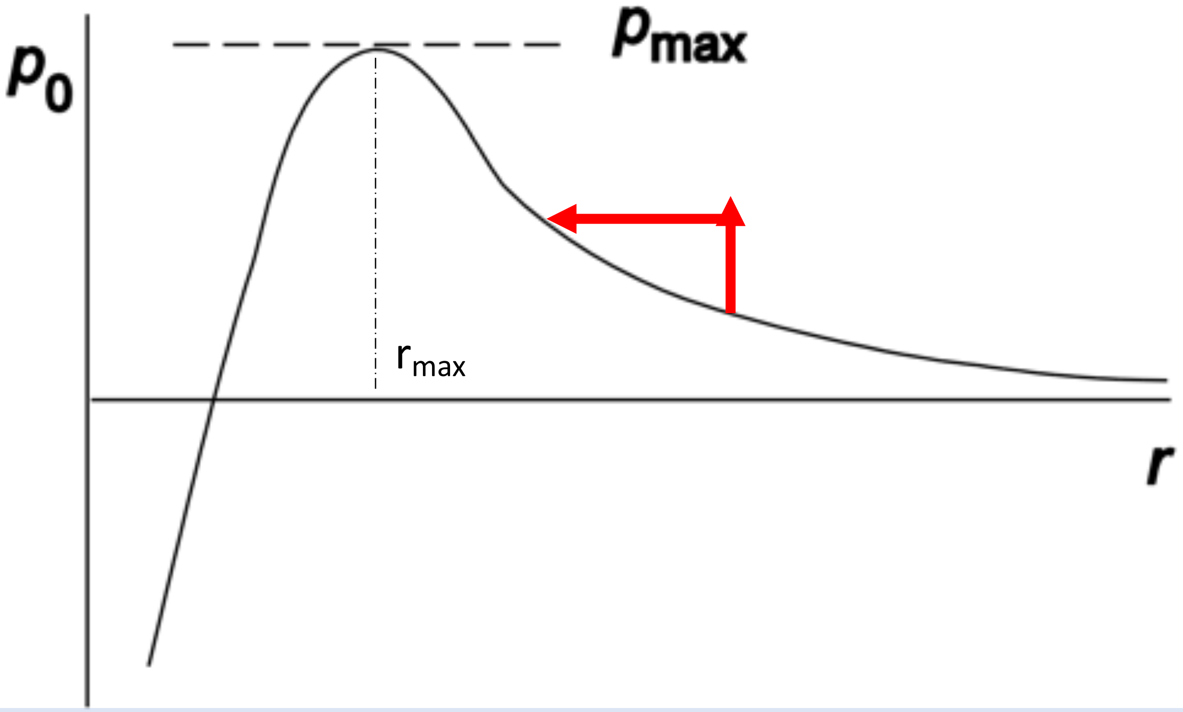
$$P_0(r) = \frac{3k_BTM_0}{4\pi r^3\mu} - \frac{3}{20\pi}\frac{GM_0^2}{r^4}$$

The maximum of this function is given by :

$$\frac{dP_0(r)}{dr} = r_{max} = \frac{4}{15}\frac{GM_0^2\mu}{k_BT}$$

with a maximum pressure of :

$$P_{max} = c_g\left(\frac{k_BT}{\mu}\right)^4 \frac{1}{G^3M_0^2}$$




Stability of the cloud of mass $M_0$ and with $r > r_{max}$ :

- If the external pressure is increased by a small amount, the system will lie above the equilibrium line, then the virial equation shows that the cloud must shrink.
- If the external pressure $P_0 > P_{max}$ the cloud is not stable and can't find any radius at which it will be in equilibrium : the cloud is collapsing.

# Stability of an isothermal cloud

$$P_{max} = c_g \left(\frac{k_B T}{\mu}\right)^4 \frac{1}{G^3 M_0^2}$$

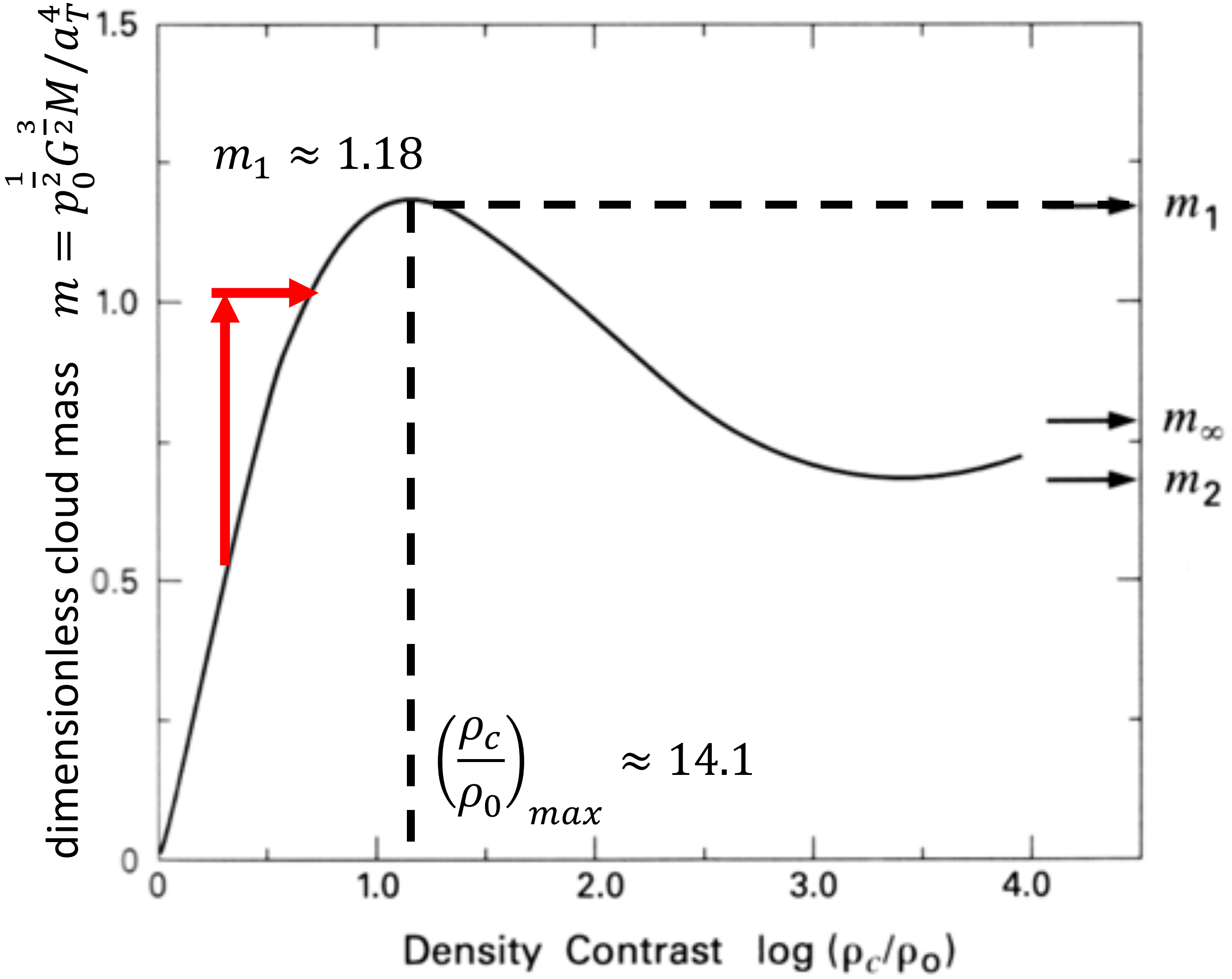
For a given external pressure, a cloud will become unstable to collapse when its mass exceeds :

$$M = c_g^{1/2} \left(\frac{k_B T}{\mu}\right)^2 \frac{1}{G^{3/2} P_0^{1/2}} = c_g^{1/2} \frac{a_T^3}{\rho_0^{1/2} G^{3/2}}$$

Bonnor-Ebert mass

Consider a cloud with a given $\rho_c/\rho_0$, if we increase the external pressure then :

- The dimensionless mass will increase ($m = p_0^{\frac{1}{2}} G^{\frac{3}{2}} M / a_T^4$ )
- For stability the internal pressure of the cloud must increase
- But at constant T, this requires $\rho$ in the cloud to increase and hence $\rho_c$



# Jeans instability

$$\rho\frac{d\vec{v}}{dt} + \rho(\overrightarrow{v}\cdot\nabla)\vec{v} = -\nabla P - \rho\nabla\Phi_g$$

$$\nabla^2\Phi_g = 4\pi G\rho$$

## PERTURBATION ANALYSIS OF FLUID EQUATIONS

Initial conditions of the system (the fluid is stationary) :

- $v_0 = 0$
- $\rho_0 = cste$
- $P_0 = cste$

Introducing perturbed quantities :

- $\rho = \rho_0 + \rho_1$
- $v = v_0 + v_1$
- $\Phi_g = \Phi_0 + \Phi_1$
- $P = P_0 + P_1$

As previously, the unperturbed potential is assumed to satisfy :

$$\nabla^2\Phi_0 = 4\pi G\rho_0$$

There is no solution when $\rho_0 = cste$

The equation of continuity given by :

$$\frac{d\rho}{dt} + \nabla.(\rho v) = 0$$

becomes (to first order):

$$\rho_0(\nabla.\overrightarrow{v_1}) = -\frac{\partial\rho_1}{\partial t}$$

Similarly, the other hydrostatic equations (Euler's & Poisson's) become :

$$\frac{\partial v_1}{\partial t} = -\nabla\phi_1 - \frac{1}{\rho_0}\nabla P_1$$

And

$$\nabla^2\phi_1 = 4\pi G\,\rho_1$$

We also assume isothermal behaviour such as :

$$P_1 = a_T^2\rho_1$$

# Jeans instability

$$\rho_0(\nabla.\overrightarrow{v_1}) = -\frac{\partial \rho_1}{\partial t}$$

$$\frac{\partial v_1}{\partial t} = -\nabla\phi_1 - \frac{1}{\rho_0}\nabla P_1$$

## PERTURBATION ANALYSIS OF FLUID EQUATIONS

Differentiating the continuity equation with respect to time, we obtain :

$$\frac{\partial}{\partial t}(\nabla.v_1) = -\frac{1}{\rho_0}\frac{\partial^2 \rho_1}{\partial t^2}$$

Taking the divergence of the Euler equation :

$$\frac{\partial}{\partial t}(\nabla.v_1) = -\nabla^2\phi_1 - \frac{a_T^2}{\rho_0}\nabla^2\rho_1$$

Combining the two previous equation gives :

$$\left(\nabla^2 - \frac{1}{a_T^2}\frac{\partial^2}{\partial t^2} + \frac{4\pi G\rho_0}{a_T^2}\right)\rho_1 = 0$$

→ similar to the wave equation !

Therefore, we should look for wave-like solutions of the form :

$$\rho_1 \propto e^{i(\vec{k}.\vec{r}-\omega t)}$$

which gives a dispersion relation :

$$a_T^2 k^2 - \omega^2 = 4\pi G\rho_0$$

The system is unstable when the modes grow (i.e. $\omega^2 < 0$).
Hence we can define a critical wave number (when $\omega^2 = 0$) :

$$k_j^2 = \frac{4\pi G\rho_0}{a_T^2} = \frac{4\pi G\mu}{k_B T}\rho_0$$

And a characteristic wavelength :

$$\lambda_J = \frac{2\pi}{k_j}$$

# Jeans instability

## PERTURBATION ANALYSIS OF FLUID EQUATIONS

The total mass within a sphere of diameter equal to the Jeans wavelength $\lambda_j$ is :

$$M_J = \frac{4}{3}\pi\left(\frac{\lambda_j}{2}\right)^3 \rho_0$$

Moreover :

$$\lambda_J^2 = \left(\frac{2\pi}{k_j}\right)^2 = \frac{\pi k_B T}{G\mu\rho_0}$$

Then :

$$\frac{\lambda_J}{2} = \frac{3}{\pi^2}\frac{GM_J\mu}{k_B T}$$

For $\lambda > \lambda_J$ or $M > M_J$ the modes grow exponentially : **the cloud is collapsing**

The Jeans Mass is the mass above which gravity dominates.

The Jeans mass is usually defined as :

$$\frac{M_J}{M_\odot} = 1.0\times\left(\frac{T}{10K}\right)^{3/2} \times\left(\frac{n_H}{2\times10^{10}m^{-3}}\right)^{-1/2}$$

Strong dependance on temperature

*Importance of cooling which could reduce the temperature and therefore allow the collapse of less massive clouds*

# Jeans instability

**PERTURBATION ANALYSIS OF FLUID EQUATIONS**

In the early Universe, the absence of metals and dust (not enough time to form ) and the much reduced molecular gas content implies very poor cooling

↓

**Formation of massive stars in the early Universe**

# Magnetic fields

Magnetic fields are important components of the ISM : these can provide additional forces which can act to stabilise clouds against gravitational collapse.

The derivation of the Euler equation in the case of magnetic fields is complex. We will just give the solution of the Euler equation :

$$P_0 = \underbrace{\frac{3k_B T M_0}{4\pi r_0^3 \mu}}_{\text{Thermal Pressure}} + \frac{1}{4\pi r_0^4}\left(\underbrace{\beta \frac{\Phi_M^2}{2\mu_0}}_{\text{Magnetic pressure}} - \underbrace{\frac{3}{5} G M_0^2}_{\text{Gravity}}\right)$$

The pressure will be a monotonically decreasing function of r if :

$$\beta \frac{\Phi_M^2}{2\mu_0} > \frac{3}{5} G M_0^2$$

The clouds will always be stable if $\Phi_M$ is a constant.

# Application to molecular clouds

Giant Molecular Clouds can be seen as swarms of more coherent clumps.

The Jeans mass for gas with $n_H \sim 2000$ and $T \sim 10K$ is $M_J \sim 3M_{\odot}$. This is well below the observed masses of the individual clouds and of order the mass of typical dense cores.

➔ There must be an additional form of support : the magnetic pressure.

Zeeman splitting provides a method for measuring the magnetic fields in clouds, although this has only be successful in a handful of dark clouds.

From the magnetic virial equation we can find the maximum cloud mass which could be supported against its own self-gravity by magnetic pressure alone :

$$M^2 = \frac{5}{3G} \beta \frac{\pi^2 r^4 B^2}{2\mu_0}$$

# Application to molecular clouds

Then :

$$M \approx \left(\frac{B}{nT}\right)\left(\frac{r}{pc}\right)^2 M_\odot$$

Inserting typical values for several cloud types, we can show that most dark clouds can be stabilised by magnetic effects :

- For dense cores $M \sim M_J$
- The density ratio is measured to $\rho_c/\rho_0 \sim 10$

| Cloud type | $n_{tot}$ [$10^6 m^{-3}$] | L [pc] | T [K] | M [$M_\odot$] | B [nT] |
|---|---|---|---|---|---|
| Giant Molecular Cloud | 100 | 50-500 | 15 | $10^5$ | 1 ? |
| Dark Cloud Complex | 500 | 10 | 10 | $10^4$ | 1 ? |
| Individual Dark Cloud | $10^3$ | 2 | 10 | 30 | 2-10 |
| Dense Core | $10^4$ | 0.1 | 10 | 10 | 2-10 |

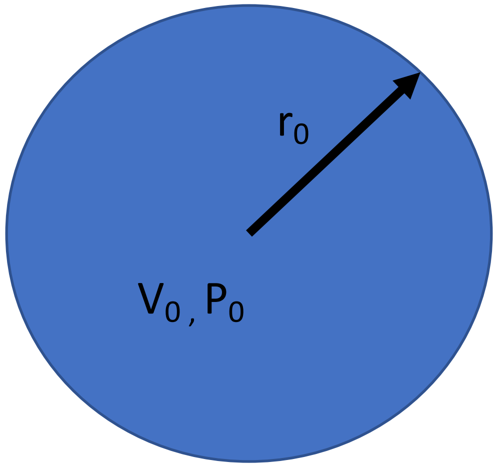
# Summary of yesterday's lecture

$$\rho \frac{d\vec{v}}{dt} + \rho(\overrightarrow{v} \cdot \nabla)\vec{v} = -\nabla P - \rho \nabla \Phi_g$$

*Hydrostatic equilibrium*
*Spherical coordinates*

$$-\frac{1}{\rho}\frac{dP}{dr} - \frac{d\Phi_g}{dr} = 0$$

Singular Isothermal sphere
*T=cst*




$$\nabla^2 \Phi_g = 4\pi G \rho$$

*Hydrostatic equilibrium*
*Spherical coordinates*

$$\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d\Phi_g}{dr} = 4\pi G \rho$$

$$\rho(r) = \frac{a_T^2}{2\pi G r^2}$$

$$P_0 = a_T^2 \rho(r_0) = \frac{a_T^4}{2\pi G r_0^2}$$

These solutions ($\rho(r)$ and $\mathrm{P}(r)$) diverge at $r \to 0$
Good description of the problem when $r \gg 0$

# Summary of yesterday's lecture

$$\rho_c = \rho(r = 0) \neq 0$$

$$\rho(r) = \rho_c \exp\left(-\frac{\Phi_g(r)}{a_T^2}\right)$$

Introducing dimensionless variables

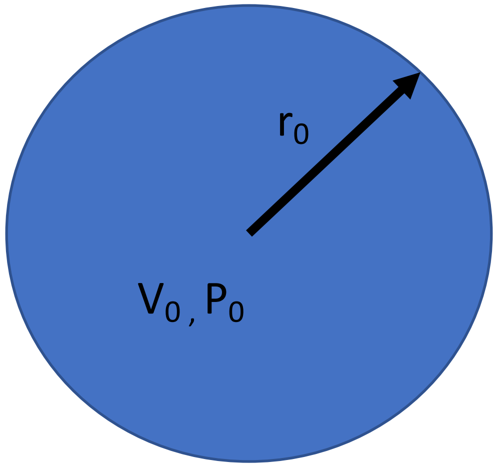
$$\psi = \frac{\Phi_g}{a_T^2} \qquad \textit{and} \;\; \xi = \left(\frac{4\pi G\rho_c}{a_T^2}\right)^{1/2} r$$

*Euler's equation*

$$\frac{1}{\xi^2}\frac{d}{d\xi}\xi^2\frac{d\psi}{d\xi} = e^{-\psi}$$

No analytical solutions ;
numerical integration needed

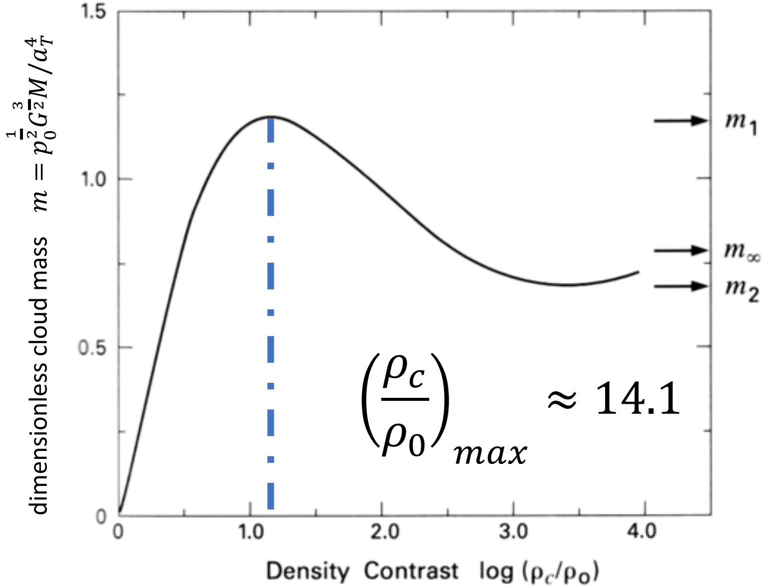
Isothermal sphere
*General Solution*




$$M(r_0) = 4\pi\rho_c\left(\frac{a_T^2}{4\pi G\rho_c}\right)^{\frac{3}{2}}\left(\xi^2\frac{d\psi}{d\xi}\right)_{\xi=\xi_0}$$

$$\mathrm{M}(\mathrm{r}_0) = \frac{2a_T^2 r_0}{G}$$

*Comparing with the mass expression obtained from the singular isothermal sphere analysis, we see that one more parameter is needed $\rho_c$*



# Summary of yesterday's lecture

*Virial equation for a spherical gas cloud*

$$3\frac{k_B T}{\mu}M_0 - \frac{3}{5}\frac{GM_0^2}{r_0} - 4\pi r_0^3 P_0 = 0$$

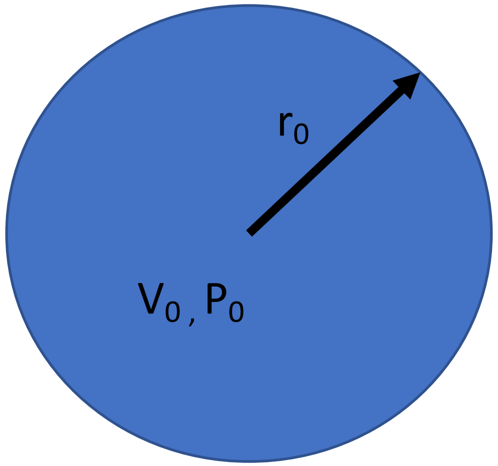
Associated with thermal pressure

Associated with gravity

Associated with external pressure

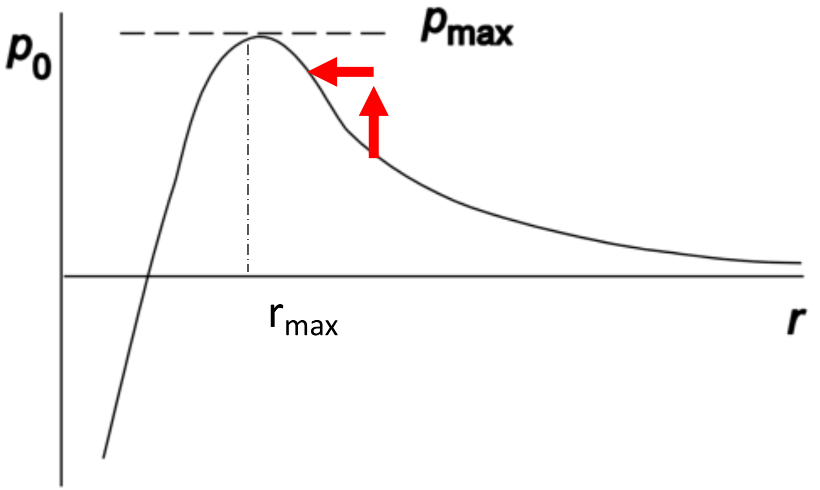
- If =0 ➔ equilibrium
- If <0 ➔ **the cloud is collapsing**
- If >0 ➔ **the cloud is expanding.**

Stability of a gas cloud




*Evolution of the pressure*

$$P_0(r) = \frac{3k_B T M_0}{4\pi r^3 \mu} - \frac{3}{20\pi}\frac{GM_0^2}{r^4}$$



# Summary of yesterday's lecture

Jeans Instability

*Introducing perturbed quantities*

$$\rho = \rho_0 + \rho_1$$
$$v = v_0 + v_1$$
$$\Phi_g = \Phi_0 + \Phi_1$$
$$P = P_0 + P_1$$

$$\frac{d\rho}{dt} + \nabla.(\rho v) = 0$$

$$\nabla^2 \Phi_g = 4\pi G \rho$$

$$\rho \frac{d\vec{v}}{dt} + \rho(\overrightarrow{v} \cdot \nabla)\vec{v} = -\nabla P - \rho \nabla \Phi_g$$

*Only keeping first order terms*

$$\rho_0(\nabla.\overrightarrow{v_1}) = -\frac{\partial \rho_1}{\partial t}$$

$$\frac{\partial v_1}{\partial t} = -\nabla\phi_1 - \frac{1}{\rho_0}\nabla P_1$$

$$\nabla^2 \phi_1 = 4\pi G\, \rho_1$$

$$\left(\nabla^2 - \frac{1}{a_T^2}\frac{\partial^2}{\partial t^2} + \frac{4\pi G \rho_0}{a_T^2}\right)\rho_1 = 0$$

# Summary of yesterday's lecture

***The Jeans Mass is the Mass above which gravity dominates => collapse !***
***Look at the strong dependence in temperature, and hence the crucial role of cooling (previous chapter) in the formation of structure !***

$$\frac{M_J}{M_\odot} = 1.0\times\left(\frac{T}{10K}\right)^{3/2}\times\left(\frac{n_H}{2\times10^{10}m^{-3}}\right)^{-1/2}$$

*Wave equation*

$$\left(\nabla^2 - \frac{1}{a_T^2}\frac{\partial^2}{\partial t^2} + \frac{4\pi G\rho_0}{a_T^2}\right)\rho_1 = 0$$

*Form of the solution*

$$\rho_1 \propto e^{i(\vec{k}.\vec{r}-\omega t)}$$

*Jeans Mass*

$$M_J = V_0\rho_0 = \frac{4}{3}\pi\left(\frac{\lambda_j}{2}\right)^3\rho_0$$

*Dispersion equation*

$$a_T^2k^2 - \omega^2 = 4\pi G\rho_0$$

$\omega = 0$

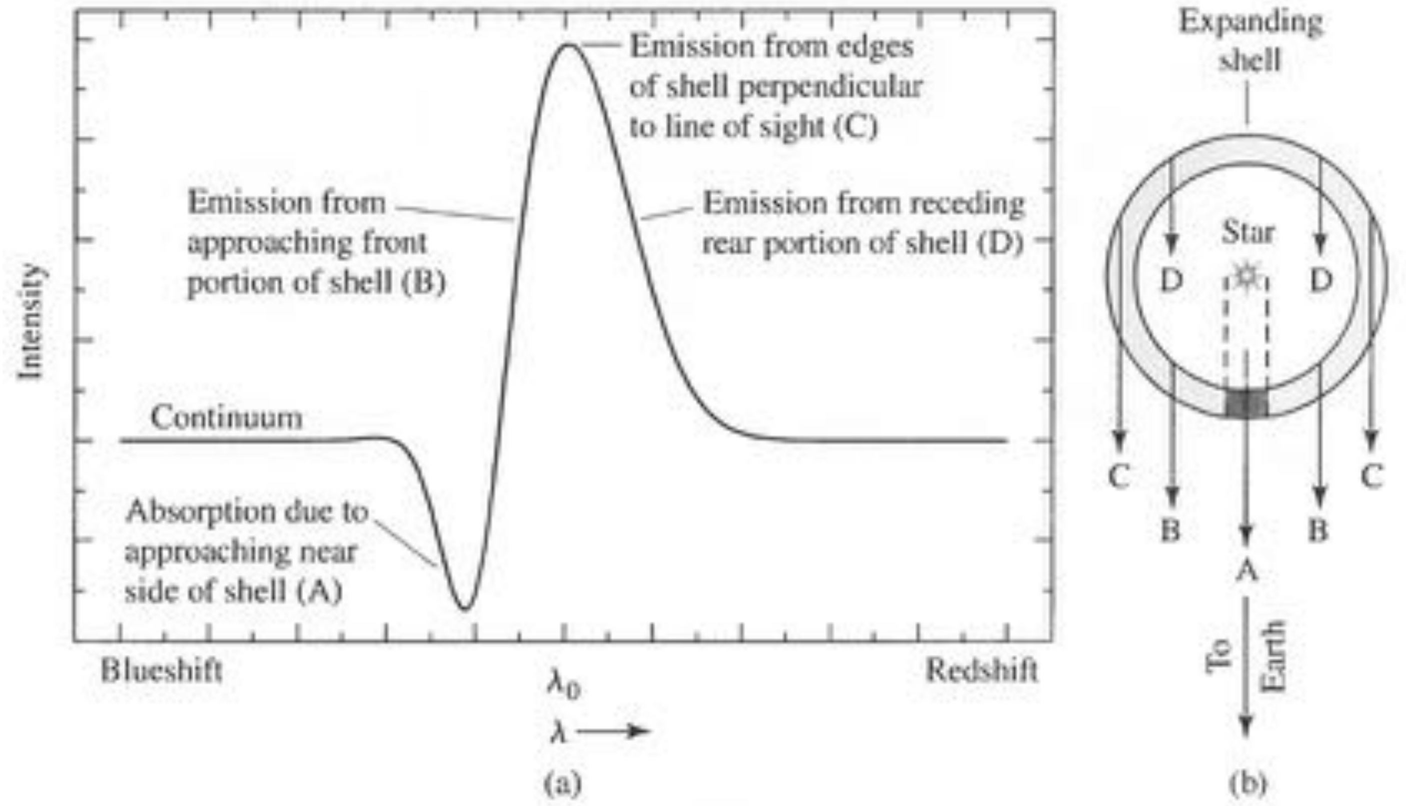
*Critical Wave Number*

$$k_j^2 = \frac{4\pi G\rho_0}{a_T^2} = \frac{4\pi G\mu}{k_BT}\rho_0$$

*Characteristic Wavelength*

$$\lambda_J = \frac{2\pi}{k_j}$$

# Question : spectra if we have as many emissions as absorptions



# Gravitational collapse

## Chapter 4

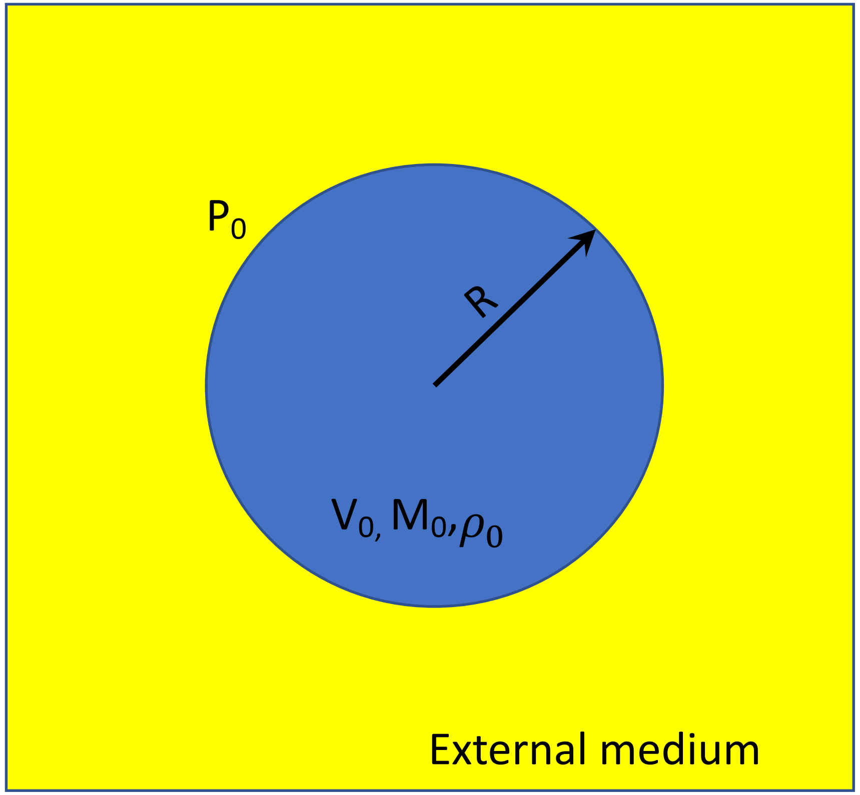
# Free-fall time

The free-fall time is the characteristic time that would take an object to collapse under its own gravity, if no other forces exist to oppose the collapse.

We will first consider the collapse of a cloud within a galaxy, but this method applies also at larger scales (i.e., galactic scale)

**INITIAL CONDITIONS OF THE COLLAPSE**

- Cloud of radius R and mass $M_0$
- Gas density $\rho_0$
- Gas molecules initially at $r_0$ will have a mass of gas M within the radius and during collapse this remains constant




From Newton gravity the equation of motion is :

$$\frac{\partial^2 r}{\partial t^2} = -\frac{GM_r}{r^2}$$

or

$$\frac{\partial}{\partial t}\left(\frac{\partial r}{\partial t}\right) = -\frac{GM_r}{r^2}$$

# Free-fall time

Multiplying by $\frac{\partial r}{\partial t}$ and integrating over dt give :

$$\frac{1}{2}\left(\frac{\partial r}{\partial t}\right)^2 = \left[\frac{GM}{r}\right]_{r_0}^{r} = \frac{GM}{r_0}\left(\frac{r_0}{r} - 1\right) = \frac{4\pi}{3} r_0^2 \rho_0 G \left(\frac{r_0}{r} - 1\right)$$

Then :

$$\frac{\partial r}{\partial t} = \sqrt{\frac{8\pi}{3} r_0^2 G \rho_0 \left(\frac{r_0}{r} - 1\right)}$$

Hence :

$$\partial t = \sqrt{\frac{3}{8\pi r_o^2 G \rho_0}} \frac{\partial r}{\sqrt{\frac{r_0}{r} - 1}}$$

Integrating :

$$t = \left(\frac{3}{8\pi r_0^2 G \rho_0}\right)^{1/2} \int_0^{r_0} \frac{\partial r}{\sqrt{\frac{r_0}{r} - 1}}$$

Substituting $\mathrm{u} = \mathrm{r}/\mathrm{r}_0$ gives :

$$t = \left(\frac{3}{8\pi G \rho_0}\right)^{1/2} \underbrace{\int_0^1 \frac{\partial u}{\sqrt{\frac{1}{u} - 1}}}_{\text{Tabulated} \rightarrow \frac{\pi}{2}}$$

Therefore, the free-fall time is given by :

$$\boxed{t_{ff} = \left(\frac{3\pi}{32 G \rho_0}\right)^{1/2}}$$

The free-fall time depends on density : if the inner region of the cloud are denser, these will collapse first➔ **inside-out collapse**

# Inside-out collapse

Adding to the previous initial conditions that the cloud has an isothermal behavior.

We also assume that there is a central sink for inflowing material (the growing central object: e.g., a protostar)

The dynamic of the problem is governed by the Euler's equation and the continuity equation (radial equations) :

$$\rho\frac{d\vec{v}}{dt}+\rho(\overrightarrow{v}\cdot\nabla)\vec{v}=-\nabla P-\rho\nabla\Phi_g$$

$$\frac{dv}{dt}+v\frac{dv}{dr}=-\frac{a_T^2}{\rho}\frac{d\rho}{dr}-\frac{GM_r}{r^2}$$

and

$$\frac{d\rho}{dt}+\frac{1}{r^2}\frac{d(r^2\rho v)}{dr}=0$$

$$\frac{d\rho}{dt}+\nabla.(\rho v)=0$$

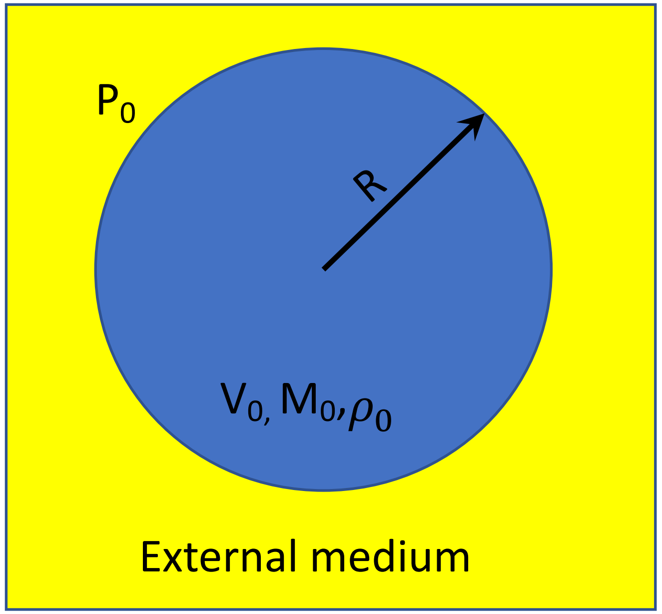
Divergence in spherical coordinates

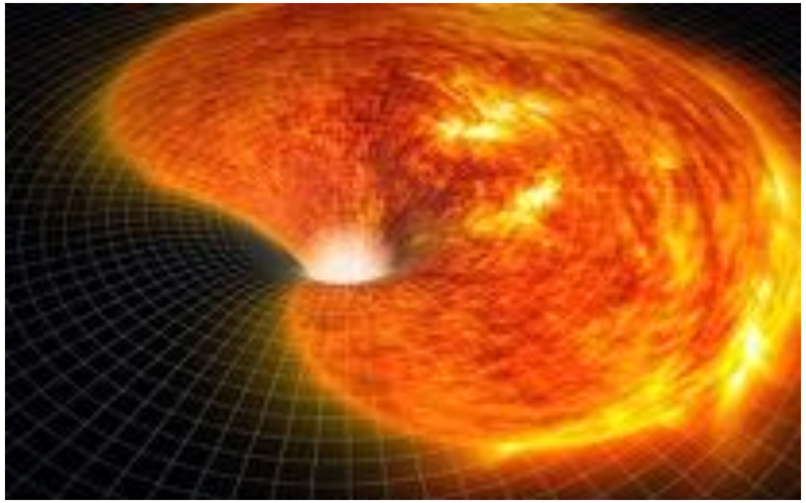
$$\nabla_r\cdot f=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)$$

where $\rho=\rho(r,t)$ and

$$M_r(t)=\int_0^r 4\pi r^2\rho(r,t)dr$$

Differentiating gives : $\frac{\partial M_r}{\partial t}=-4\pi r^2\rho v$



# Inside-out collapse

$$\frac{dv}{dt} + v\frac{dv}{dr} = -\frac{a_T^2}{\rho}\frac{d\rho}{dr} - \frac{GM_r}{r^2}$$

$$\frac{d\rho}{dt} + \frac{1}{r^2}\frac{d(r^2\rho v)}{dr} = 0$$

## SIMILARITY ANALYSIS

In the previous equations we can identify :

- The independent variables : $r$ and $t$
- The constants : $G$ and $a_T$
- The variables : $\rho(r,t), v(r,t)$ and $M(r,t)$

The only way to form a dimensionless length is :

$$x = \frac{r}{a_T t}$$

We are doing a similarity analysis; therefore, we are searching for solutions in the form :

$$M_r(r,t) = \frac{a_T^3 t}{G} m(x)$$

$$\rho(r,t) = \frac{1}{4\pi G t^2}\alpha(x)$$

$$v(r,t) = a_T \beta(x)$$

$m(x), a(x)$ and $\beta(x)$ are dimensionless

Then :

$$dx = \frac{1}{a_T t}dr - \frac{r}{a_T\, t^2}dt$$

$$\left(\frac{\partial}{\partial r}\right)_t = \frac{1}{a_T t}\frac{\partial}{\partial x}$$

$$\left(\frac{\partial}{\partial t}\right)_r = \left(\frac{\partial}{\partial t}\right)_x + \left(\frac{\partial x}{dt}\right)_r \left(\frac{\partial}{\partial x}\right) = \left(\frac{\partial}{\partial t}\right)_x - \frac{x}{t}\left(\frac{\partial}{\partial x}\right)$$

# Inside-out collapse

## SIMILARITY ANALYSIS

The equations we must solve are :

$$\frac{dv}{dt} + v\frac{dv}{dr} = -\frac{a_T^2}{\rho}\frac{d\rho}{dr} - \frac{GM_r}{r^2}$$

$$\frac{d\rho}{dt} + \frac{1}{r^2}\frac{dr^2\rho v}{dr} = 0$$

$$\frac{\partial M}{\partial r} = 4\pi r^2 \rho$$

Knowing that :

$$\left(\frac{\partial}{\partial r}\right)_t = \frac{1}{a_T t}\frac{\partial}{\partial x}$$

$$\left(\frac{\partial}{\partial t}\right)_r = \left(\frac{\partial}{\partial t}\right)_x + \left(\frac{\partial x}{dt}\right)_r \left(\frac{\partial}{\partial x}\right) = \left(\frac{\partial}{\partial t}\right)_x - \frac{x}{t}\left(\frac{\partial}{\partial x}\right)$$

The equations become :

$$m = x^2\alpha(x-\beta)$$

$$[(x-\beta)^2 - 1]\frac{1}{\alpha}\frac{d\alpha}{dx} = \left[\alpha - \frac{2}{x}(x-\beta)\right](x-\beta)$$

$$[(x-\beta)^2 - 1]\frac{d\beta}{dx} = \left[\alpha(x-\beta) - \frac{2}{x}\right](x-\beta)$$

These equations must be solved numerically, but we can learn a lot from their form.

# Inside-out collapse

## SIMILARITY ANALYSIS

$$m = x^2\alpha(x-\beta)$$

$$[(x-\beta)^2 - 1]\frac{1}{\alpha}\frac{d\alpha}{dx} = \left[\alpha - \frac{2}{x}(x-\beta)\right](x-\beta)$$

$$[(x-\beta)^2 - 1]\frac{d\beta}{dx} = \left[\alpha(x-\beta) - \frac{2}{x}\right](x-\beta)$$

We are looking for solutions with the form :

$$M_r(r,t) = \frac{a_T^3 t}{G} m(x)$$

$$\rho(r,t) = \frac{1}{4\pi G t^2}\alpha(x)$$

$$v(r,t) = a_T\beta(x)$$

**<u>An exact solution is the singular isothermal sphere</u>**, where we demonstrated that :

$$\mathrm{M}(\mathrm{r}_0) = \frac{2a_T^2 r_0}{G} = \frac{a_T^3 t}{G}\ 2x$$

then $m = 2x$, hence :

$$m = 2x = x^2\alpha(x-\beta)$$

or

$$\alpha = \frac{2}{x(x-\beta)}$$

and $\beta = \frac{v(r,t)}{a_T}$. In the singular isothermal sphere, the system is in equilibrium ($v(r,t) = 0$) then $\beta = 0$ and

$$\alpha = \frac{2}{x^2}$$

# Inside-out collapse

## SIMILARITY ANALYSIS

$$m = x^2\alpha(x-\beta)$$

$$[(x-\beta)^2-1]\frac{1}{\alpha}\frac{d\alpha}{dx} = \left[\alpha - \frac{2}{x}(x-\beta)\right](x-\beta)$$

$$[(x-\beta)^2-1]\frac{d\beta}{dx} = \left[\alpha(x-\beta) - \frac{2}{x}\right](x-\beta)$$

We are looking for solutions with the form :

$$M_r(r,t) = \frac{a_T^3 t}{G} m(x)$$

$$\rho(r,t) = \frac{1}{4\pi G t^2}\alpha(x)$$

$$v(r,t) = a_T\beta(x)$$

Another singular solution is given by :

$$x-\beta = 1$$
$$\alpha = \frac{2}{x}$$

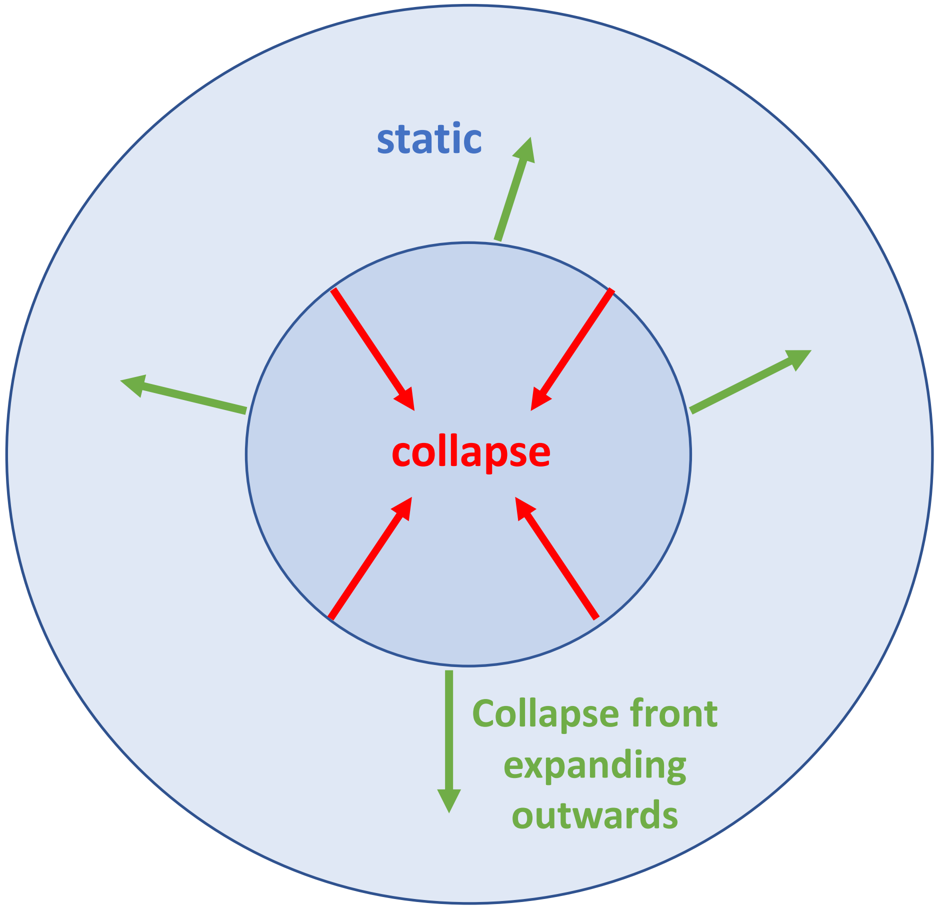
$x = 1$ is the crucial transition point :

- At $x > 1$: the solution is the singular isothermal sphere
- At $x < 1$ : then $\beta < 0$, hence $v(r,t) < 0$ ➔ **infall**

The transition critical point between infall and static isothermal solution $(x_c = 1)$ translates into $r_c = a_T t$

This is a wave moving outwards at the sound speed $a_T$

# Inside-out collapse



# Inside-out collapse

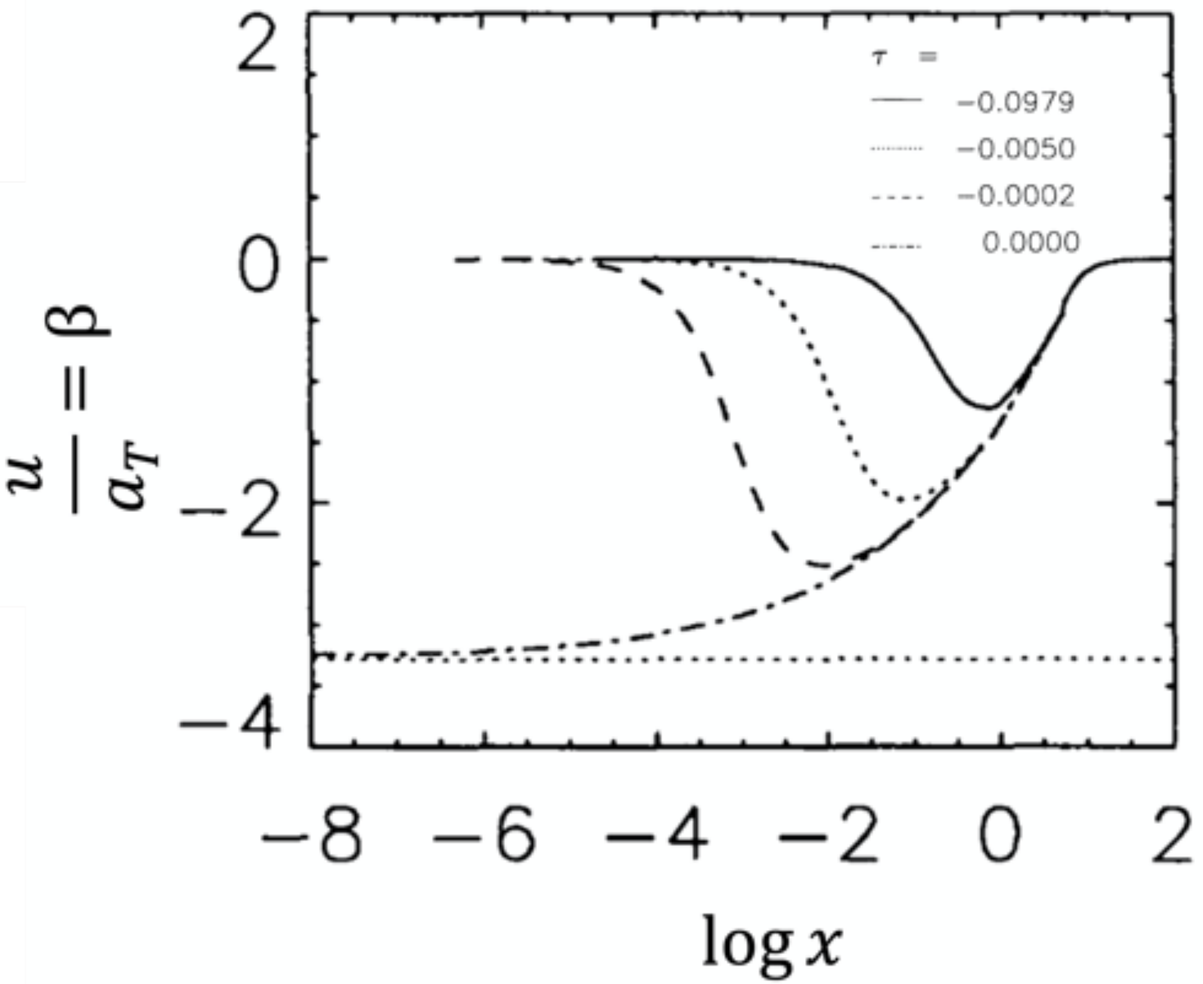
To do the previous analysis, we assumed that :

- The system is the equilibrium isothermal sphere
- The boundary conditions are those for that initial state
- There is a sink for matter reaching the origin : this will turn into a protostar

Another interesting case to consider is that of a cloud which is marginally unstable, for example with a mass slightly larger than the Bonnor-Ebert mass. We then perturbate the system and follow the evolution.

$\tau$=0 : start of the creation of the protostar as mass starts to flow into the sink

<u>Example of numerical solution</u> : *velocity during the collapse of an isothermal sphere with mass slightly above the Bonnor-Ebert mass*



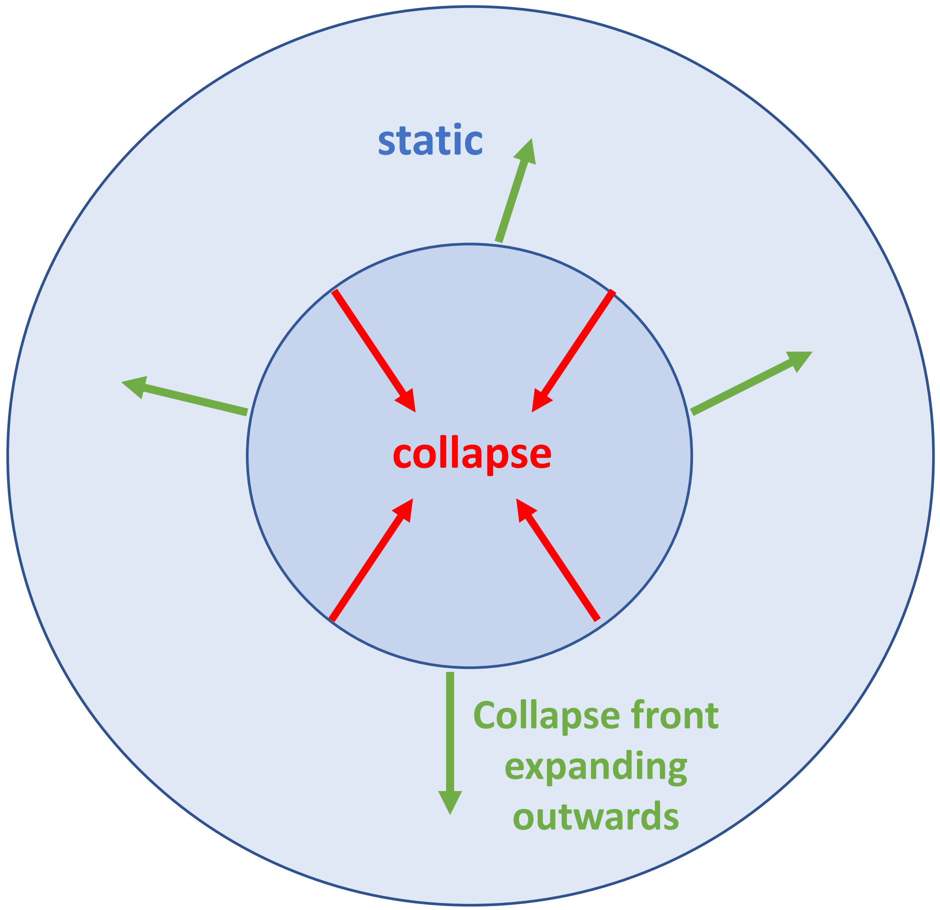

$$x = \frac{r}{a_T t}$$

# Physics Analysis

The transition point moves outwards as a rarefaction wave with only the gas inside of the radius $R_{ff} \approx a_T t$ moving inward.

After a short fraction of a free-fall time a large fraction of the gas within this radius is moving supersonically with the velocity increasing to the centre :

$r/|v|$ is less than the sound crossing time.




Gas is falling onto a growing central object, the protostar, with a mass $M_*$ :

- close to this protostar, gas is approximately in free fall

- $v_{ff} \approx \left(\frac{2GM_*}{r}\right)^{1/2}$

$$\frac{1}{2}v_{ff}^2 = \frac{GM}{r}$$

At the transition point, the gas moves approximately sonically :

- $v_{ff} \approx a_T$

$$r = a_T t$$

- $a_T^2 \sim M_* G/R_{ff}$

# Physics Analysis

The rate of growth of $M_*$ is determined by accretion at a rate :

$$\frac{dM}{dt} = \lim_{r\to 0} -4\pi r^2 v\rho$$

Assuming constant accretion rate $M_* = \frac{dM}{dt} t$ and

$$\frac{dM_*}{dt} \approx \frac{M_*}{t} \approx \frac{a_T^2}{G}\frac{R_{ff}}{t} \approx \frac{a_T^3}{G}$$

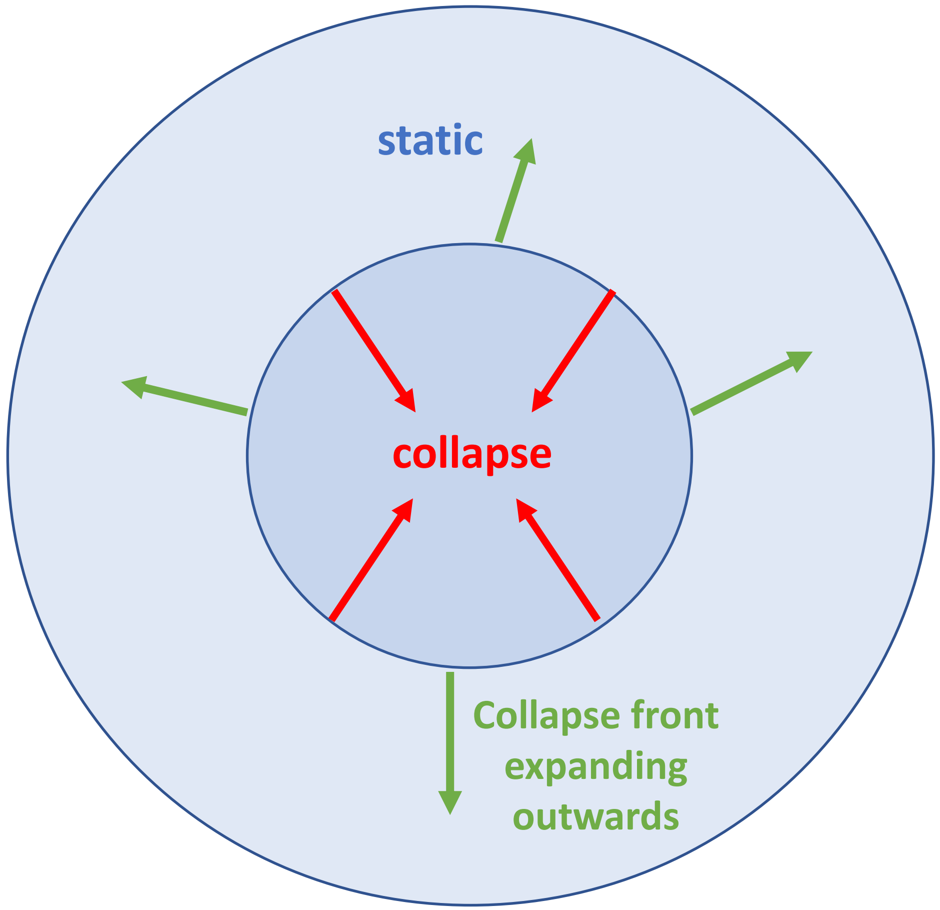
$a_T^2 \sim M_* G / R_{ff}$

$R_{ff} \approx a_T t$

Inserting values, the accretion rate for the growth of the protostar is :

$$\frac{dM_*}{dt} \approx 2\times 10^{-6} \left(\frac{T}{10K}\right)^{\frac{3}{2}} M_\odot \, yr^{-1}$$

$$a_T^2 = \frac{k_B T}{\mu}$$

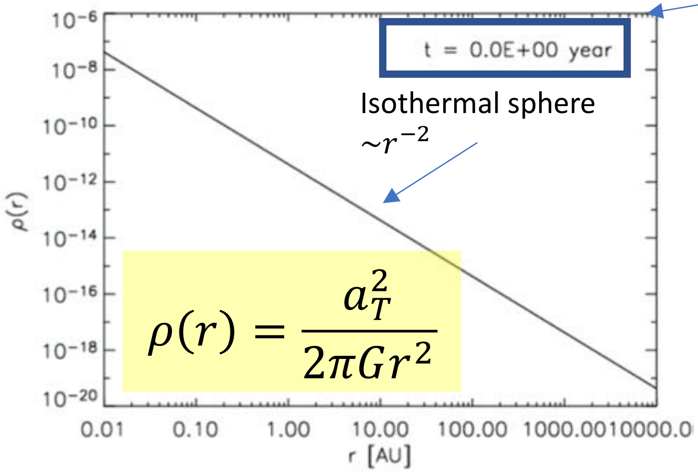



The density profile in the collapse region must satisfy :

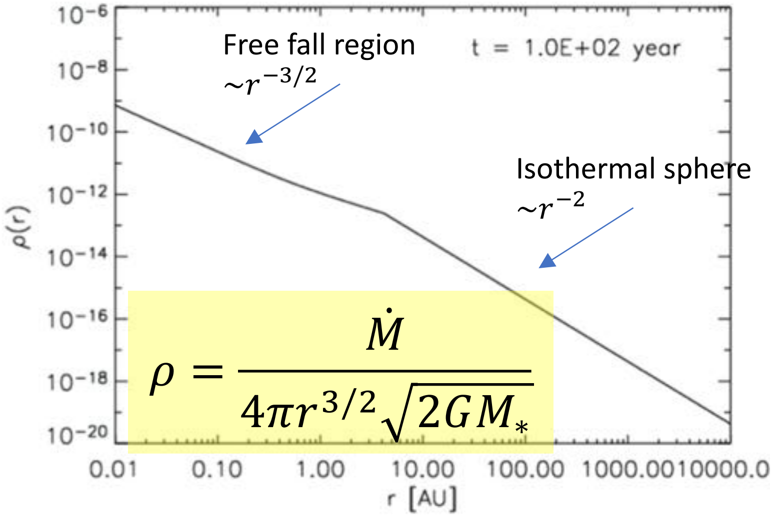
$$\rho = \frac{\dot{M}}{4\pi r^2 |v|} = \frac{\dot{M}}{4\pi r^2 v_{ff}} = \frac{\dot{M}}{4\pi r^{3/2}\sqrt{2GM_*}}$$

# Physics Analysis

The collapse starts at t=0




$$\rho(r) = \frac{a_T^2}{2\pi G r^2}$$

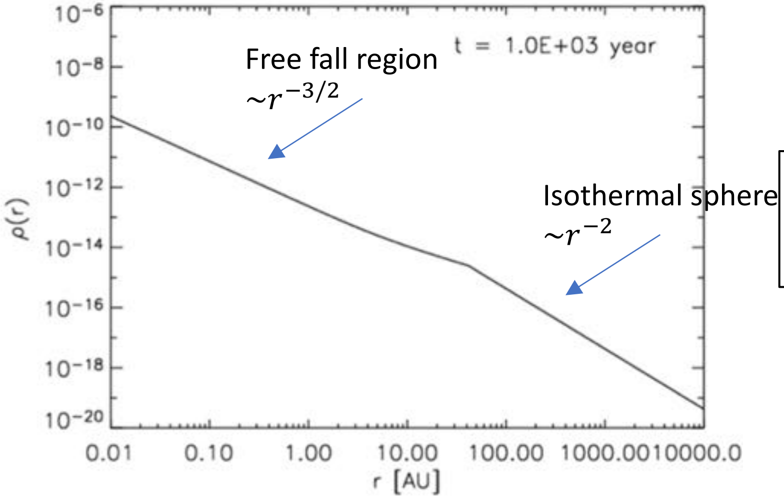



$$\rho = \frac{\dot{M}}{4\pi r^{3/2}\sqrt{2GM_*}}$$

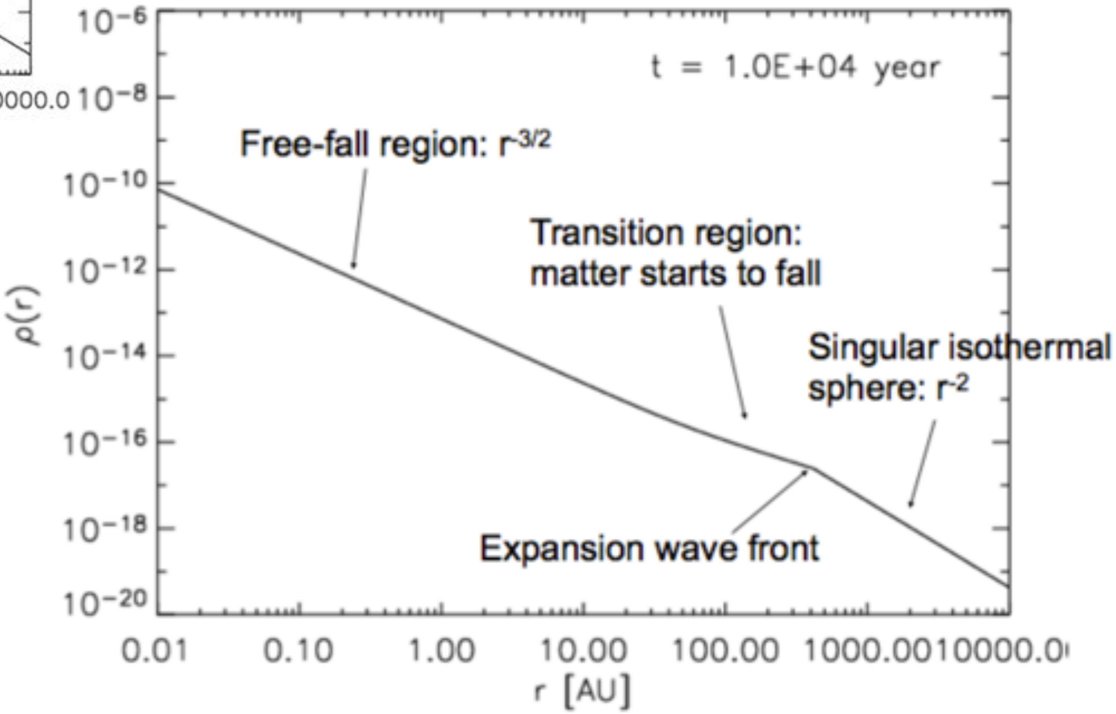
Since the Jean Mass is $M_j \propto \rho^{-1/2}$ inner regions have smaller Jeans Mass

➔ smaller regions will collapse into smaller sub-clumps

➔ **fragmentation**




**Evolution of the density profile with time**

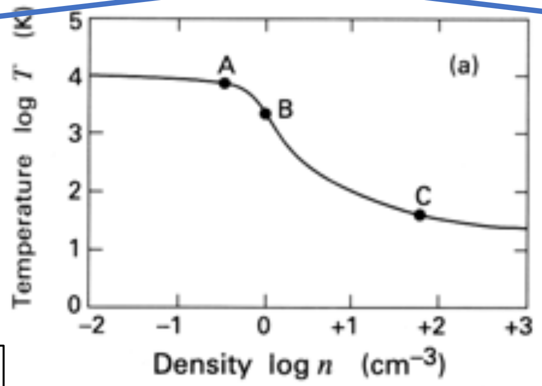


# Summary of the formation of structures in the Universe

**FROM THE FIRST LECTURE**

$$\frac{M_J}{M_\odot} = 1.0\times\left(\frac{T}{10K}\right)^{3/2}\times\left(\frac{n_H}{2\times10^{10}m^{-3}}\right)^{-1/2}$$

Gas clouds with multi-phases




dense regions

fast cooling ↓ collapse

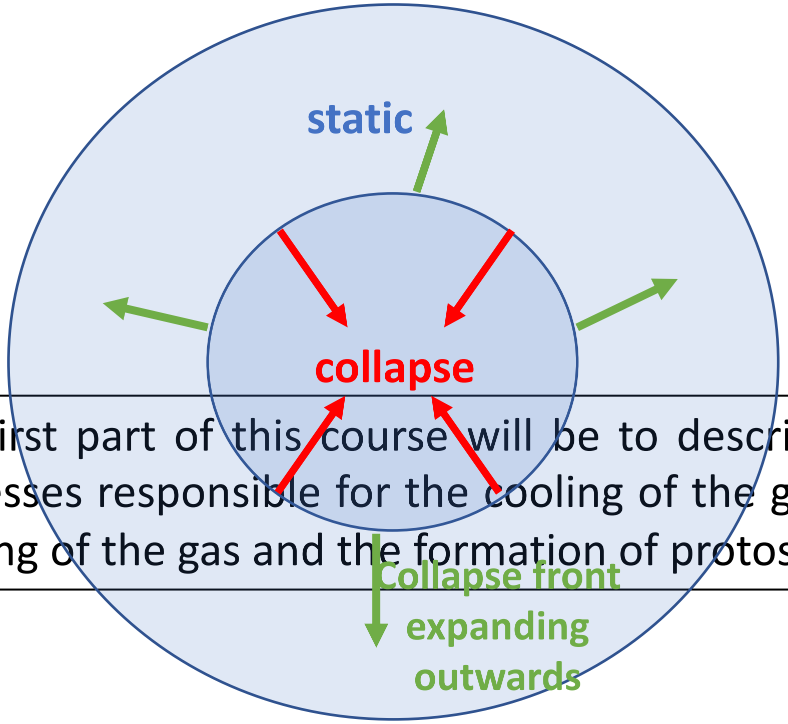
Gas clouds | Gas clouds | Gas clouds

$$t_{ff} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2}$$

Heating of collapsing gas

Protostar | Protostar | Protostar

Low-density regions




The first part of this course will be to describe the processes responsible for the cooling of the gas, the heating of the gas and the formation of protostars.

# Summary of Friday's lecture

We defined the free-fall time as the characteristic time that would take an object to collapse under its own gravity

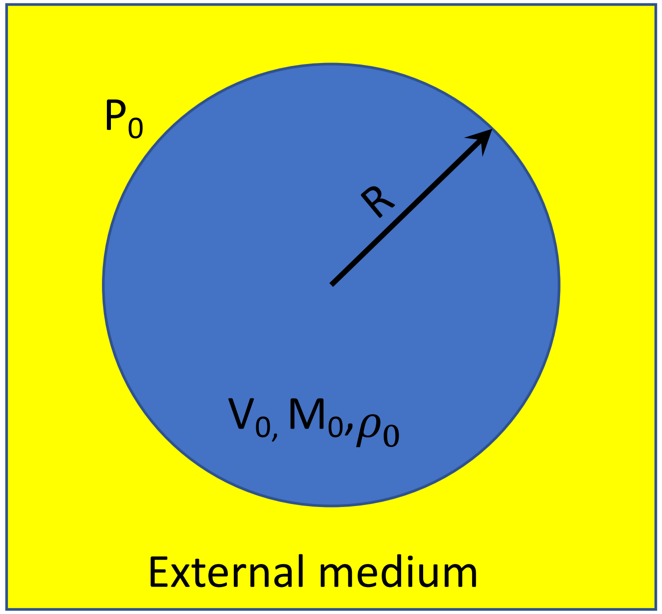
$$t_{ff} = \left(\frac{3\pi}{32 G \rho_0}\right)^{1/2}$$

*Only depends on the density, suggesting that denser region will collapse first*

➔ ***inside-out collapse***

$$\rho(r) = \frac{a_T^2}{2\pi G r^2}$$

# Summary of Friday's lecture



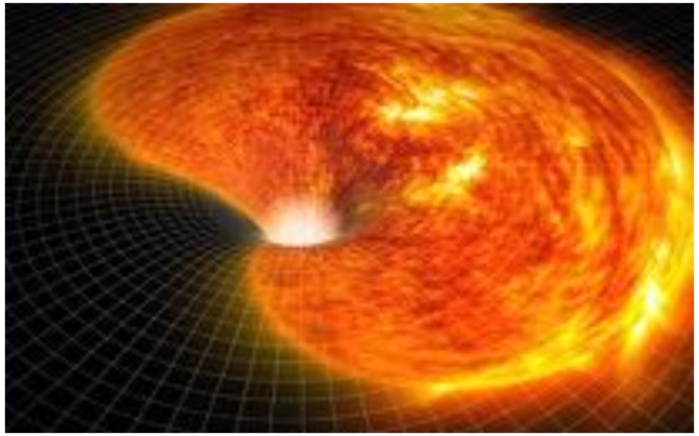

We studied the case of a collapsing gas cloud with a sink at the center for the inflowing material, and do a similarity analysis to solve the Euler's and continuity equations

$$\frac{dv}{dt} + v\frac{dv}{dr} = -\frac{a_T^2}{\rho}\frac{d\rho}{dr} - \frac{GM_r}{r^2}$$

$$\frac{d\rho}{dt} + \frac{1}{r^2}\frac{d(r^2\rho v)}{dr} = 0$$

Dimensionless variable

$$x = \frac{r}{a_T t}$$



We were looking for solutions with the form :

$$M_r(r,t) = \frac{a_T^3 t}{G} m(x)$$

$$\rho(r,t) = \frac{1}{4\pi G t^2}\alpha(x)$$

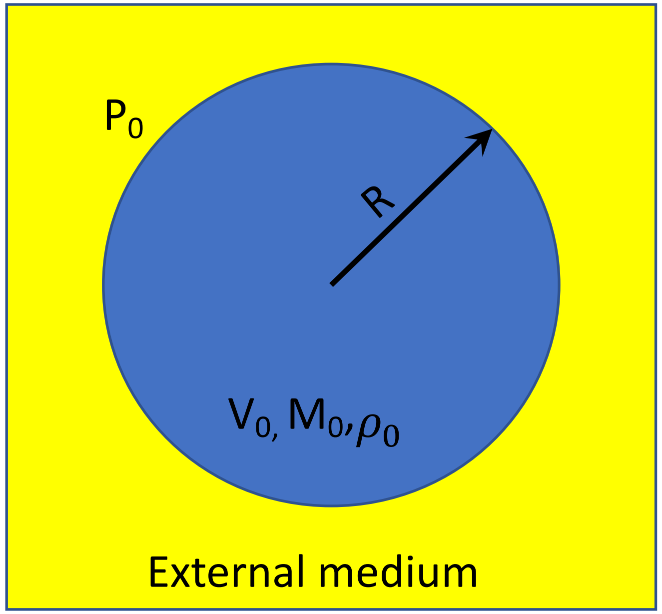
$$v(r,t) = a_T \beta(x)$$

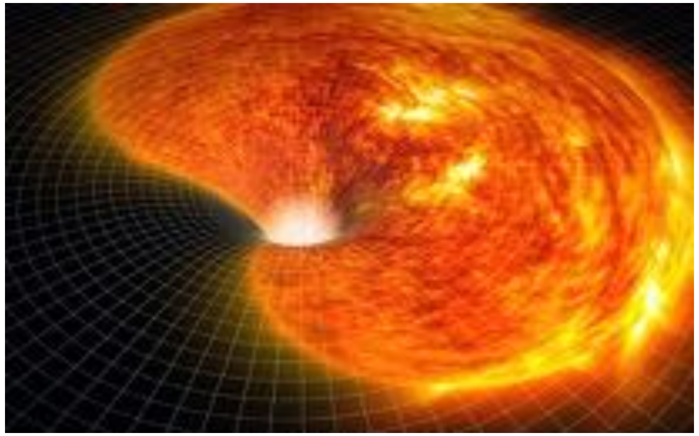
$$m = x^2\alpha(x-\beta)$$

$$[(x-\beta)^2 - 1]\frac{1}{\alpha}\frac{d\alpha}{dx} = \left[\alpha - \frac{2}{x}(x-\beta)\right](x-\beta)$$

$$[(x-\beta)^2 - 1]\frac{d\beta}{dx} = \left[\alpha(x-\beta) - \frac{2}{x}\right](x-\beta)$$

# Summary of Friday's lecture



One exact solution of previous solution is the isothermal sphere, for which we found :

$$\mathrm{M(r_0)} = \frac{2a_T^2 r_0}{G} = \frac{a_T^3 t}{G} 2x$$

$$M_r(r,t) = \frac{a_T^3 t}{G} m(x)$$

$$x = \frac{r}{a_T t}$$

$$m(x) = 2x$$

$$m(x) = x^2 \alpha(x - \beta)$$

$$\alpha = \frac{2}{x(x-\beta)}$$

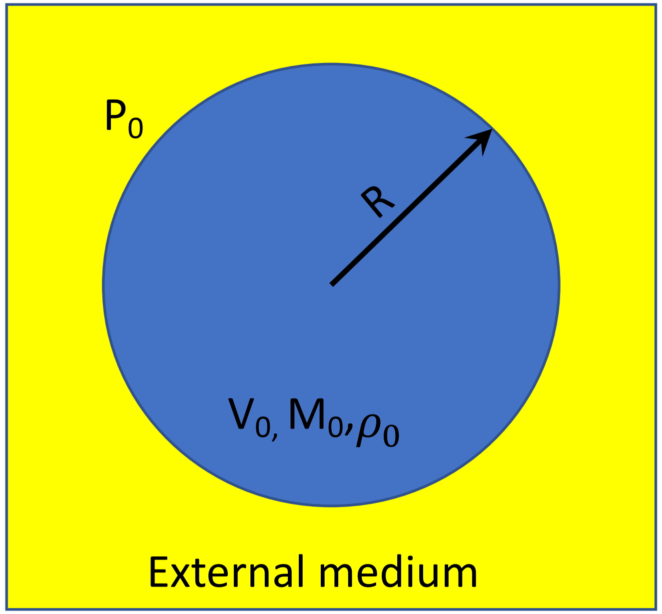
We also defined : $v(r,t) = a_T \beta(x)$

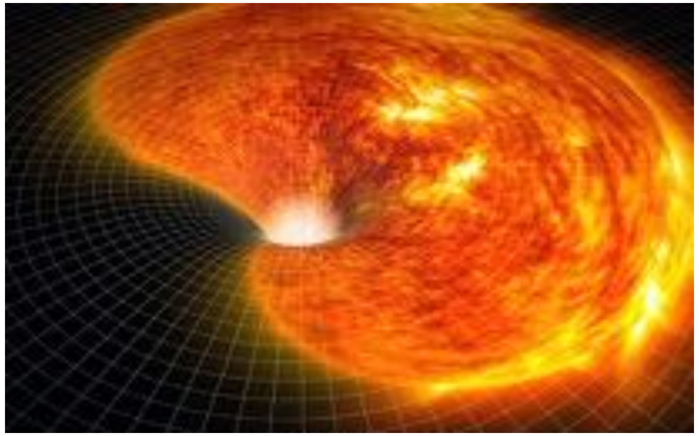
In the case of the singular isothermal sphere, the system is in equilibrium : $v(r,t) = 0 \rightarrow \beta = 0$

$$\alpha = \frac{2}{x^2}$$

$$\rho(r,t) = \frac{1}{2\pi G t^2 x^2} = \frac{a_T^2}{2\pi G r^2}$$

# Summary of Friday's lecture



$$m = x^2\alpha(x-\beta)$$

$$[(x-\beta)^2-1]\frac{1}{\alpha}\frac{d\alpha}{dx} = \left[\alpha - \frac{2}{x}(x-\beta)\right](x-\beta)$$

$$[(x-\beta)^2-1]\frac{d\beta}{dx} = \left[\alpha(x-\beta) - \frac{2}{x}\right](x-\beta)$$

Another singular solution is obtained when $x - \beta = 1$

$$x = \frac{r}{a_T t}$$

There is a transition point when $x = 1$ (i.e. when $r = a_T t$)

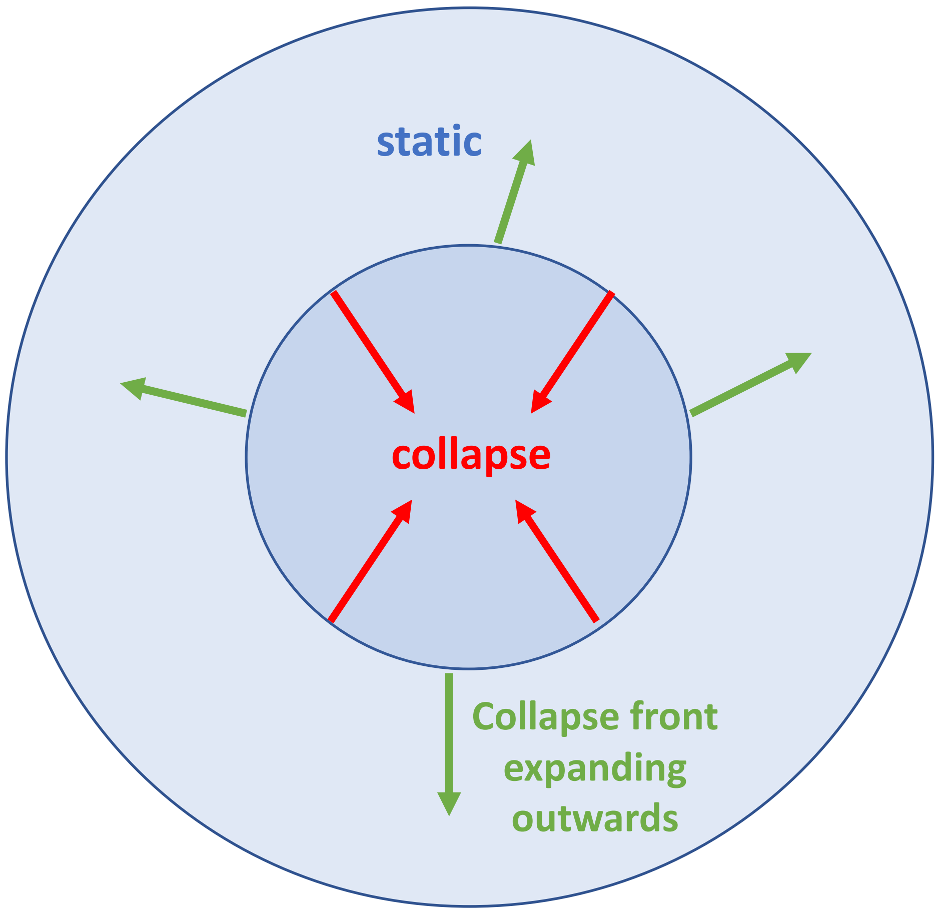
$x < 1$

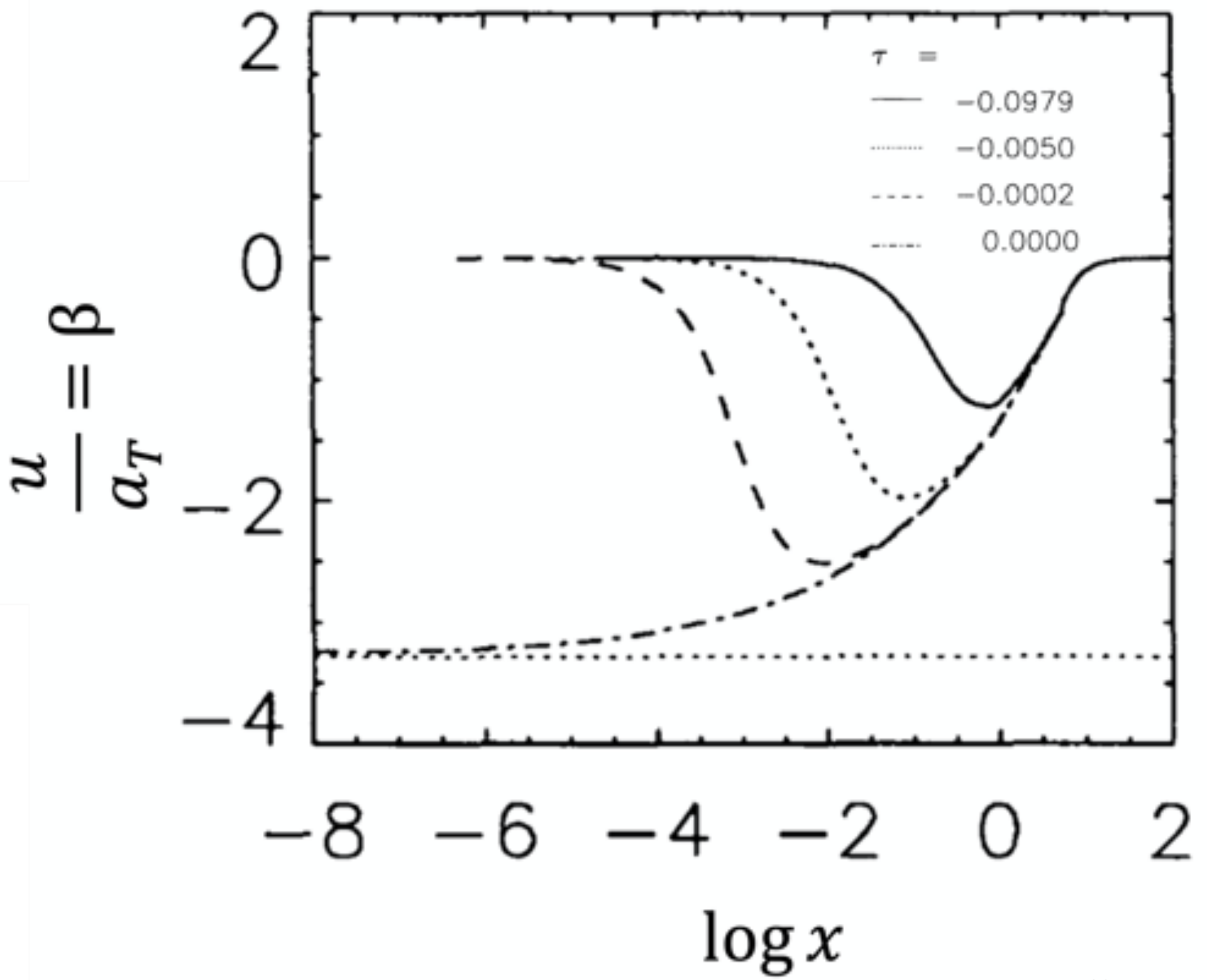
$a_T t > r$
then $\beta < 0$
=> Infall

$x > 1$

$a_T t < r$
then $\beta > 0$
=> Isothermal sphere

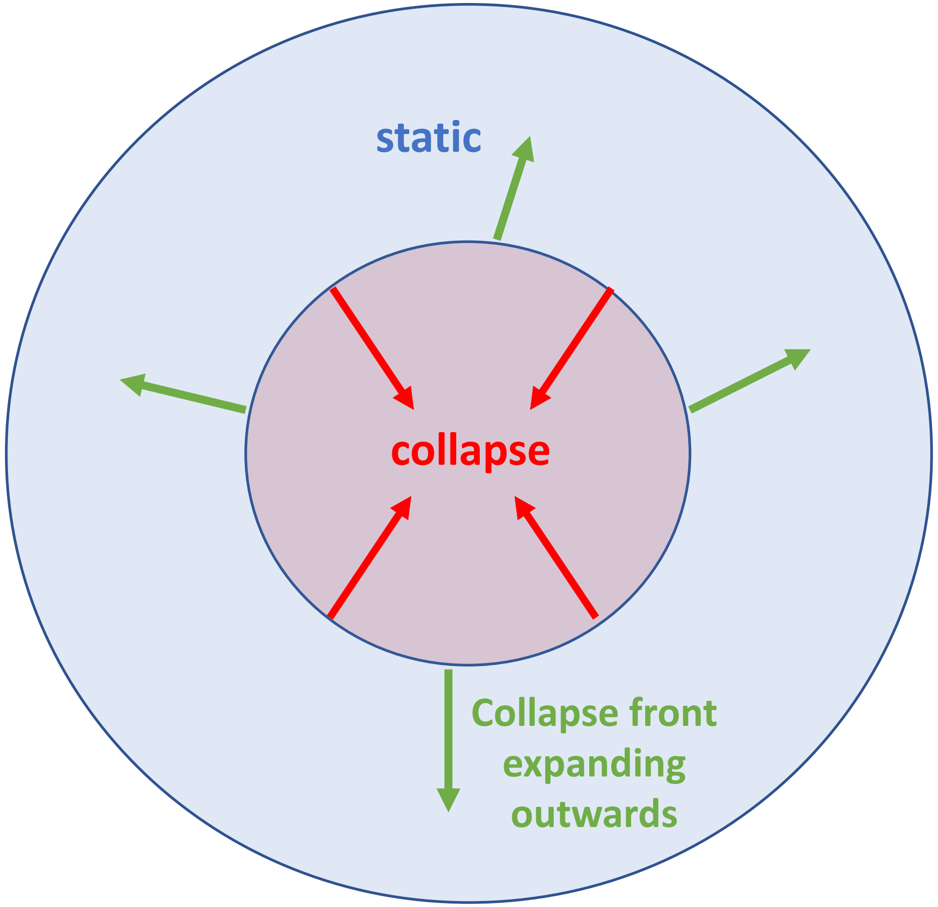
# Summary of Friday's lecture

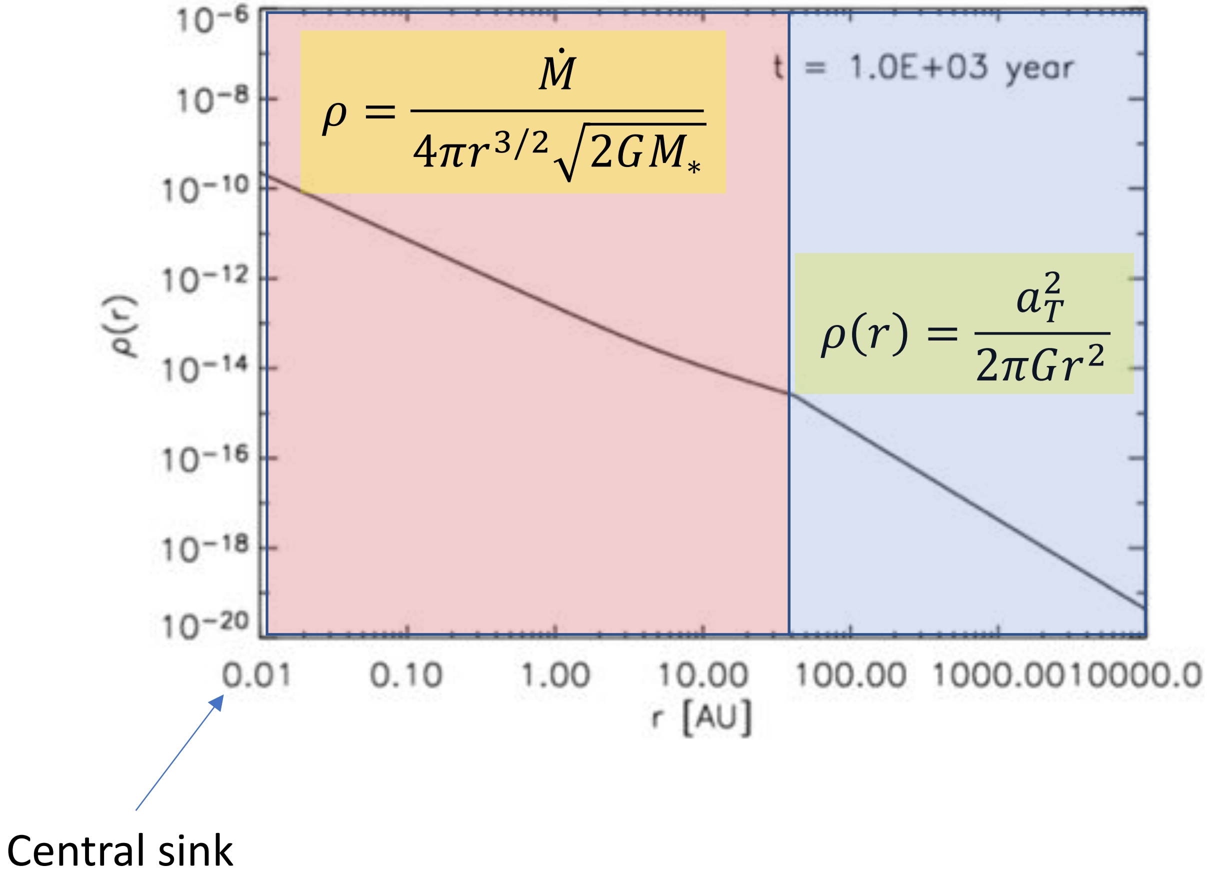



$$x = \frac{r}{a_T t}$$

# Summary of Friday's lecture




# Supervision

| GROUP 1 | | | GROUP 2 | | | GROUP 3 | | |
|---|---|---|---|---|---|---|---|---|
| 13/02 14h-15h | 27/02 14h-15h | 13/03 14h-15h | 14/02 14h-15h | 28/02 14h-15h | 14/03 15h-16h | 15/02 14h-15h | 01/03 14h-15h | 15/03 16h-17h |
| SPO Meeting Room (Kavli) | SPO Meeting Room (Kavli) | SPO Meeting Room (Kavli) | SPO Meeting Room (Kavli) | SPO Meeting Room (Kavli) | SPO Meeting Room (Kavli) | SPO Meeting Room (Kavli) | SPO Meeting Room (Kavli) | SPO Meeting Room (Kavli) |
| Stephanie Buttigieg | | | Oscar Windrath-Carr | | | Fred Thompson | | |
| Stefan Ivanov | | | Nachiket Girish | | | Tingjing Xing | | |
| Marco Loncar | | | Camille Fontaine | | | Rhys Bennett | | |
| Magnus Handley | | | Daniil Lozner | | | Zixiao Hu | | |
| Susanna Power | | | Gwilym Price | | | Amanda Stoffers | | |
| Aidan Joyce | | | Hanyuan Zhang | | | Robson Tebbutt | | |
| Elena Cates | | | Archie Herrod | | | Cole Meldorf | | |
| | | | | | | Aria Bamllari | | |
| | | | | | | Daniel Gibson | | |

# From gas cloud to collapsed object

Chapter 5

# Basic physics of object formation

Cooling time

If $\tau_c \ll t_{collapse}$ $\rightarrow$ the system will evolve isothermally

The shortest collapse time is the free-fall time (*by definition*).

If $\tau_c \ll t_{ff}$ $\rightarrow$ the cloud will collapse isothermally

The Jeans Mass is given by :

$$\frac{M_J}{M_\odot} = 1.0 \times \left(\frac{T}{10K}\right)^{3/2} \times \left(\frac{n_H}{2\times10^{10} m^{-3}}\right)^{-1/2}$$

then if the cooling is efficient, the Jeans Mass decreases and, as we see in the previous chapter, the density increases at small R.

This results in the collapse of smaller region of the cloud in a process called fragmentation.

Consequence of an increase in density :

- The optical depth : $\tau \propto \rho^{2/3}$

$$\tau = \alpha R = \rho \kappa R$$

Mass absorption coefficient

$R \propto \rho^{1/3}$

- The collapsed object (the first core) becomes optically thick ($\tau > 1$) making the cooling inefficient.
- The collapse will continue, and the temperature of the core rises as the collapse proceeds adiabatically.

# Evolution of the first core

The first core is mainly composed.

We have demonstrated that the virial equation for an isothermal sphere is :

$$3\frac{k_B T}{\mu}M_0 - \frac{3}{5}\frac{GM_0^2}{r_0} - 4\pi r_0^3 P_0 = 0$$

Neglecting the external pressure, gives :

$$T_{FC} \approx \frac{\mu}{5k_B}\frac{GM}{R} \approx 850\left(\frac{M}{5\times10^{-2}M_\odot}\right)\left(\frac{R}{5AU}\right)^{-1} K$$

Radius of the First core

and the accretion rate to this core is (demonstrated in the last chapter) :

$$\frac{dM_*}{dt} \approx 2\times10^{-6}\left(\frac{T_{cloud}}{10K}\right)^{\frac{3}{2}} M_\odot\, yr^{-1}$$

Further mass addition to this first core will lead to the increase of temperature.

collisional dissociation of $H_2$ begins, part of the potential energy goes into $H_2$ dissociation not into temperature increase.

As soon as the dissociation of $H_2$ begins, the rise in temperature happens with a slower rate.

The central core has a significant density gradient and eventually undergoes collapse to form a very dense central core.

# Structure around the protostar

The collapse of the first core gives a protostar with the following properties :

- $M \sim 0.1\, M_\odot$
- $R \sim 5 R_\odot$
- $\mathrm{T} \sim 10^5 K$
- $\rho \sim 10^{-2} g\ cm^{-3}$

**ACCRETION LUMINOSITY**

The energy balance of the protostar is given by :

$$U_{gr} + U_{th} + E_p + L_r t = 0$$

Gravitational potential

Internal energy

Potential energy

Energy radiated over the formation time of the protostar

With :

- $U_{gr} \approx -GM_*^2/R_*$
- $U_{th} \approx -U_{gr}/2$
- $E_p = \frac{XM_*}{m_H}\left[\frac{E_d(H)}{2} + E_i(H)\right] + \frac{YM_* E_i(He)}{4m_H}$

Dissociation energy of $H_2$ (4.2eV)

Fraction of Helium

Fraction of Hydrogen

Ionisation potential of Hydrogen (13.6 eV)

Ionisation potential of Helium (75 eV)

*Do we really need to include the energy radiated ?* If $L_r = 0$, then :

$$U_{gr} - \frac{1}{2}U_{gr} + E_p = 0$$

$$-\frac{1}{2}U_{gr} = E_p$$

Hence

$$R_{max} = \frac{GM_*^2}{2E_p} = 60\left(\frac{M_*}{M_\odot}\right)R_\odot$$

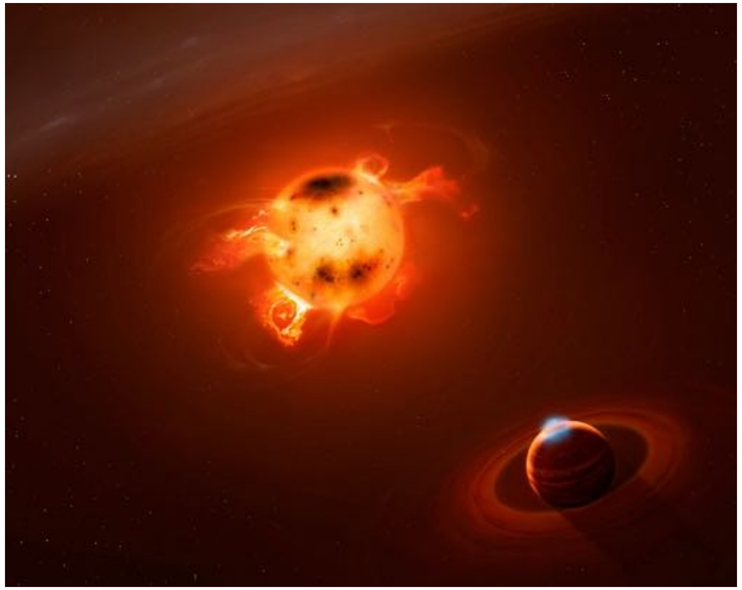
which is much larger than what is observed for T Tauri stars ($\approx 8M_\odot$) .

# Structure around the protostar

From the previous discussion, it implies that $L_r \neq 0$.

Indeed, $L_r$ must be close to the accretion luminosity :

$$L_r \approx L_{acc} = \frac{GM_*\dot{M}}{R_*} = 61\left(\frac{\dot{M}}{5\times10^{-5}M_\odot yr^{-1}}\right)\left(\frac{M_*}{M_\odot}\right)\left(\frac{R_*}{5R_\odot}\right)$$



**Artist view of a T Tauri star**

# Structure around the protostar

## ACCRETION SHOCK AND DUST ENVELOPPE

The accreting gas is infalling with a velocity close to the free-fall velocity :

$$v_{ff} \approx \left(\frac{2GM_*}{r}\right)^{\frac{1}{2}} = 280\left(\frac{M_*}{M_\odot}\right)^{\frac{1}{2}}\left(\frac{R_*}{5R_\odot}\right)^{-\frac{1}{2}} km/s$$

This is much larger than the sound speed in the gas

A strong shock forms with $v_{ss} \approx v_{ff}$, giving a post-shock temperature of $10^6$K

The whole region is optically thick with an effective temperature such that :

$$L_{acc} \approx 4\pi R_*^2 \sigma_B T_{eff}^4$$

$$L_{acc} = \frac{GM_*\dot{M}}{R_*}$$

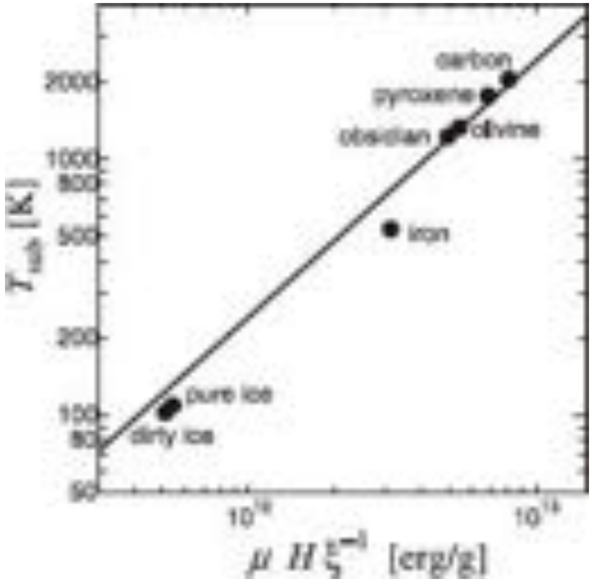
Or :

$$T \approx \left(\frac{GM_*\dot{M}}{4\pi\sigma_B R_*^3}\right)^{\frac{1}{4}} = 7300\left(\frac{\dot{M}}{5\times10^{-5}M_\odot yr^{-1}}\right)^{-\frac{1}{4}}\left(\frac{M_*}{M_\odot}\right)^{\frac{1}{4}}\left(\frac{R_*}{5R_\odot}\right)^{-3/4} K$$

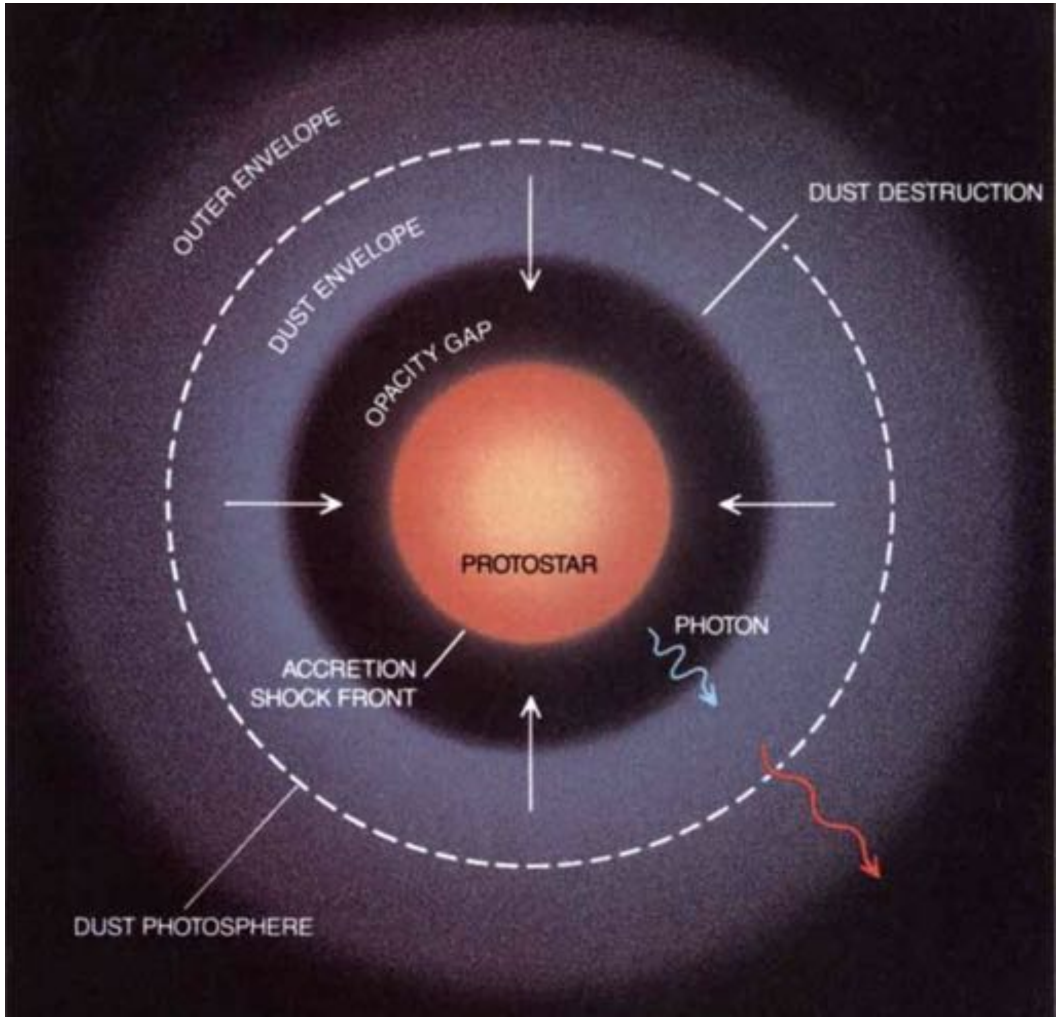
At this temperature, UV and X-Rays photons are produced

$$\lambda_{max} = \frac{b}{T}$$

*Wien's displacement law*



# Structure around the protostar




Surrounding the protostar, the gaseous envelope also contains dust.

The UV flux produced by the young protostar can vaporize dust grains within a region called the ***opacity gap*** out to a radius known as the dust destruction front.

Outside of this radius, the dust absorbs the radiation and re-radiates. In the dusty layer, the dust temperature must drop until the layer becomes optically thin to re-emission of infrared radiation from the dust. This occurs at (dust photosphere):

$$R_{phot} = \frac{1}{\rho\kappa}$$

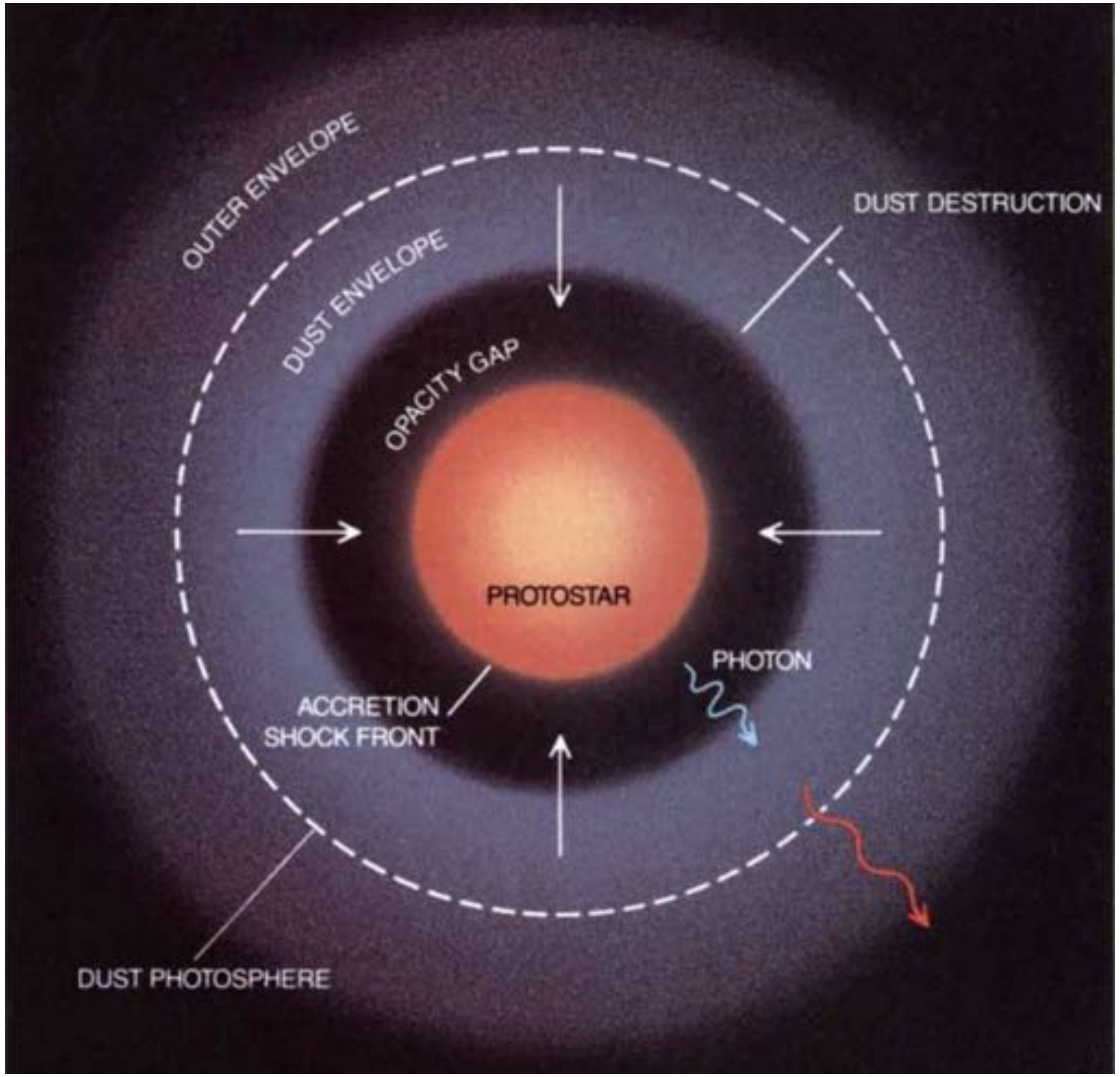
Limit : $\tau = 1$

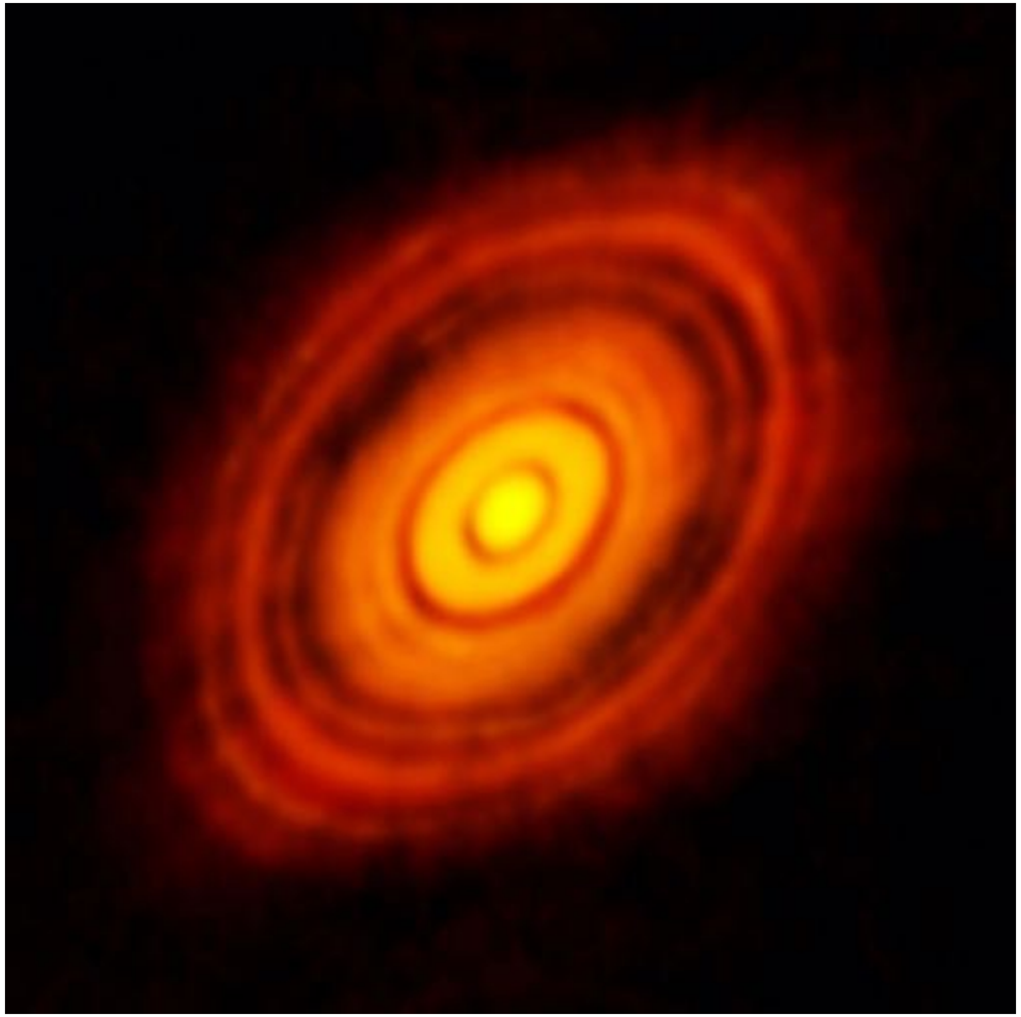
$\tau = \alpha R$

$\alpha = \rho\kappa$

*Mass Absorption Coefficient*

# Structure around the protostar



**ALMA observation of HL Tauri**
**(distance : 450 light-years)**

# Structure around the protostar

# Evolution of the protostar

## PROTOSTELLAR STRUCTURE DURING ACCRETION

The structure of the protostar will be governed by the equations of hydrostatic equilibrium plus the equations describing the thermal structure (cf. last year course).

The equations governing the evolution are :

$$-\frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{\partial \Phi_g}{\partial r} = 0 \qquad \text{Euler's equation}$$

And the definition of mass :

$$\frac{\partial M}{\partial r} = 4\pi r^2 \rho$$

Also

$$P = \frac{\rho k_B T}{\mu} \qquad \text{Equation of state}$$

Applying the first law of thermodynamics gives :

$$\rho T \frac{\partial S}{\partial t} = \rho\epsilon - \vec{\nabla}.\vec{F}$$

where F is the radiative flux defined as :

$$F = \frac{L}{4\pi r^2}$$

Hence :

$$\frac{\partial L}{\partial r} = 4\pi r^2 \rho\epsilon - 4\pi r^2 \rho T \frac{\partial S}{\partial t}$$

If the heat is transported by radiative diffusion we also have :

$$\frac{L}{4\pi r^2} = -\frac{4acT^3}{3\rho\kappa}\frac{\partial T}{\partial r}$$

These equations can not be solved analytically, and we must rely on numerical simulations to guide our understanding of the physics.

# Evolution of the protostar

The boundary conditions of previous equations are :

$$\rho = \frac{\dot{M}}{4\pi r^{3/2}\sqrt{2GM_*}}$$

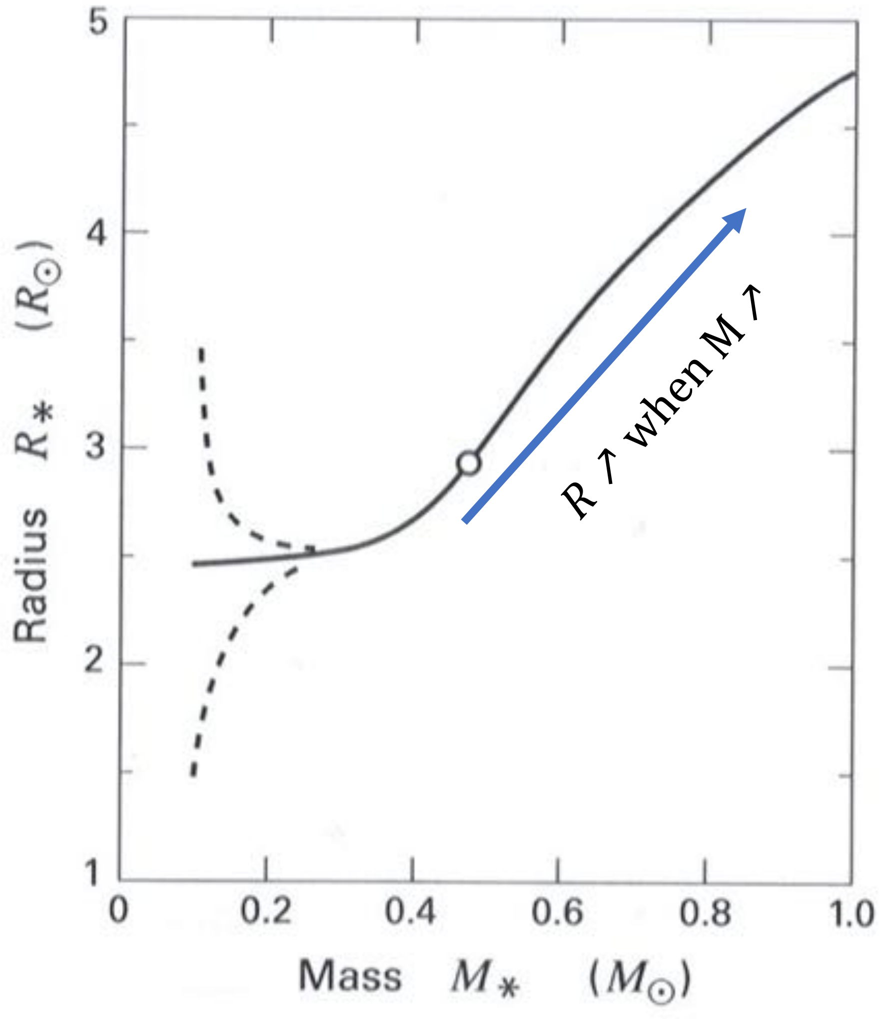
$$v_{ff} \approx \left(\frac{2GM_*}{r}\right)^{1/2}$$

- $M(R \to 0) = 0$
- $L(R \to 0) = 0$
- $L_{surface} = L_* - L_{acc}$
- The surface pressure must balance the momentum flux, or ram pressure of the infalling gas which is $\sim \rho v_{ff}^2$ giving :

$$P(r_0) = \frac{\dot{M}}{4\pi}\left(\frac{2GM_*}{R_*^5}\right)^{1/2}$$

Numerical integration gives for the accretion rate $\dot{M} = 1{\times}10^{-5} M_\odot/yr^{-1}$

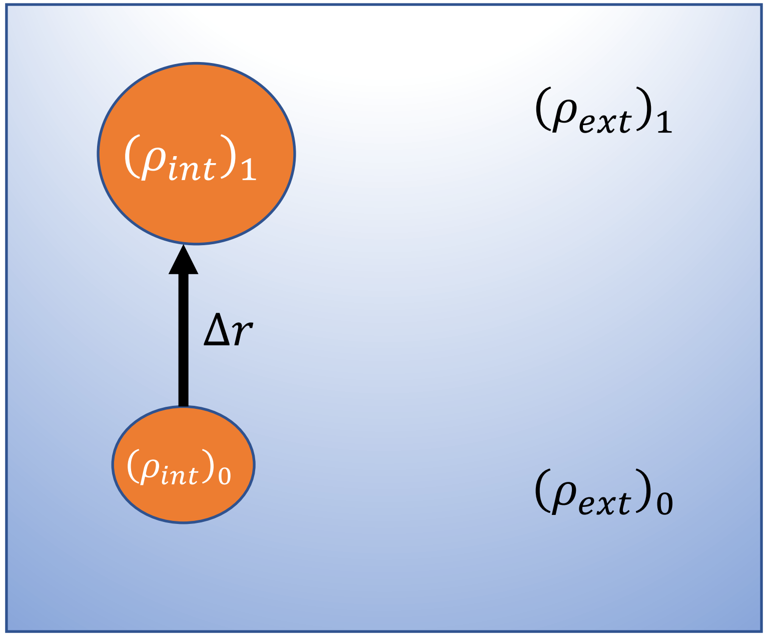
- If initially $R_*$ is large, the entropy of the gas added is low and the protostar shrinks under gravity. The converse is true is $R_*$ is small
- The protostar is characterized by its entropy profile S(r)
- The results show that $\frac{M_*}{R_*}$ is an increasing function in the early stages which gives an increasing entropy distribution with radius.



# Evolution of the protostar

## ONSET OF DEUTERIUM BURNING AND CONVECTION

Consider a fluid element which moves a small distance through the atmosphere $\Delta r$ so that it remains in pressure balances with the surrounding gas.




- The element expands adiabatically to a lower density ($\rho_{int}^1 < \rho_{int}^0$)
- For stability, the density of this element must be greater than the density of the surrounding gas
- If the entropy is increasing with radius, the element is of lower entropy than the surrounding
- For an ideal gas :

$$S = c_v \log\left(\frac{P}{\rho^\gamma}\right)$$

where $\gamma = c_p/c_v$, is the heat capacity ratio.

Then if the density increases, the entropy decreases.

The atmosphere is convectively stable if :

$$\frac{dS}{dr} > 0$$

# Summary of Monday's lecture

When approaching the central sink = the first core, the density increases (see the density profile), then :

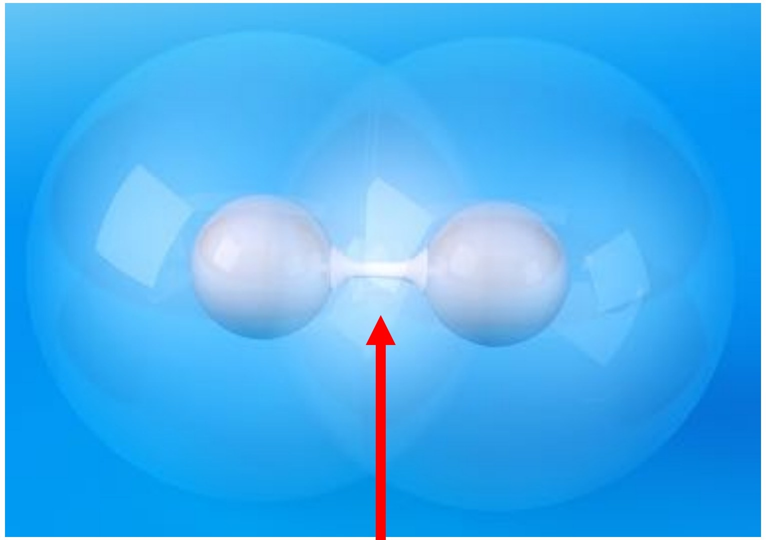
$$\tau = \alpha R = \rho \kappa R \propto \rho^{2/3}$$

The optical depth increases

Photons can not escape the optically thin medium
➔ ***cooling becomes inefficient***

The temperature of the first core increases

The temperature becomes sufficient to start the dissociation of $H_2$



Dissociation energy : 4.2 eV

As soon as the $H_2$ dissociation begins, the rise in temperature happens at a slower rate (most of the energy goes into the $H_2$ dissociation)

# Summary of Monday's lecture

The energy balance of the protostar is given by :

$$U_{gr} + U_{th} + E_p + L_r t = 0$$

Internal energy

Gravitational potential

Potential energy

Energy radiated over the formation time of the protostar

The collapse of the first core gives a protostar with the following properties :

- $M \sim 0.1\ M_\odot$
- $R \sim 5R_\odot$
- $\mathrm{T} \sim 10^5 K$
- $\rho \sim 10^{-2} g\ cm^{-3}$

Close to the first core, the accretion luminosity gives a "surface" temperature of :

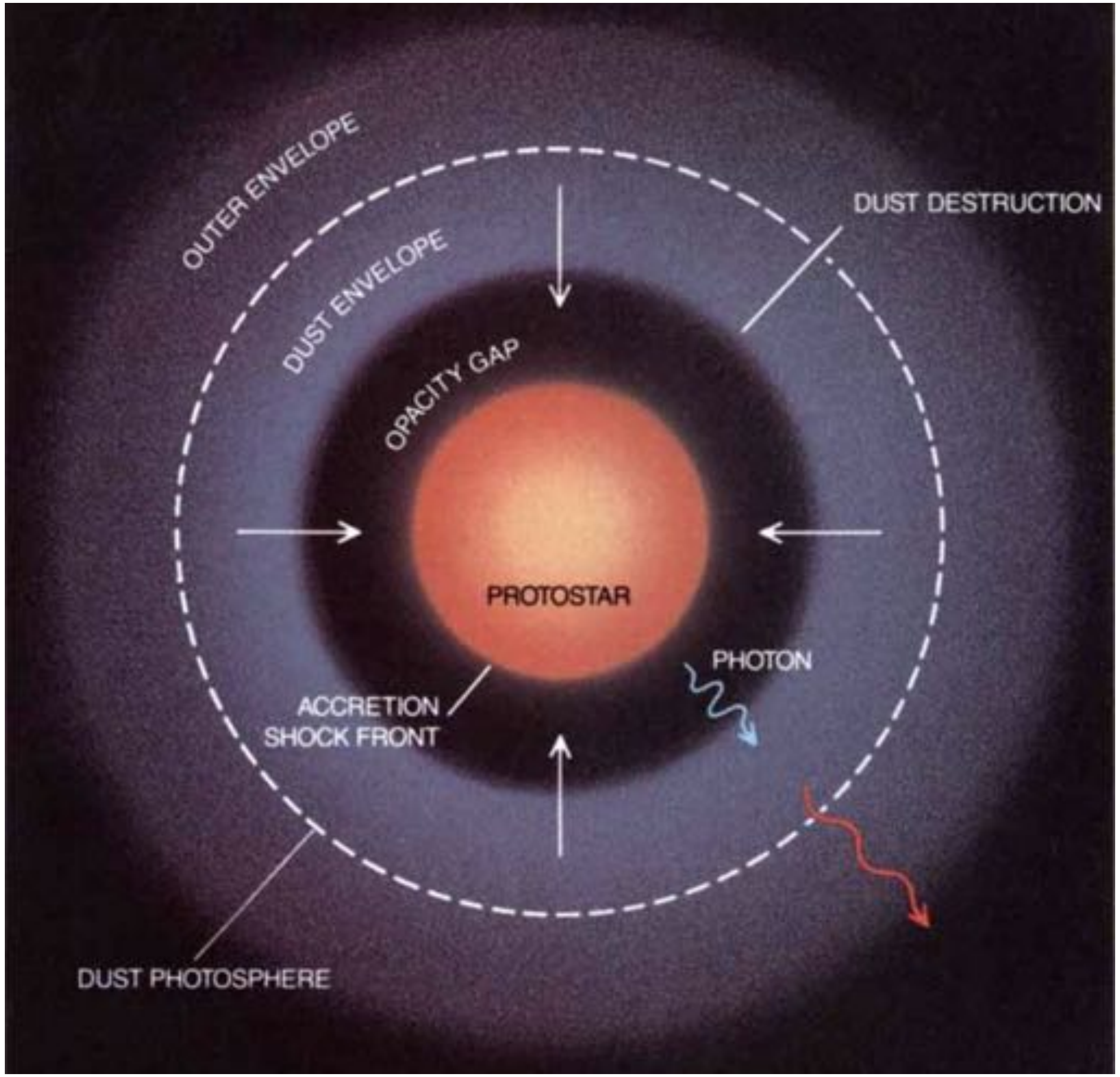
$$T \approx \left(\frac{GM_*\dot{M}}{4\pi\sigma_B R_*^3}\right)^{\frac{1}{4}} = 7300 \left(\frac{\dot{M}}{5\times10^{-5} M_\odot yr^{-1}}\right)^{-\frac{1}{4}} \left(\frac{M_*}{M_\odot}\right)^{\frac{1}{4}} \left(\frac{R_*}{5R_\odot}\right)^{-3/4} K$$

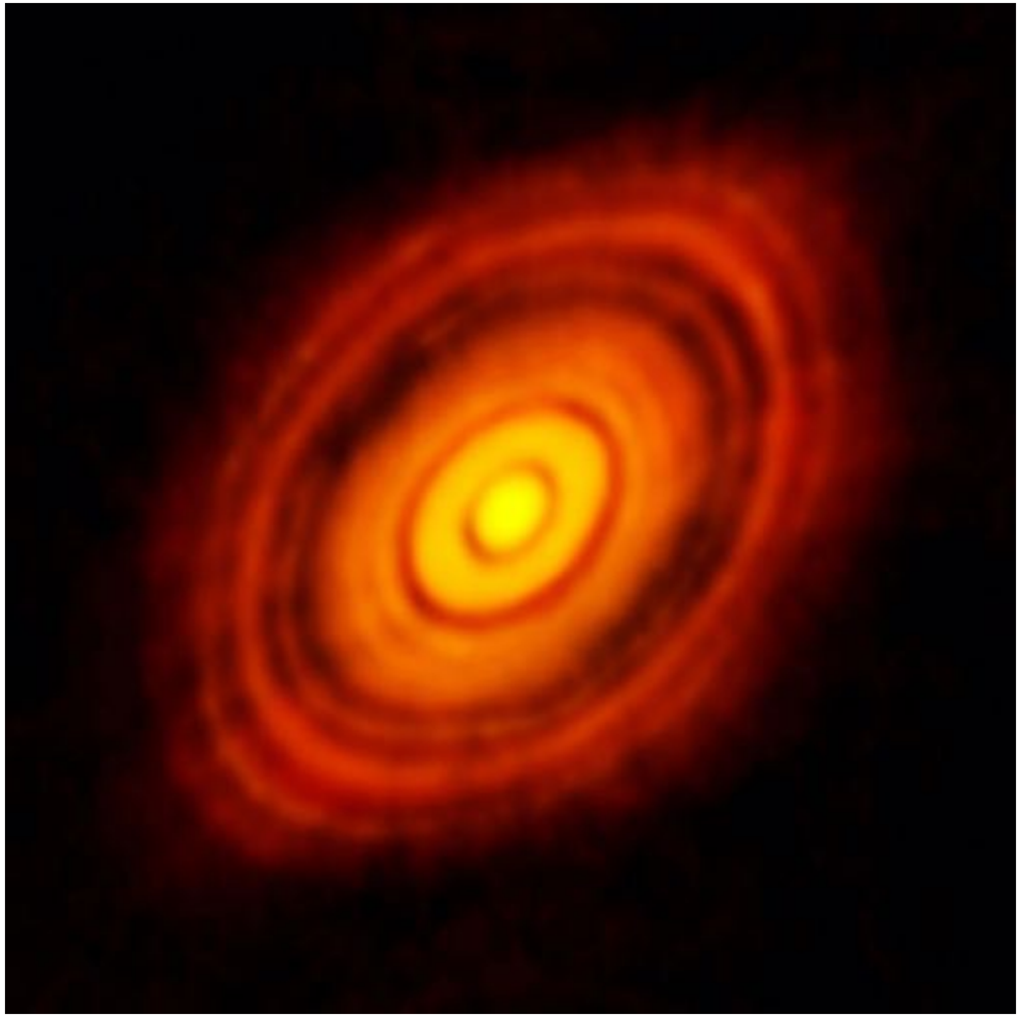
This temperature is sufficient to vaporize the dust grains surrounding the protostar ➔ **opacity gap** !

We defined the photosphere radius as the radius at which light re-emitted by dust grain can escape :

$$R_{phot} = \frac{1}{\rho\kappa}$$

# Summary of Friday's lecture



**ALMA observation of HL Tauri**
**(distance : 450 light-years)**

# Evolution of the protostar

At a temperature of about $T \sim 10^6$K, the first nuclear fuel to ignite is the deuterium following :

$$^2H + \ ^1H \rightarrow \ ^3He + \gamma$$

which releases 5.5 MeV.

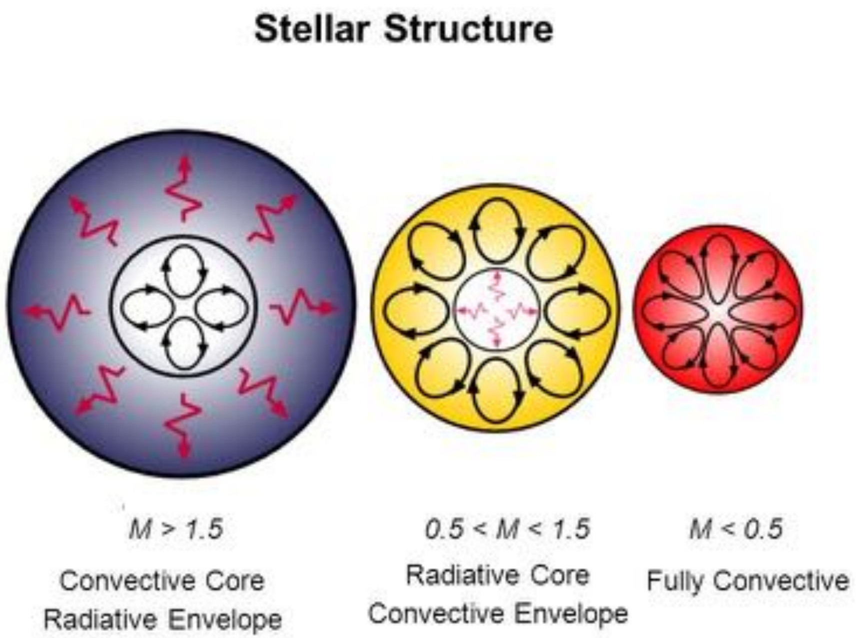
The heating rate due to this process is very dependent on temperature with $\epsilon_D \propto T^{11.8}$:

- The protostar is unable to effectively transport the large luminosity produced in the core via radiative transport
- The core heats up, reversing the entropy gradient and the protostar becomes convective

We can calculate the maximum energy flux which can be carried by pure radiative flux. This occurs when the entropy is constant. From the equation of protostellar structure we get :

$$L_{crit} = 4\pi r^2 \frac{4acT^3}{3\rho\kappa}\left(\frac{\mathrm{dT}}{dr}\right)_s = -\frac{GM16\pi acT^3}{3\rho\kappa}\left(\frac{dT}{dP}\right)_s$$

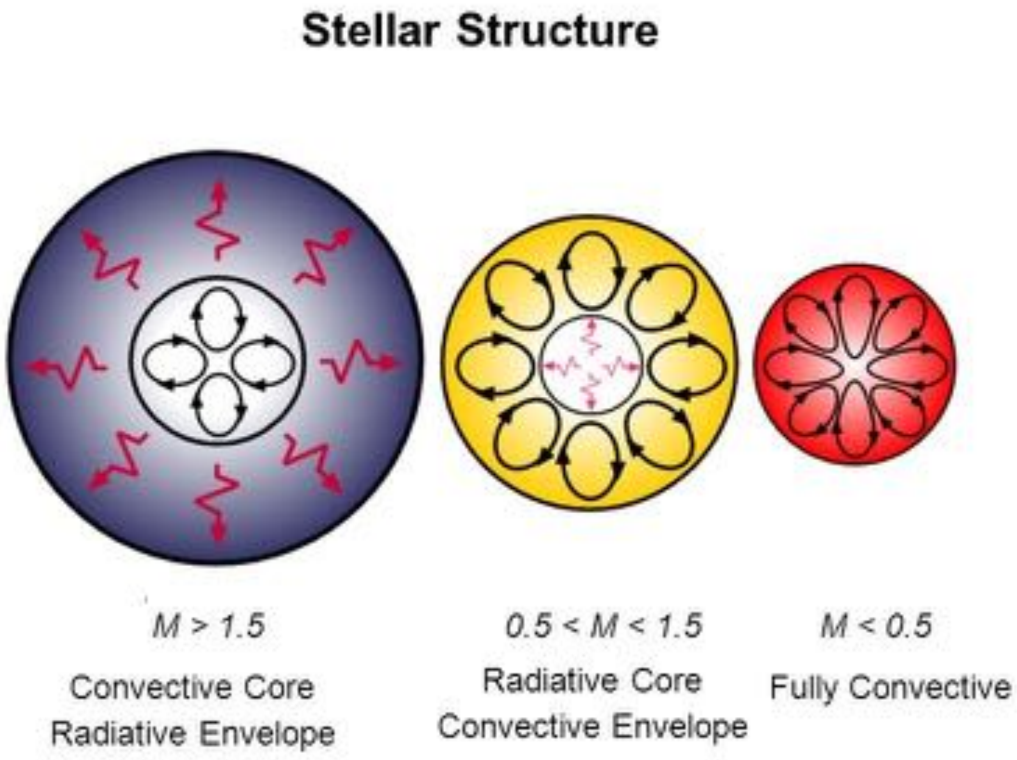
$$\frac{L}{4\pi r^2} = -\frac{4acT^3}{3\rho\kappa}\frac{\partial T}{\partial r}$$



# Evolution of the protostar

- Although the amount of deuterium is small, convection helps to bring new fuel to the core from the accreting gas
- This deuterium burning phase acts as a thermostat – the ***deuterium thermostat***
- Any rise in $M_*/R_*$ increases the stellar entropy which, via convection, increases $T_c$; this leads to a substantial increase in $\epsilon_D$ which inflates the star, reducing $M_*/R_*$

As the protostar mass continues to grow via accretion, the energy production from the deuterium burning remains approximately constant and determined by the rate of supply of new fuel from the accreting gas.



# Evolution of the protostar

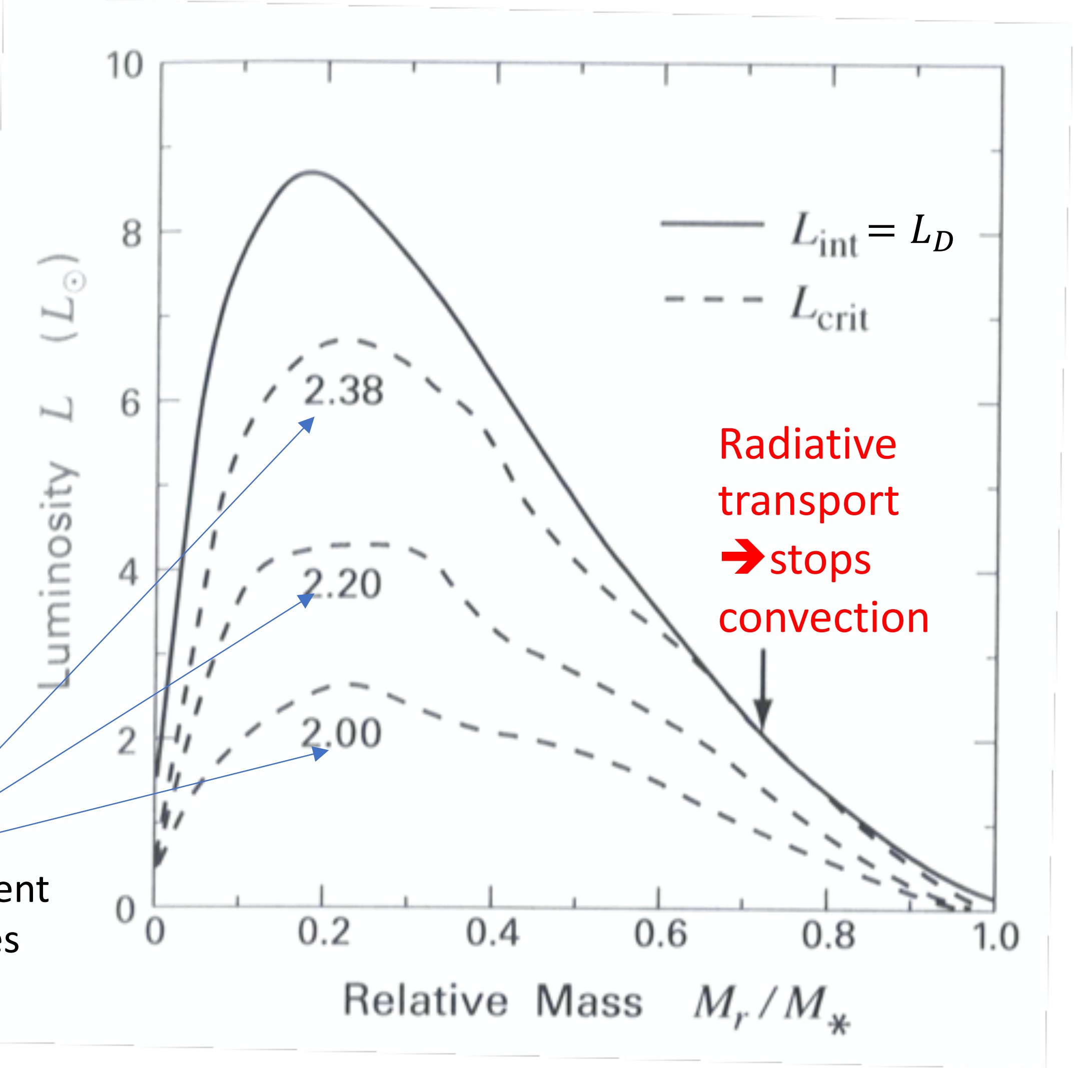
The maximum energy flux that can be carried by a pure radiative flux is :

$$L_{crit} = 4\pi r^2 \frac{4acT^3}{3\rho\kappa}\left(\frac{\mathrm{dT}}{dr}\right)_s = -\frac{GM16\pi acT^3}{3\rho\kappa}\left(\frac{dT}{dP}\right)_s$$

Initially $L_{crit}$ is much lower than the luminosity produced by deuterium burning and keeps rising.

We can show that :

- $L_{crit}$ scales as $M_*^{11/2}R_*^{-1/2}$
- Eventually $L_{crit} = L_D$ and radiative energy transport can again remove energy from the core




$L_{crit}$ for different stellar masses

# Evolution of the protostar

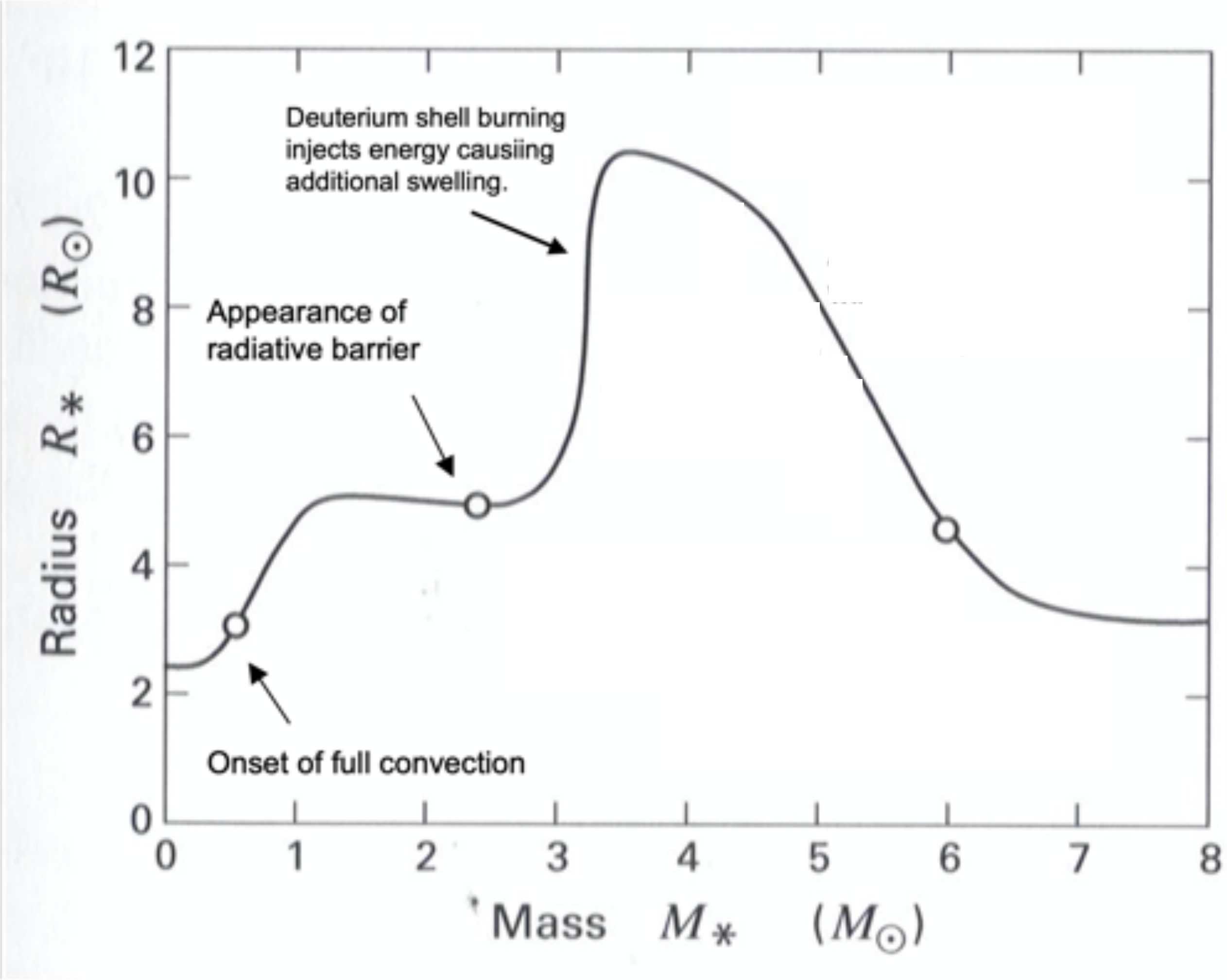
Effectively the radiative transport acts as a **barrier** preventing deuterium reaching the core.

Deuterium in the core is quickly depleted.

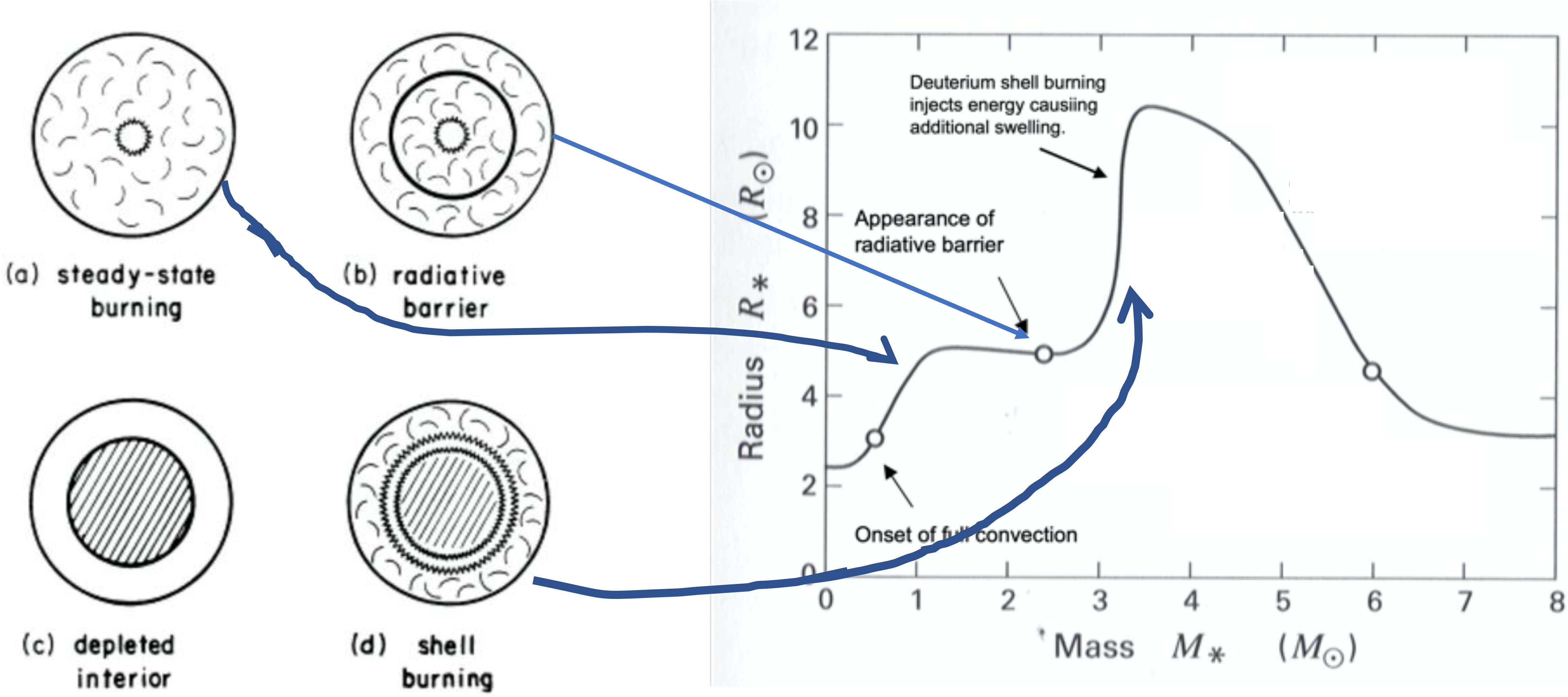
Without convection new deuterium accreted onto the protostar accumulates in a shell.

Eventually the temperature of this shell reaches $10^6$K and the shell ignites

The hot outer shell leads to a substantial increase in the stellar radius

# Evolution of the protostar



# Evolution of the protostar

**CONTRACTION AND HYDROGEN BURNING**

The final stage of protostellar evolution is the contraction of the star.

Without deuterium burning in the core, the self-gravity of the protostar drives the gravitational contraction of the star.

The rate at which the star contracts is determined by the rate at which the star loses internal energy via radiation : the ***Kelvin-Helmholtz timescale :***

$$t_{KH} = \frac{GM_*^2}{R_* L_*} = 3\times10^7 \left(\frac{M_*}{M_\odot}\right)^2 \left(\frac{R_*}{R_\odot}\right)^2 \left(\frac{L_*}{L_\odot}\right)^2 yr$$

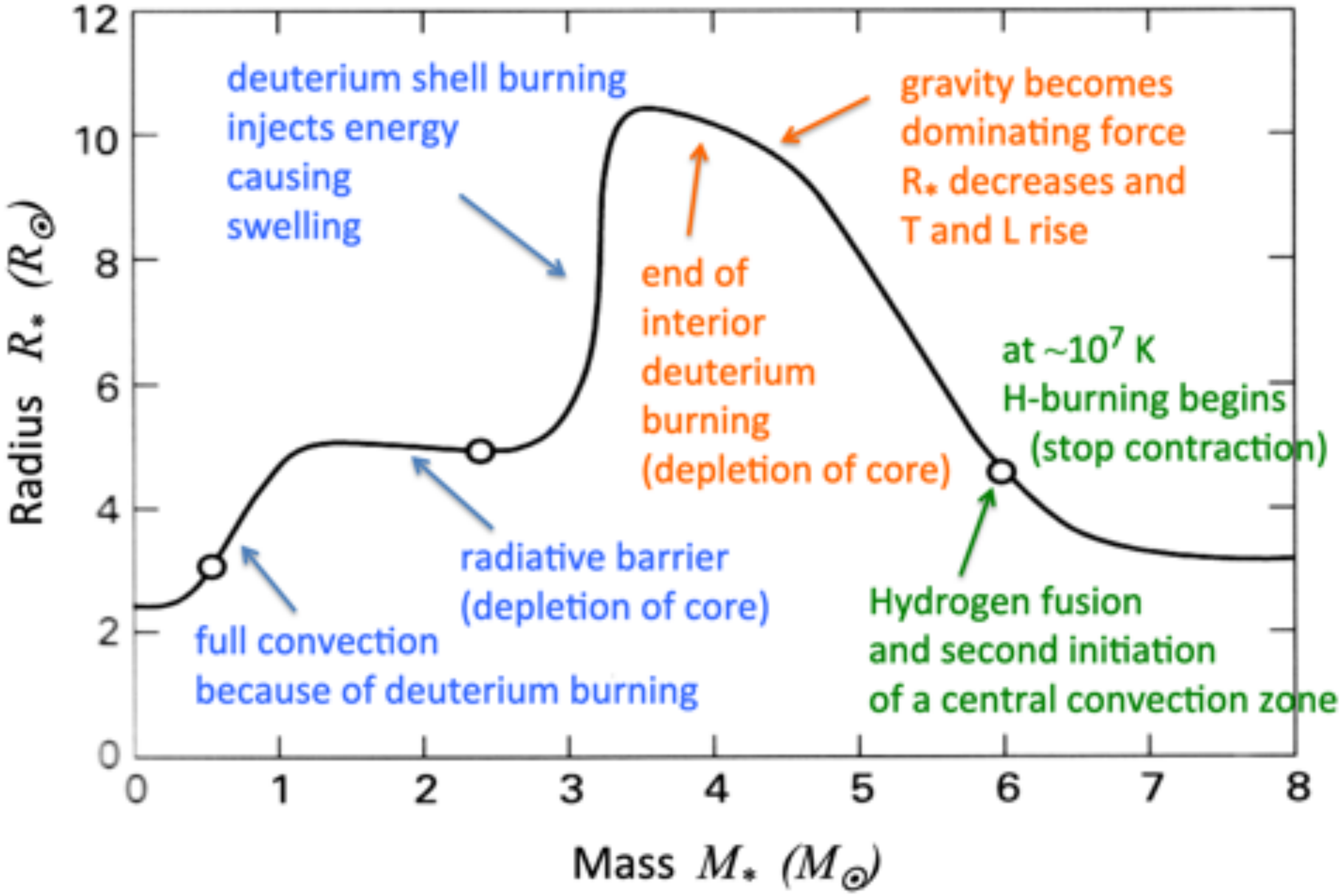
As the contraction proceeds, the core temperature continues to rise until eventually $10^7$ K.

At this temperature, hydrogen burning commences and halts the contraction ←restarts central convection

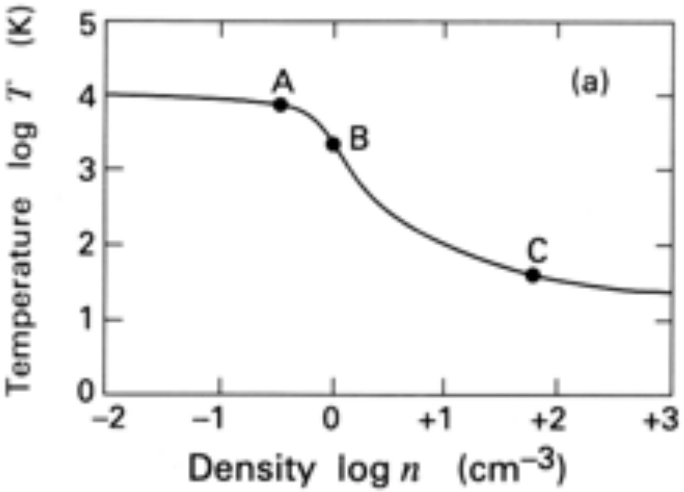
Further temperature rise enables the CNO cycle

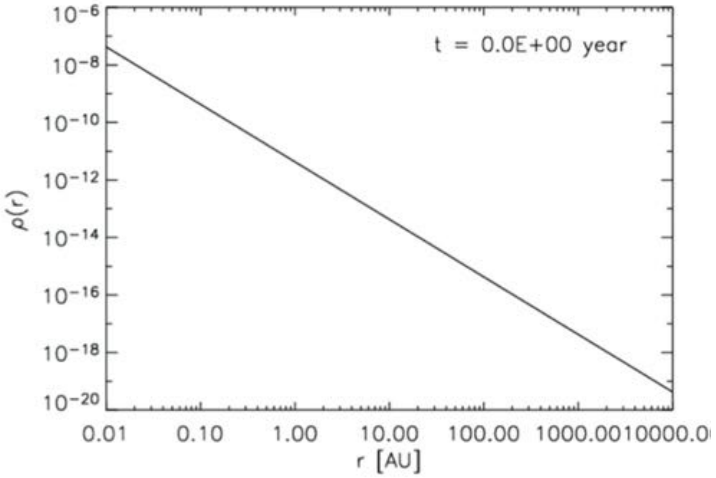
At this stage, the protostar is regarded as a pre-main sequence star

# Evolution of the protostar



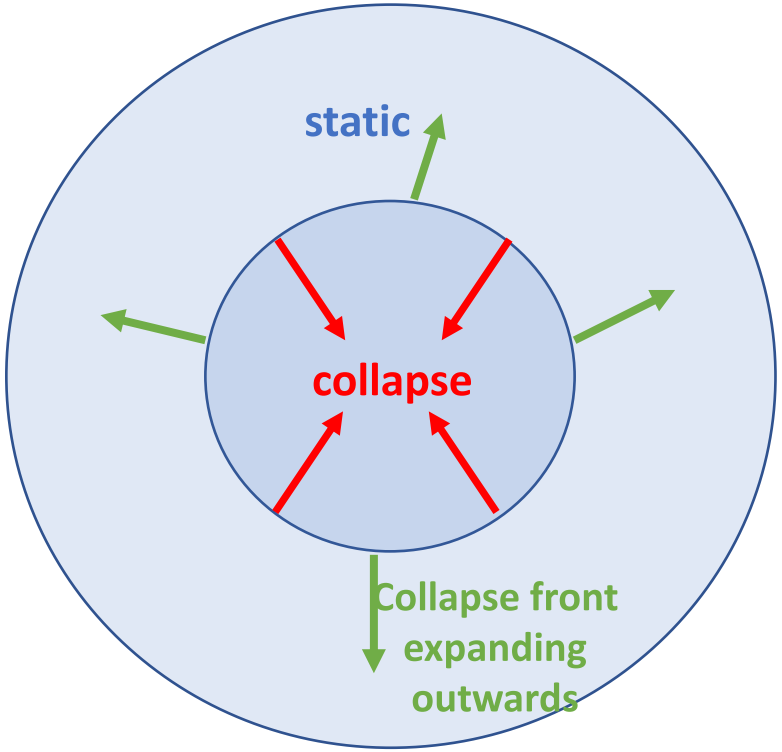
# Summary of the first half of this course



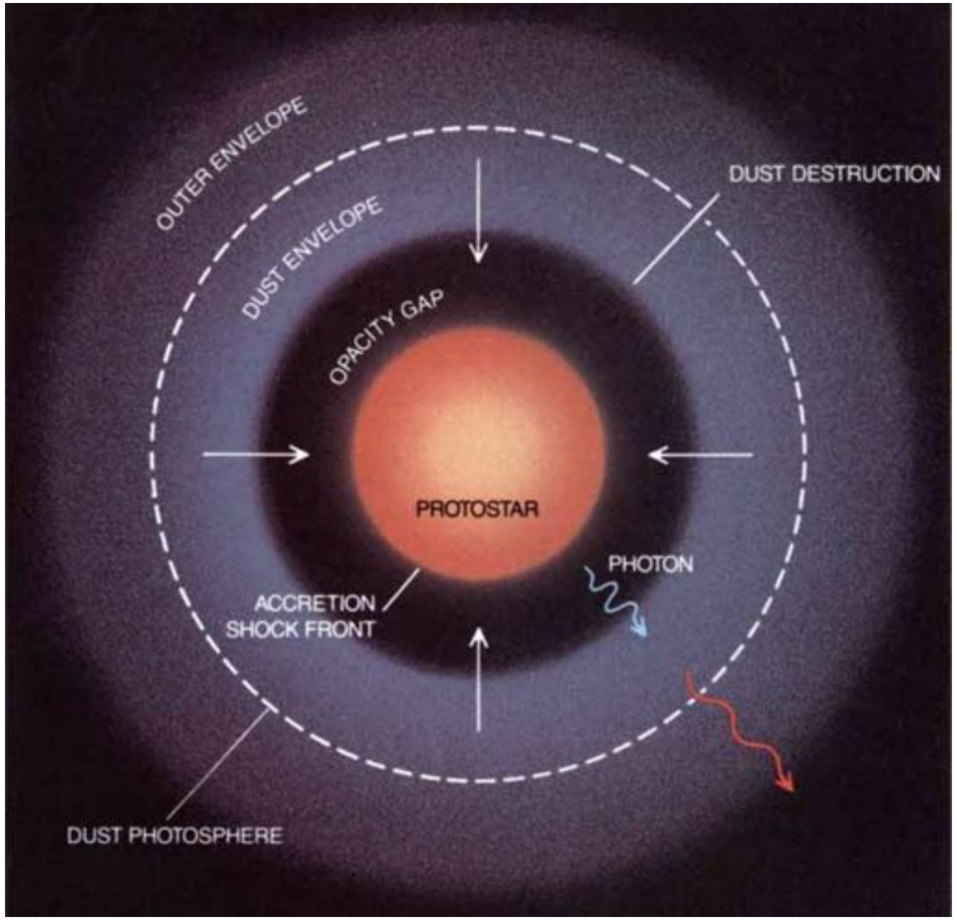



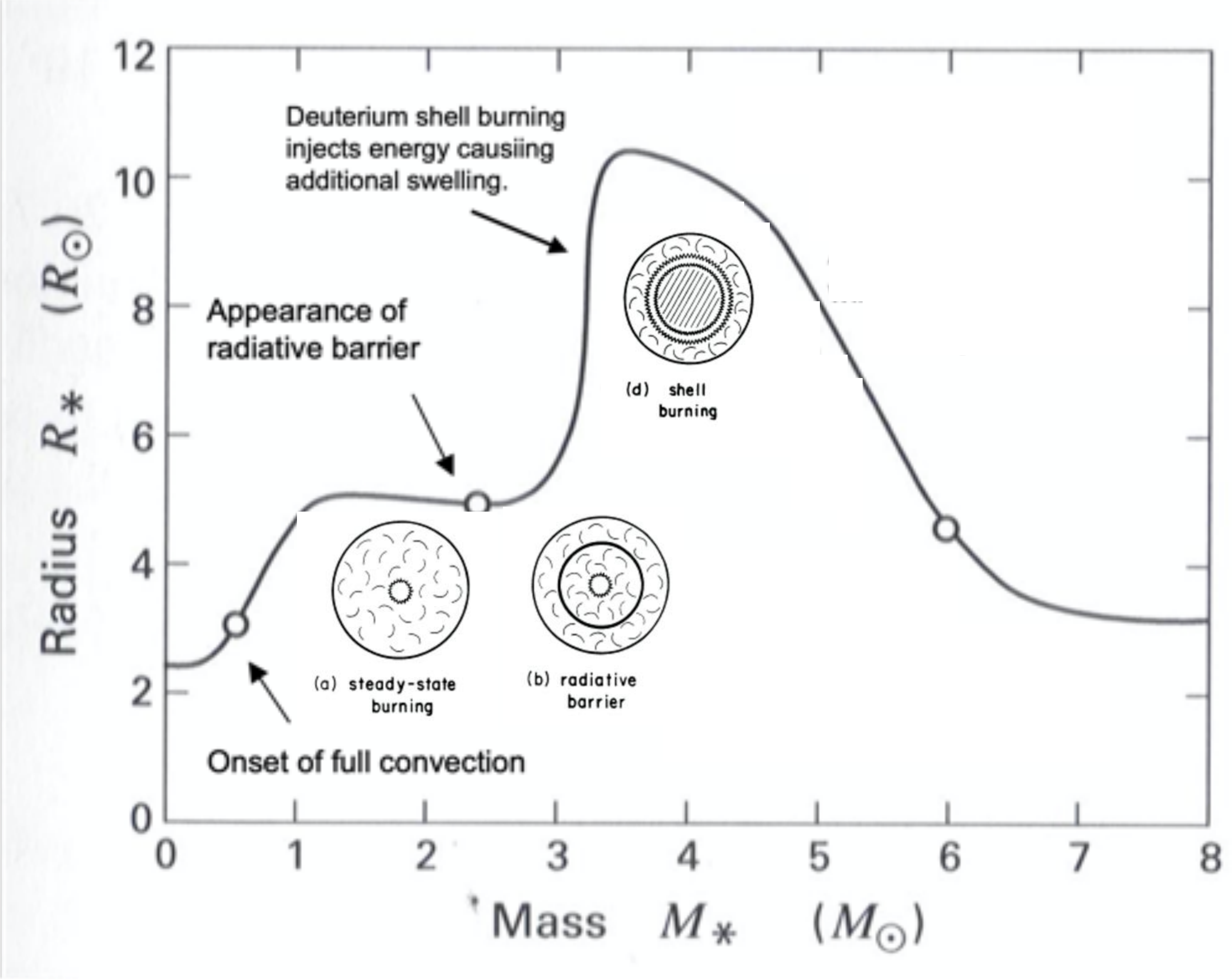

Jeans Mass : mass above which gravity dominates

$$\frac{M_J}{M_\odot} = 1.0 \times \left(\frac{T}{10K}\right)^{3/2} \times \left(\frac{n_H}{2\times 10^{10} m^{-3}}\right)^{-1/2}$$



# Summary of the first half of this course



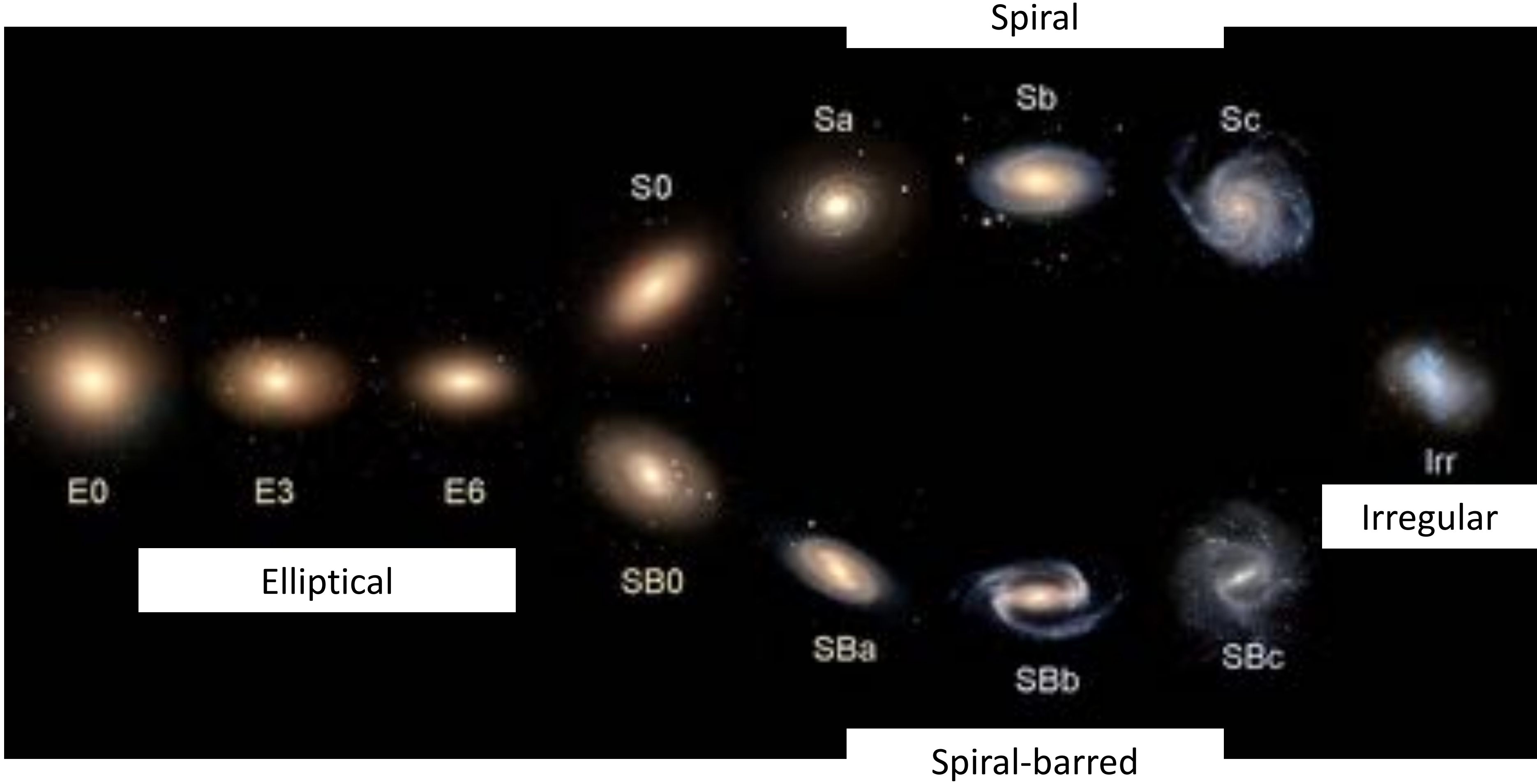
# Galaxies and star-formation on galactic scales

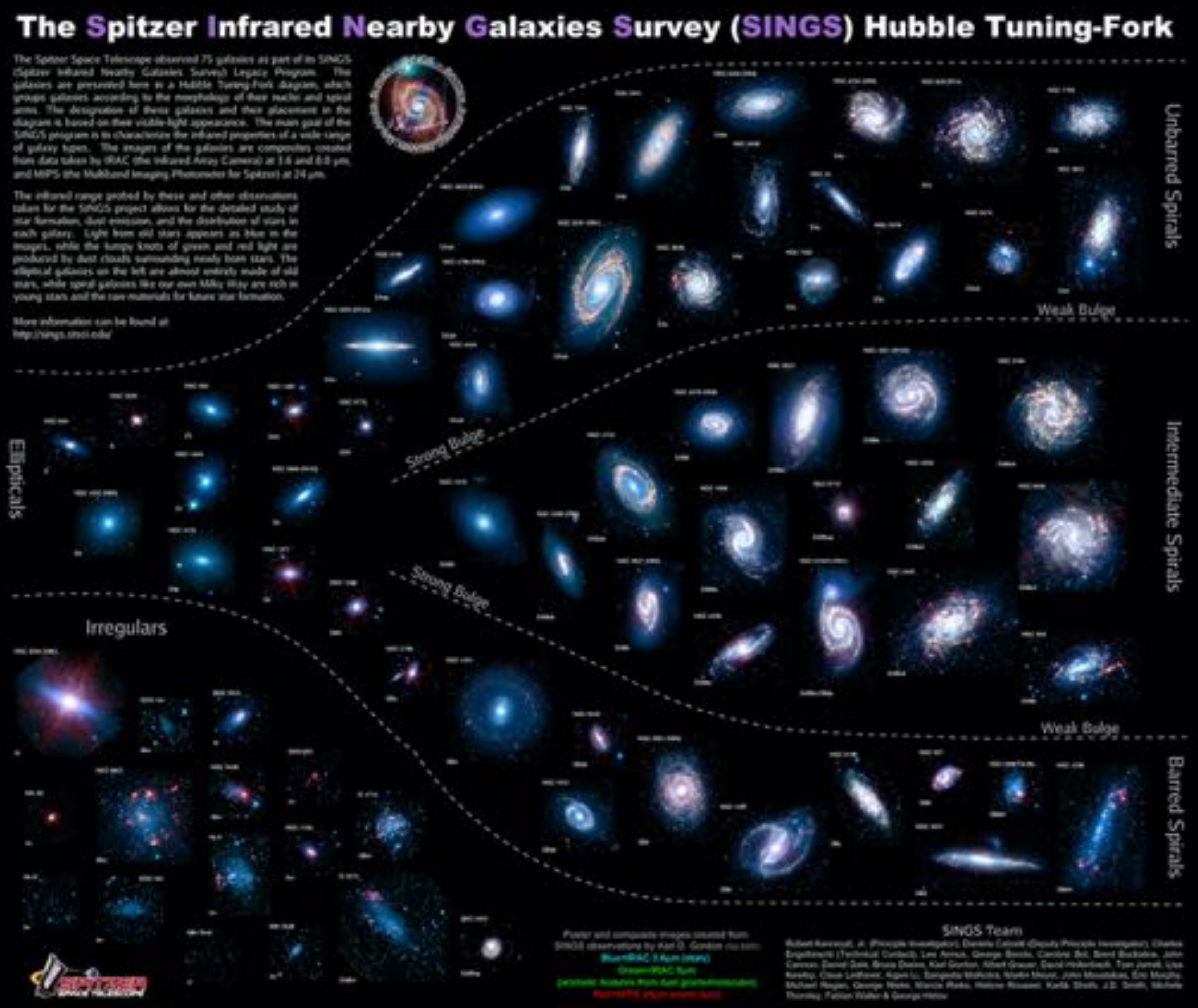
Chapter 6 (part 2)

# Properties of Galaxies in the Local Universe

**THE GALAXY ZOO**

Edwin Hubble
1889-1953



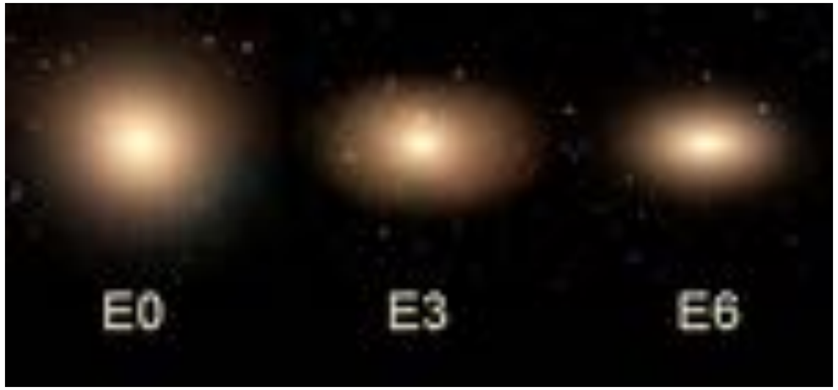

The Spitzer Infrared Nearby Galaxies Survey (SINGS) Hubble Tuning-Fork
Unbarred Spirals
Weak Bulge
Intermediate Spirals
Strong Bulge
Strong Bulge
Weak Bulge
Barred Spirals
Ellipticals
Irregulars
SINGS Team

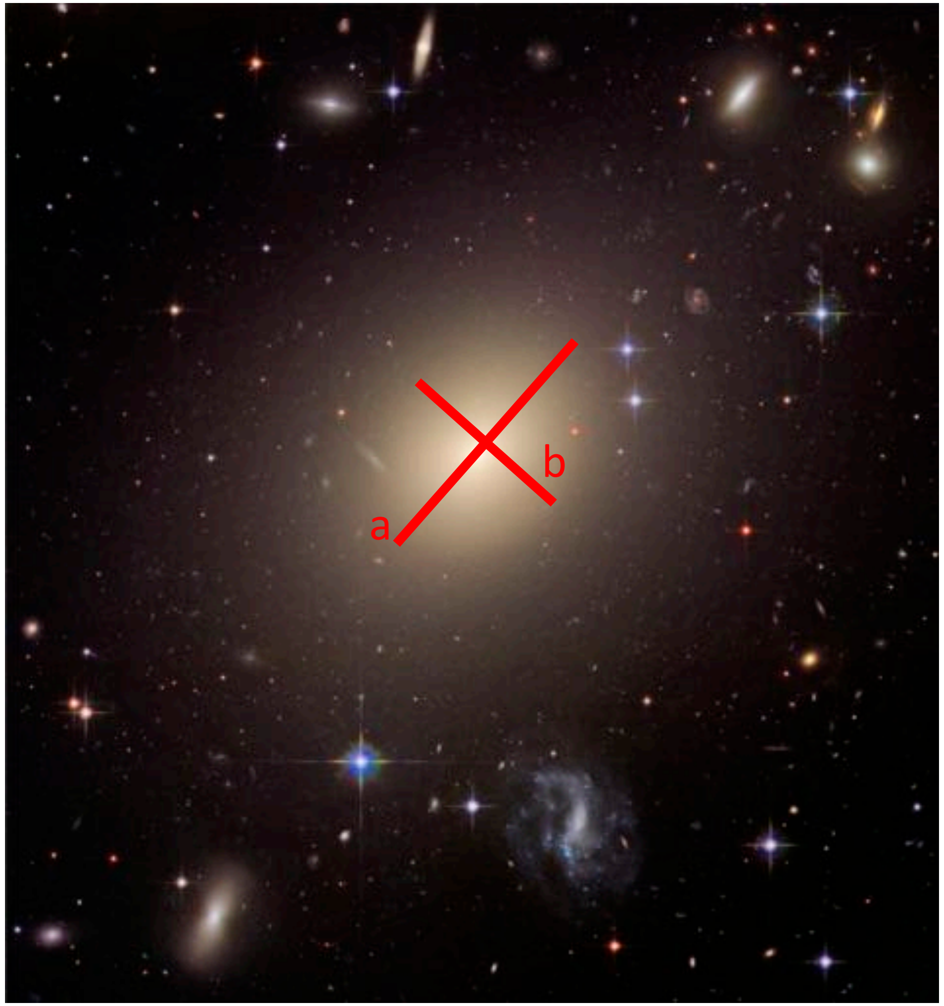
# Properties of Galaxies in the Local Universe

**ELLIPTICALS / EARLY TYPE GALAXIES**

The elliptical galaxies are classified following the ratio between their major (a) and minor (b) axis, and named $E_n$ with $n$ :

$$n = 10\ \frac{a-b}{a}$$



# Properties of Galaxies in the Local Universe

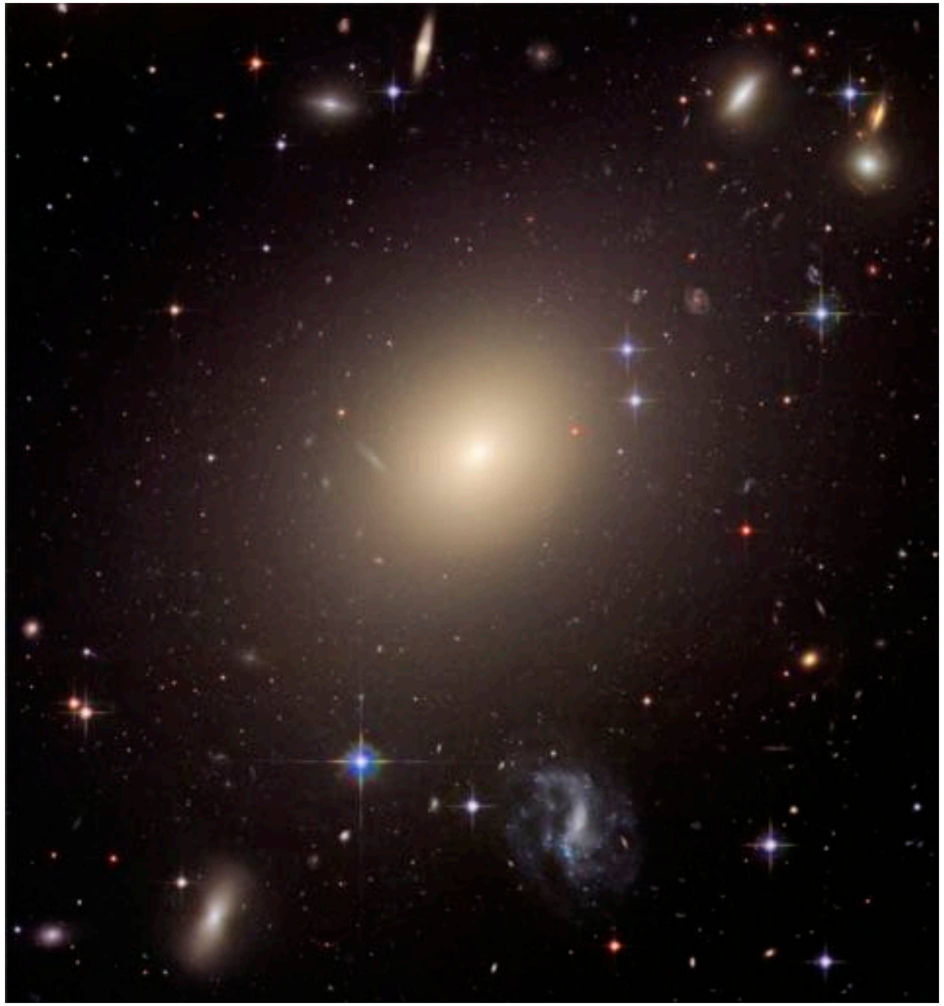
**ELLIPTICALS / EARLY TYPE GALAXIES**

The main properties of ellipticals are :

- Gas poor
- No star formation
- Old stars
- Stellar mass ranging between $10^{11}$ and $10^{13}$ $M_\odot$

The surface brightness of an elliptical galaxy is given by :

$$I(R) = I_0 \exp\left[-\left(\frac{R}{a}\right)^{\frac{1}{4}}\right]$$



# Properties of Galaxies in the Local Universe

**SPIRALS / LATE TYPE GALAXIES**

A spiral galaxy is always composed of a bright bulge and a disk

# Properties of Galaxies in the Local Universe

**SPIRALS / LATE TYPE GALAXIES**

The classification of spirals galaxies depends on the openness of arms and the prominence of the bulge.

From Sa to Sc : the openness of the arms increases, and the bulge prominence decreases.


# Properties of Galaxies in the Local Universe

**SPIRALS / LATE TYPE GALAXIES**

The classification of spirals galaxies depends on the openness of arms and the prominence of the bulge.

From Sa to Sc : the openness of the arms increases, and the bulge prominence decreases.

The main properties of spirals are :

- Gas rich
- star formation on-going
- Gas in both low density neutral hydrogen and dense molecular hydrogen
- The fractional mass of neutral hydrogen to the total is <0.03 for a Sa, and goes up to 0.1 for a Sc

$$f_{HI} = \frac{M_{HI}}{M_{tot}}$$

# Properties of Galaxies in the Local Universe

**SPIRALS / LATE TYPE GALAXIES**

We can refine the classification by adding :
- The presence of a ring ( r )
- The definition of the arms : from well defined ‘I’ to fuzzy ‘V’


SBb(r)III

# Properties of Galaxies in the Local Universe

**SPIRALS / LATE TYPE GALAXIES**

To describe the brightness profile of a spiral (or spiral-barred) galaxy, we need to consider the two components : the disc and the bulge.

A spiral seen face-on has a light profile given by :

$$I(r) = I_0 \exp\left(-\frac{r}{a}\right)$$

The overall distribution of mass in the disc is given by :

$$\rho(r,z) = \rho_0 \exp\left(-\frac{r}{a}\right) \exp\left(-\frac{|z|}{h}\right)$$

<ins>Scale height</ins> : the height at which the density falls of by a factor *e*

# Properties of Galaxies in the Local Universe

## THE GALAXY LUMINOSITY FUNCTION

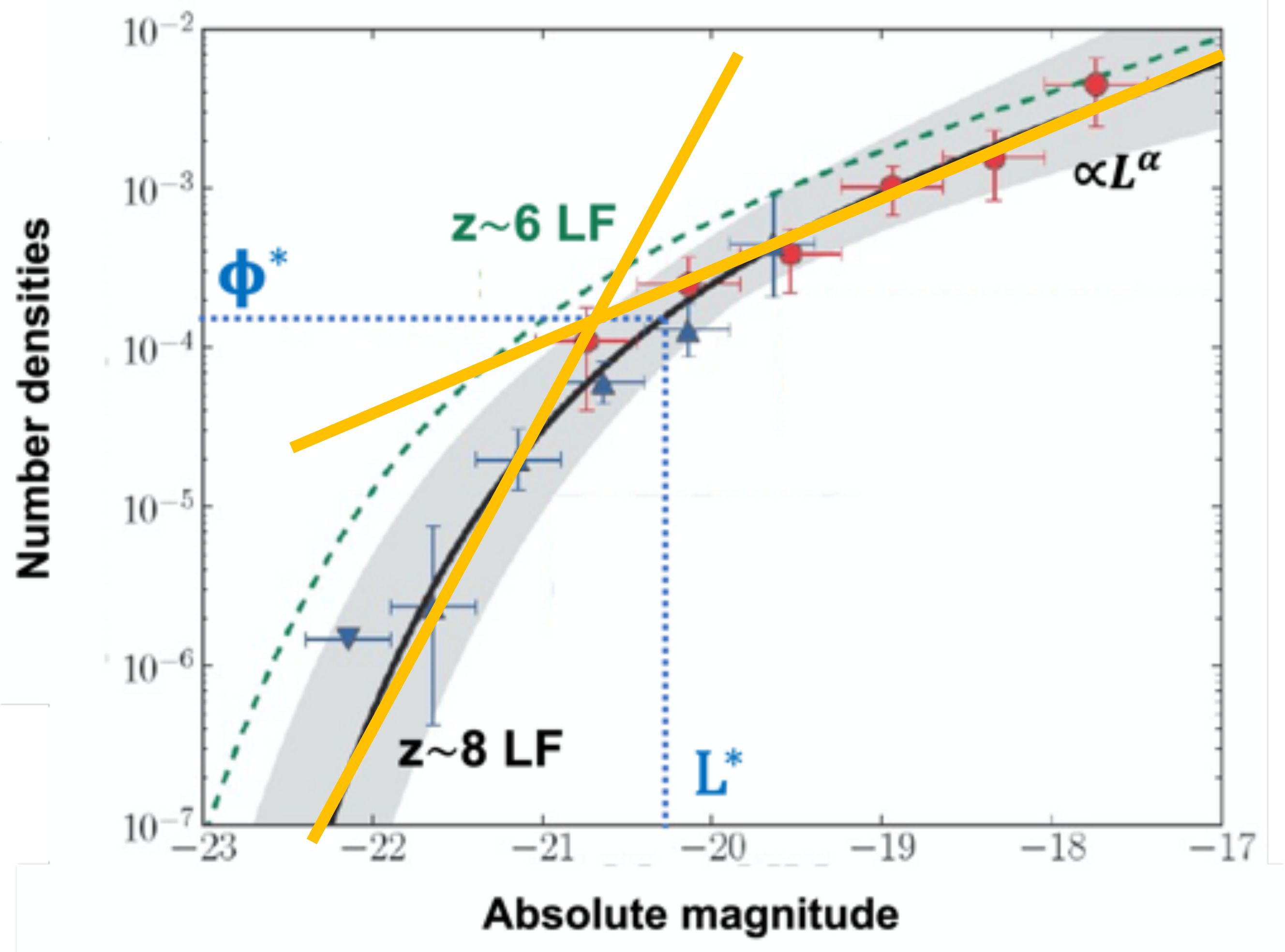
The galaxy Luminosity Function is the distribution in luminosity of the number density of galaxies at a given redshift. Its form has been described empirically by Schechter (1976) :

$$\Phi\left(\frac{L}{L^*}\right) d\left(\frac{L}{L^*}\right) = \Phi^* \left(\frac{L}{L^*}\right)^{\alpha} \exp\left(-\frac{L}{L^*}\right) d\left(\frac{L}{L^*}\right)$$

which can be written as :

$$\Phi(x)dx = \Phi^* x^{\alpha} e^{-x} dx$$

Where $\Phi^*$ and $L^*$ are the density and luminosity where there is a change in the shape of the function, and $\alpha$ is the slope at the faint-end.



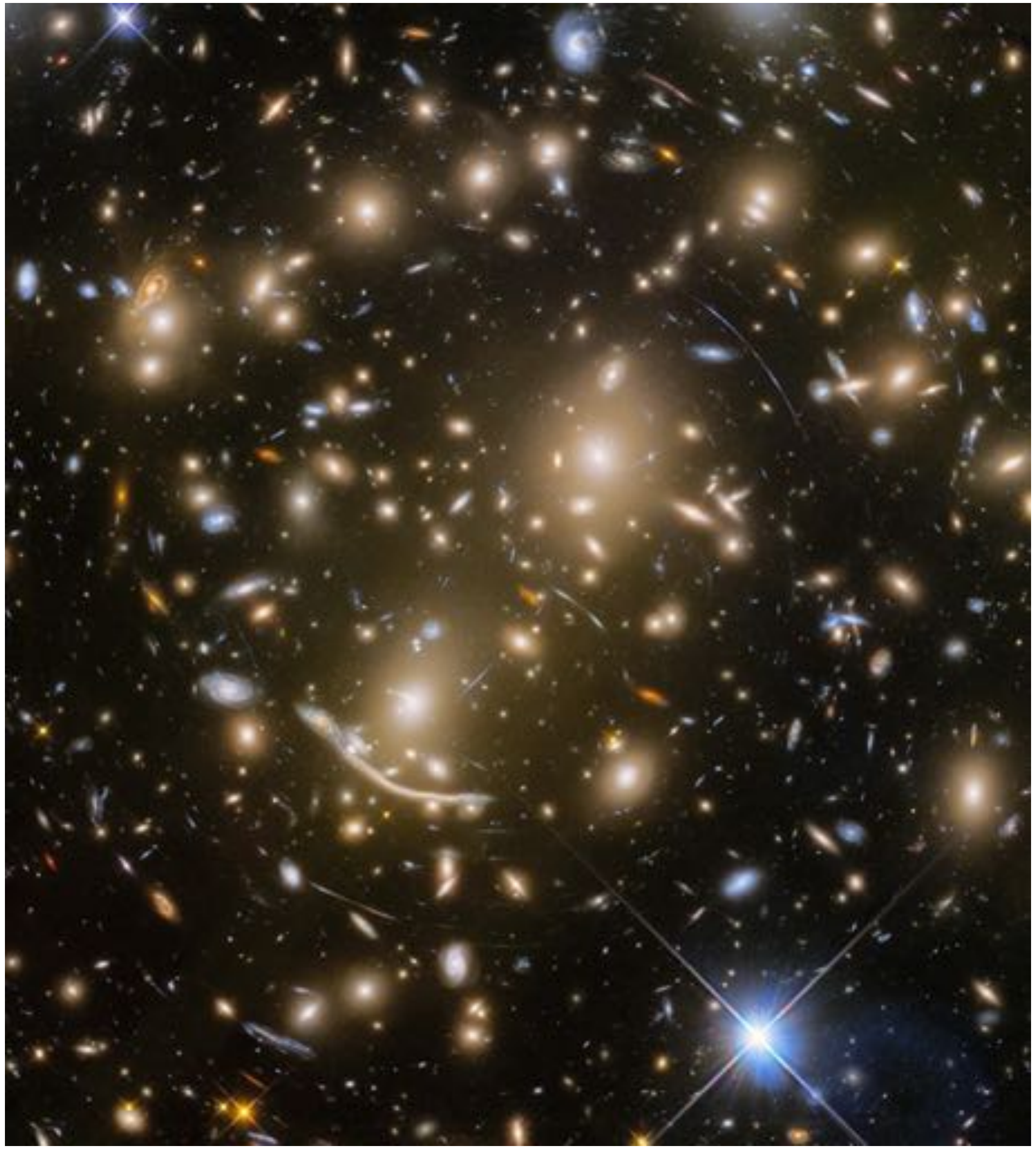
# Properties of Galaxies in the Local Universe

**THE GALAXY LUMINOSITY FUNCTION**

To measure the LF, we need a photometric survey with a redshift for each detected galaxy.

One can naively say that the number of galaxies per given luminosity is the number density of galaxies $\Phi(L)$. This is not the case because of the *Malmquist* bias (preferential detection of intrinsically bright galaxies).

We need to correct this effect to obtain an unbiased determination of the LF.

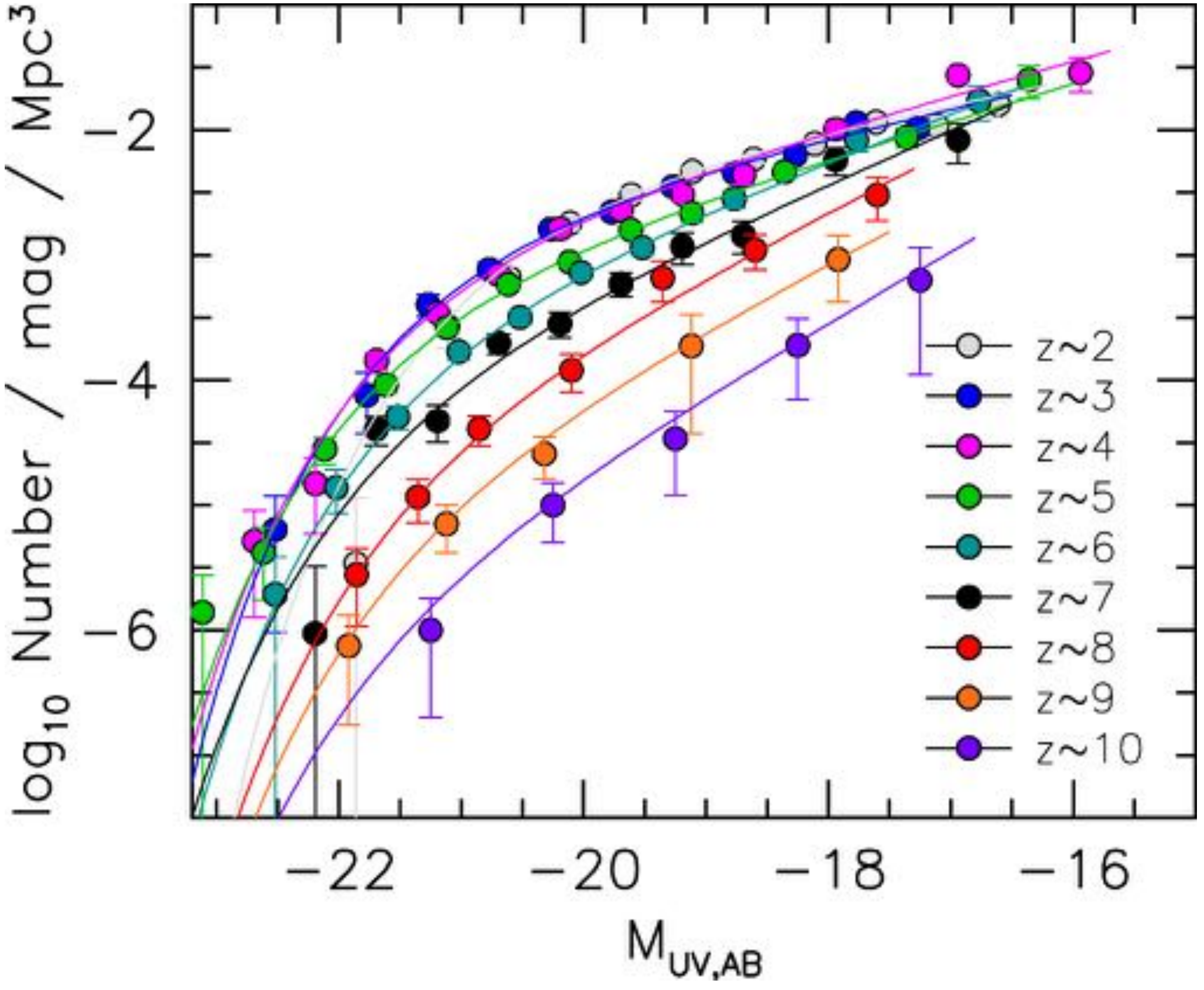


# Properties of Galaxies in the Local Universe

**THE GALAXY LUMINOSITY FUNCTION**

The V/$V_{max}$ method (also named the volume luminosity test) is the most successful method to determine the LF.

Taking into account the luminosity limit of the survey $L_{lim}$ we define :
$V$: volume within which each source is distributed
$V_{max}$ : maximum volume within which each source could still be detected



# Properties of Galaxies in the Local Universe

**STELLAR POPULATION**

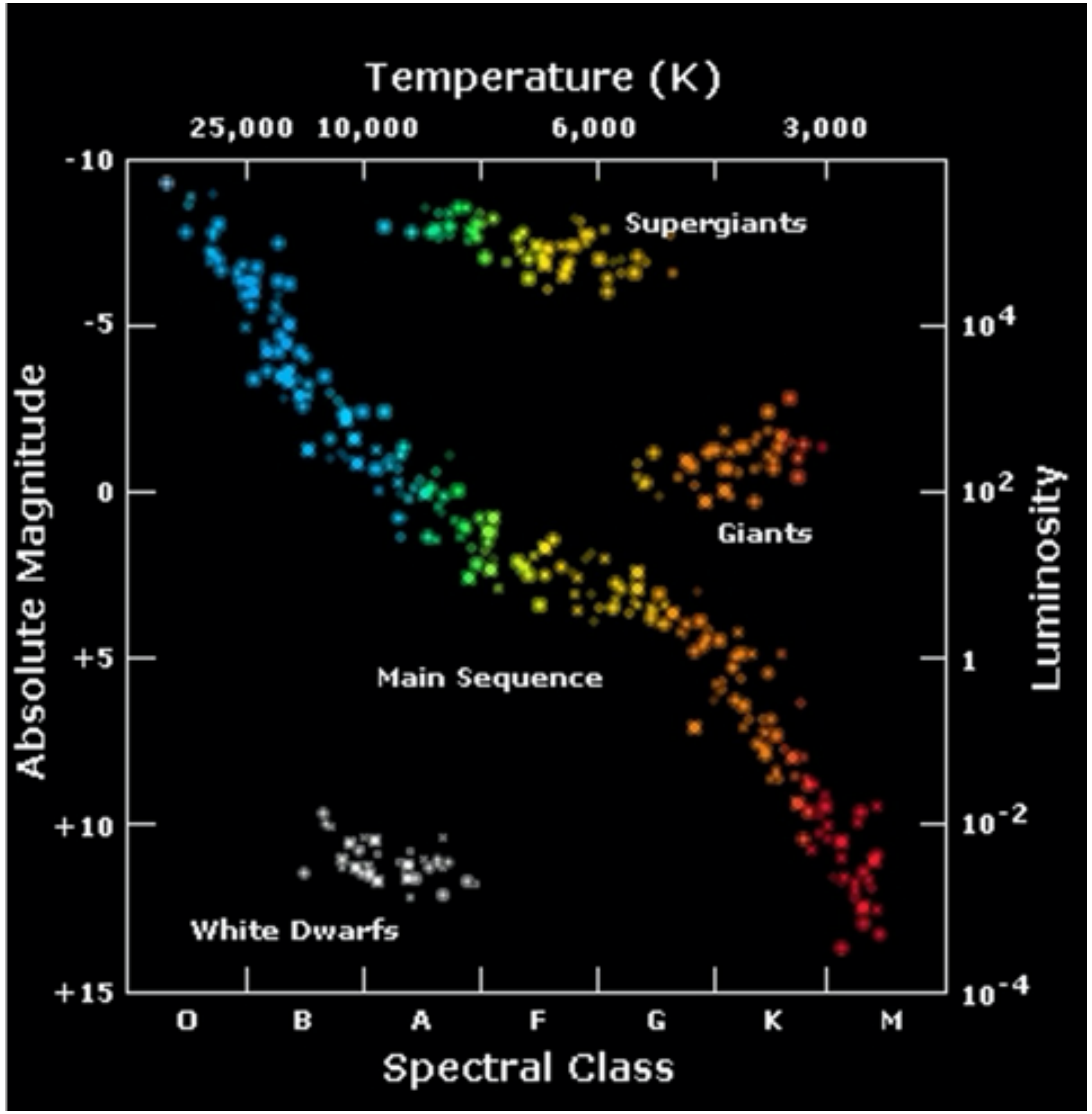
Stars are mainly characterised by their luminosity and surface temperature. They are classified according to their spectral type :

*With decreasing T* : O, B, A, F, G, K, M

A diagram showing the luminosity of a star as a function of its temperature is a *Hertzsprung-Russel* (HR) diagram.

Most of the stars are along the line called the Main Sequence where they are fusing hydrogen in their core.

The Giant branch is when the stars are fusing the Helium in their core.



# Properties of Galaxies in the Local Universe

**STELLAR POPULATION**

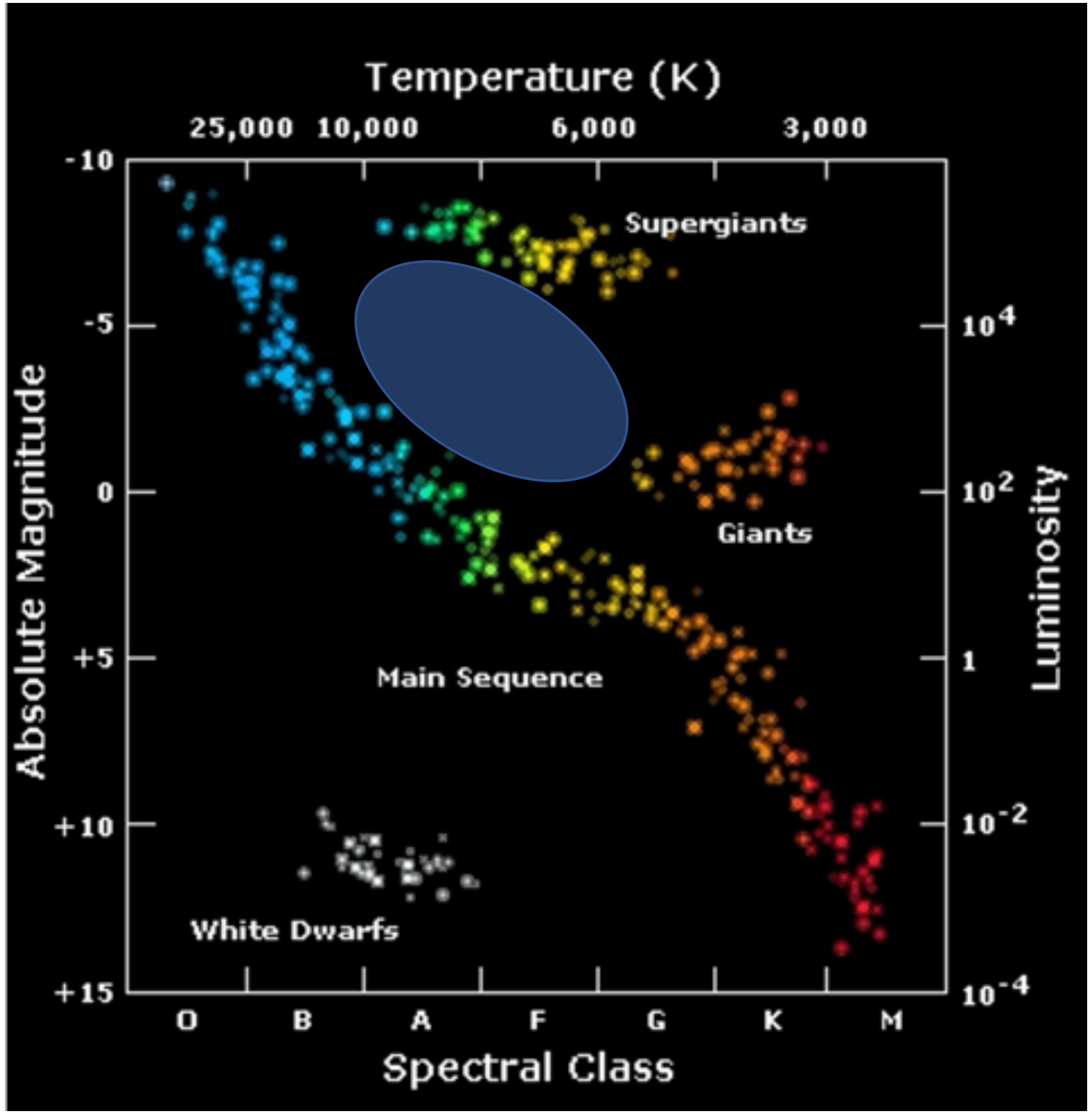
We can identify a gap in the HR diagram : that is the place where we can find variable stars (such as RR Lyrae or Cepheid).

Cepheids are evolved variable stars (helium burning stars). Their visual magnitudes vary between a ~0.01 and ~2 mag, with a period of a few days to a few weeks.

**The longer the period of their variability, the brighter their intrinsic luminosity**

$$d_{pc} = 10^{0.2\left(\beta\left[(B-V)_0 + \frac{m}{\beta}\right] - \alpha \log P + \gamma'\right)}$$

If we can measure the period of a Cepheid, we can deduce its intrinsic luminosity and therefore its distance.



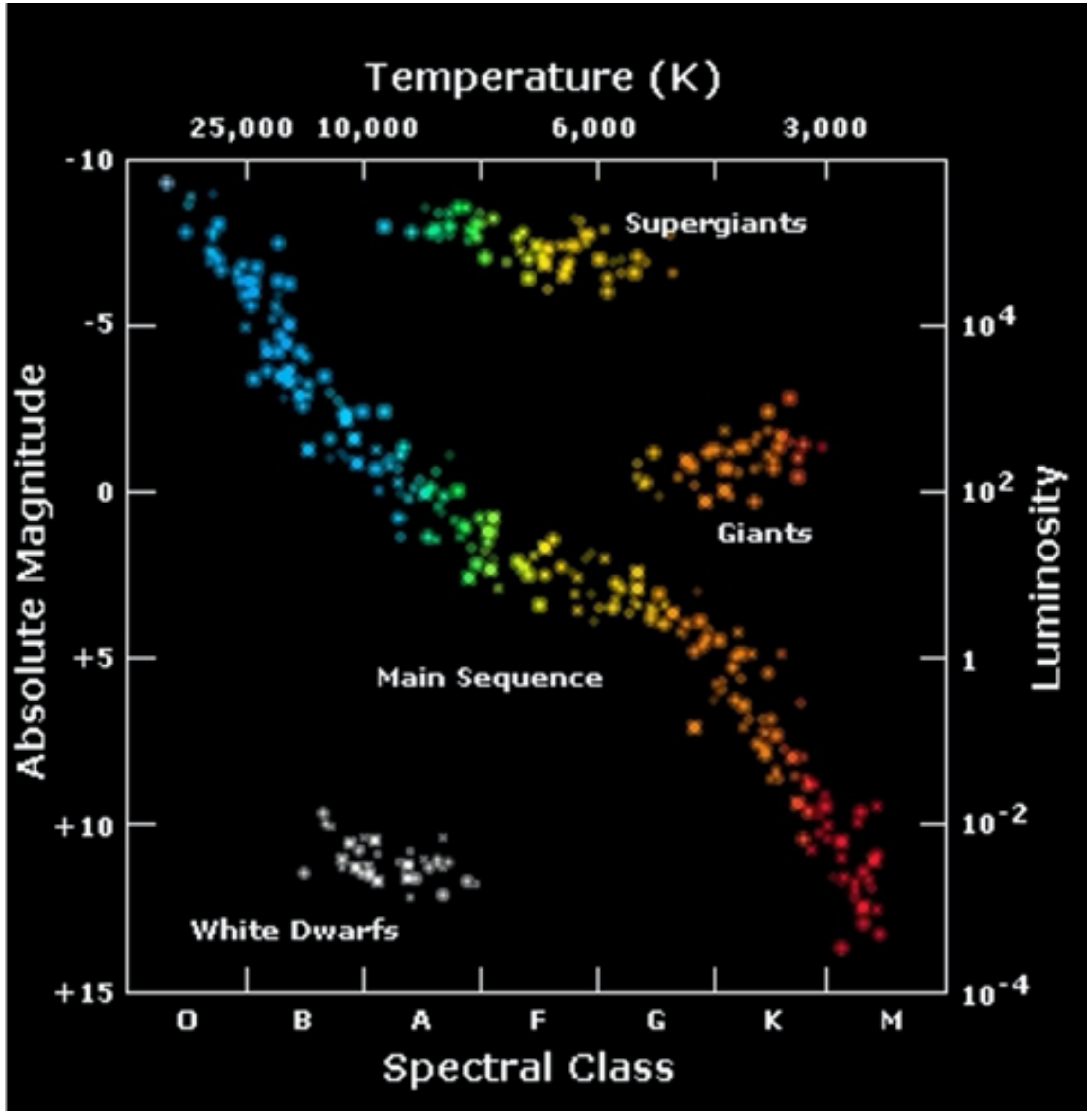
# Properties of Galaxies in the Local Universe

The HR diagram ca be used to measure how far away a star cluster/galaxy is from Earth.

This can be done by comparing the apparent magnitudes of a star clusters with distance unknown, with the absolute magnitude of stars with known distance.

The observed group is then shifted in the vertical direction until it reaches the main sequence.

The different in magnitude (m-M) is a direct measure for the distance



# Properties of Galaxies in the Local Universe

Stellar luminosity scales approximately as

$$L \propto M^{\alpha}$$

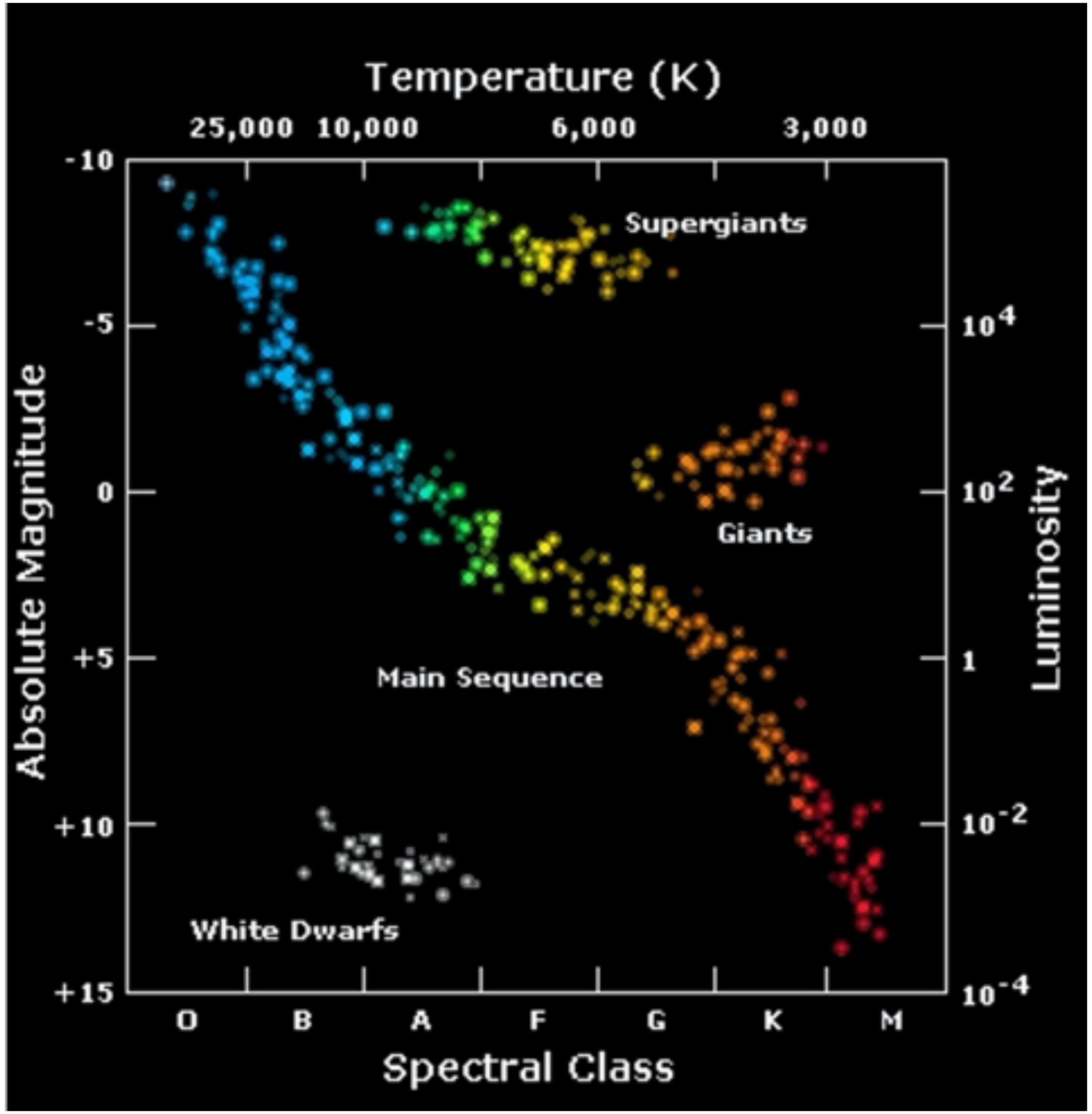
with $\alpha \sim 3$ for stars with $M < 0.5\ M_{\odot}$

and $\alpha \sim 4$ for stars with $M > 0.5\ M_{\odot}$

The time a star can remain on the Main Sequence is given by :

$$\tau_{ms} \propto \frac{M}{L} \propto M^{1-\alpha}$$

From previous equation, we clearly see that massive stars have shorter lifetime. Therefore, the most massive stars are excellent tracers of recent star formation. According to the HR diagram the most massive stars are O and B.



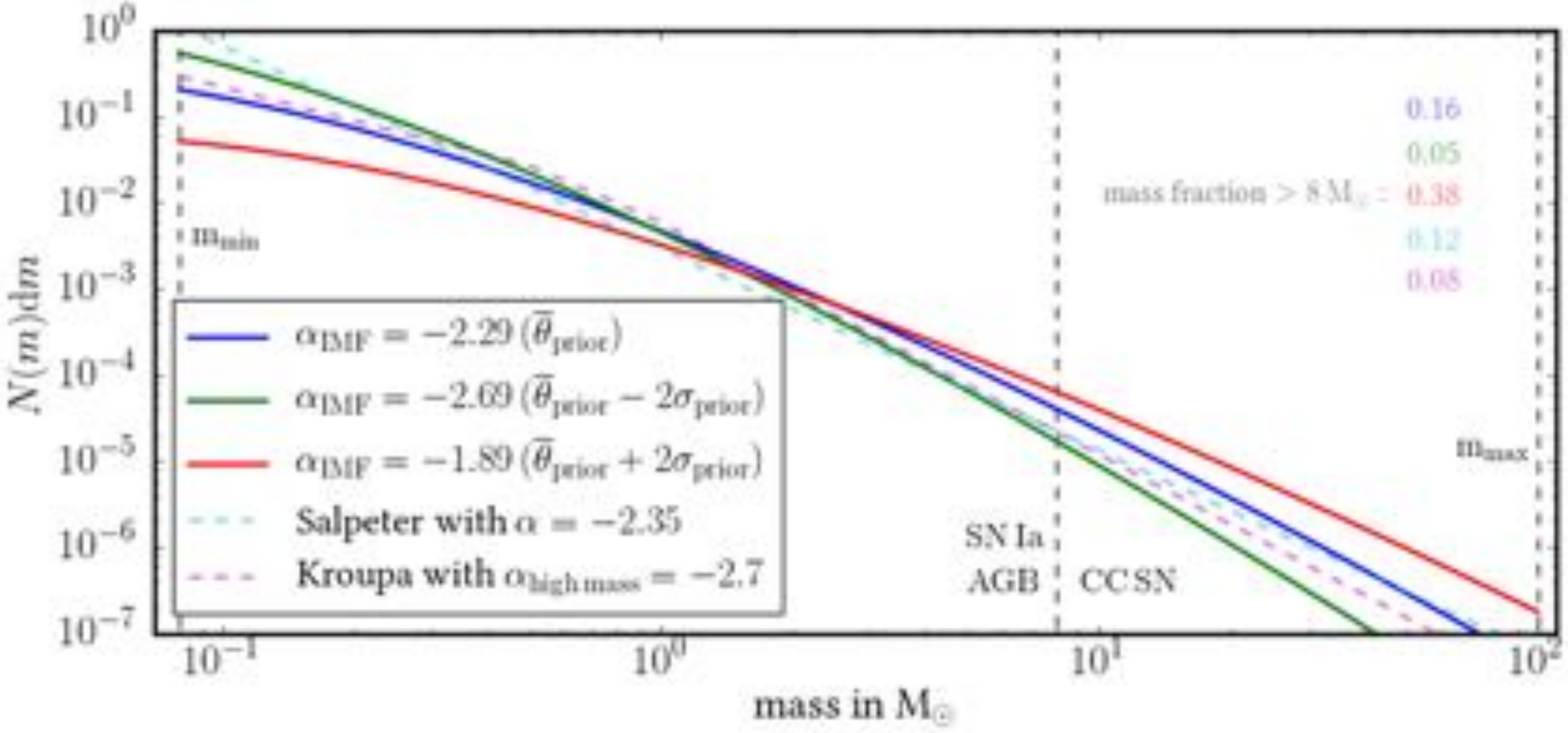
# Properties of Galaxies in the Local Universe

The number of newly formed star with masses in the range $M \rightarrow M + dM$ is given by the **Initial Mass Function (IMF)**.

The commonly used IMF is the Salpeter IMF which has the shape of a power-law :

$$\frac{dN}{dM} \propto M^{\theta}$$

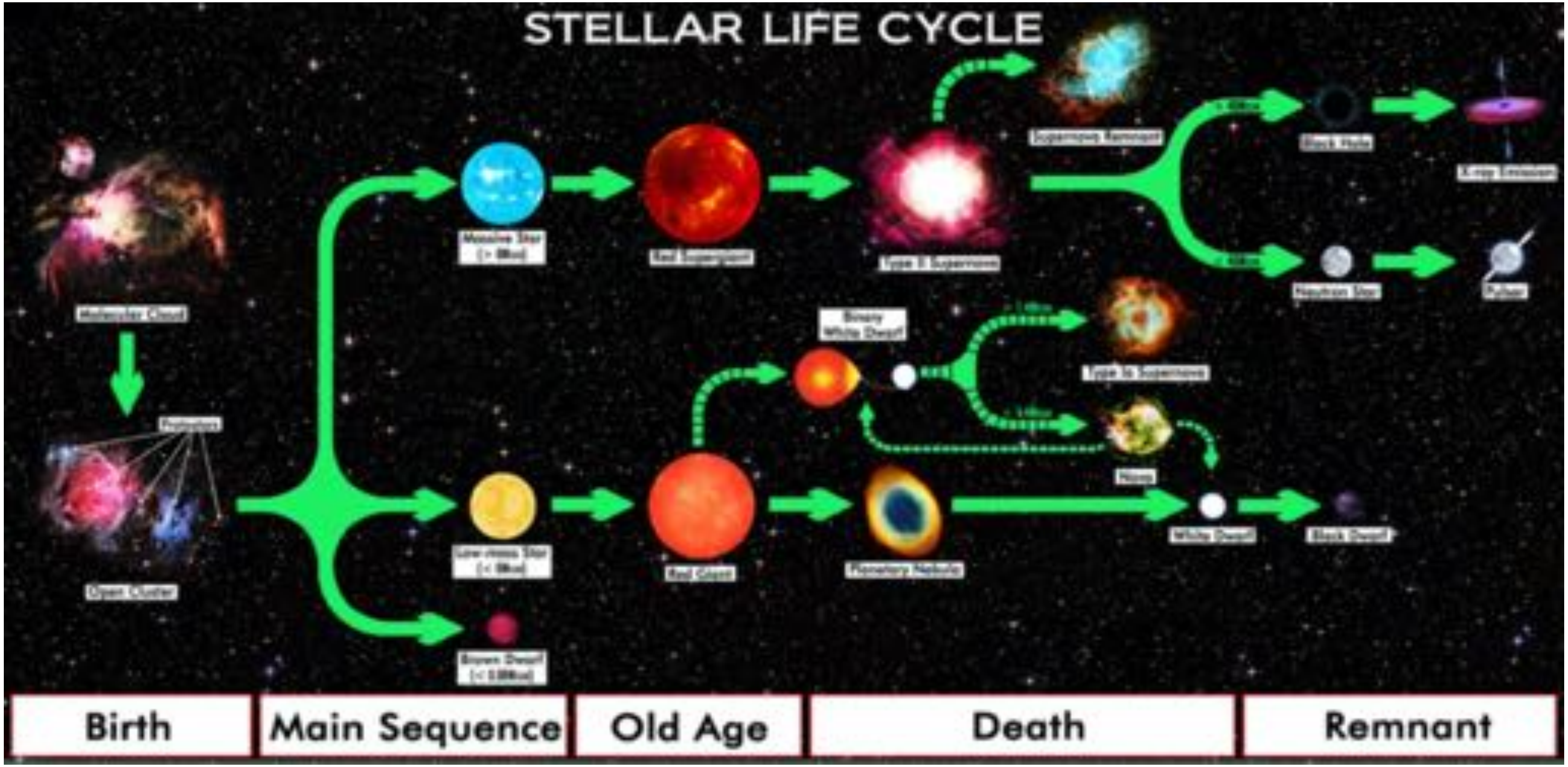
for a standard Salpeter IMF $\theta = -2.35$



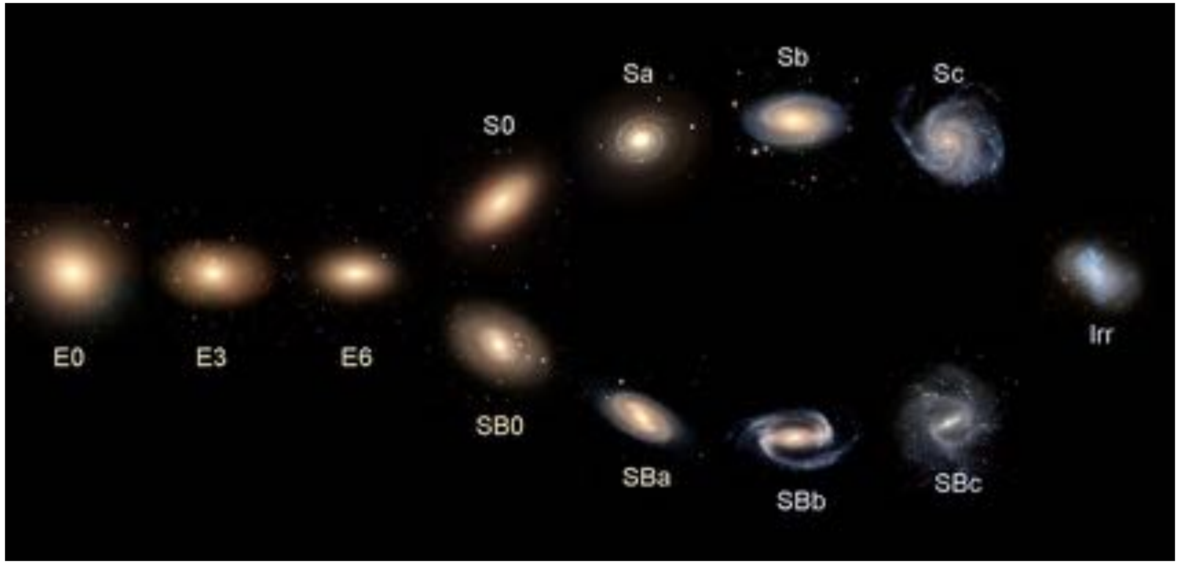

Typo in the handout

The mass budget is dominated by the low-mass stars. But according to the relation between Luminosity and Mass $L \propto M^{\alpha}$, the most massive stars dominate the luminosity budget.

# Properties of Galaxies in the Local Universe

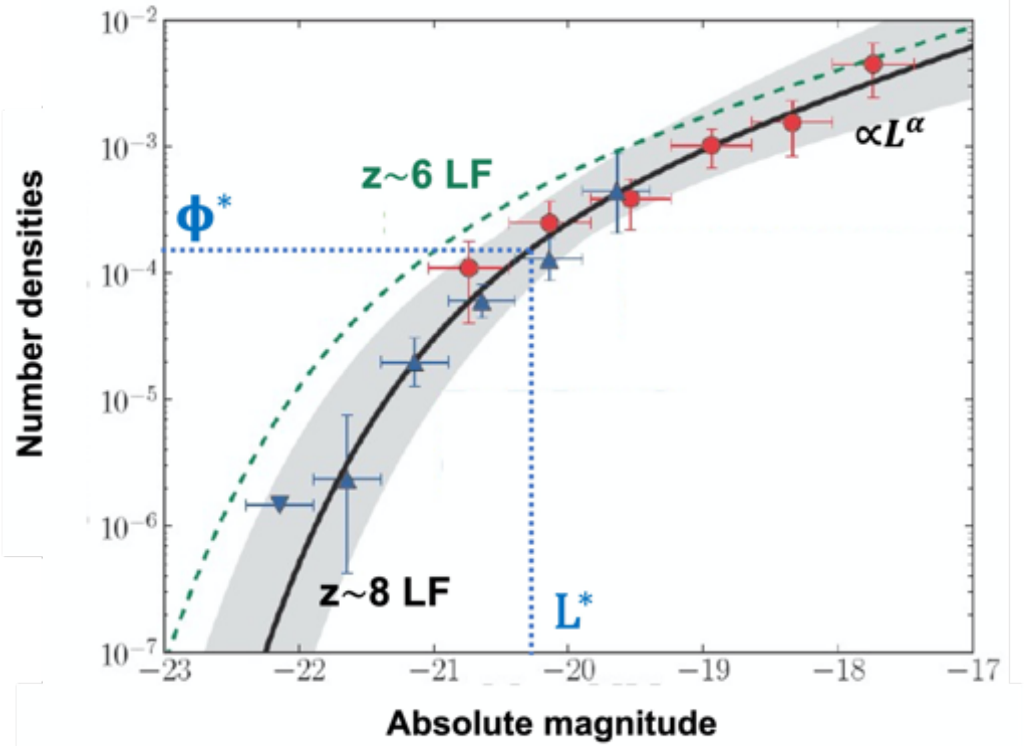


# Summary of Friday's lecture



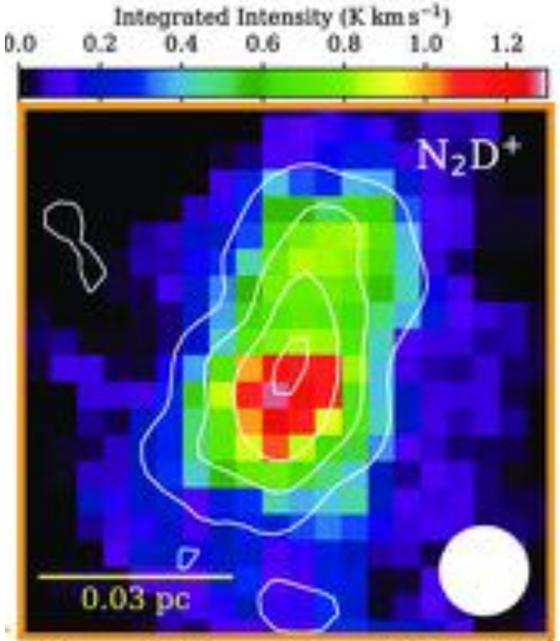

$$\frac{dN}{dM} \propto M^{\theta}$$

Initial Mass Function




$$L \propto M^{\alpha}$$

# Cloud fragmentation

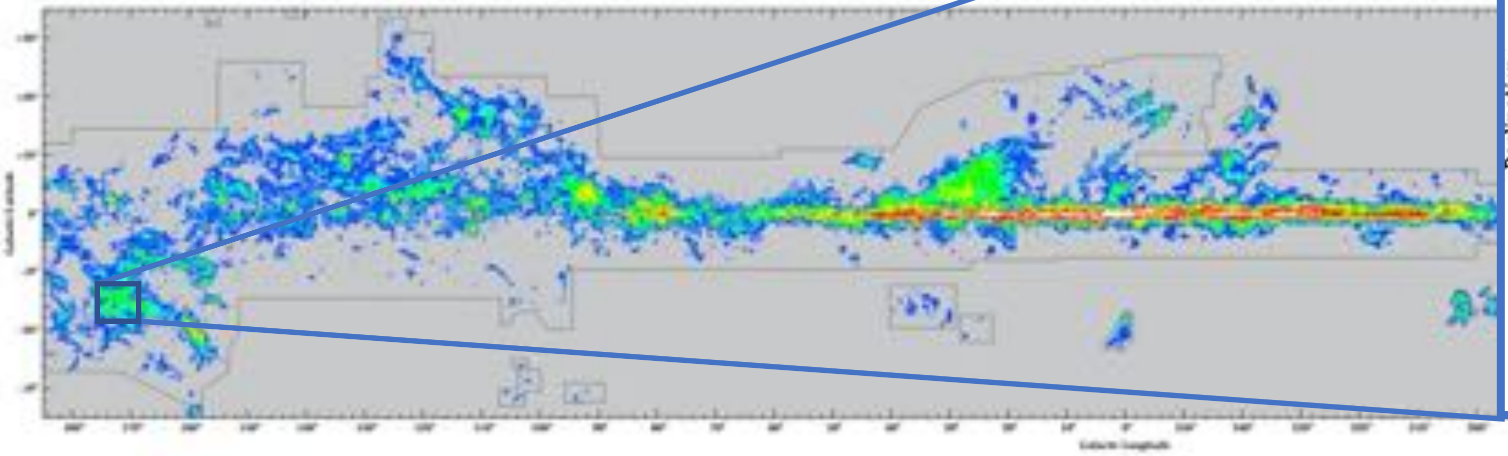


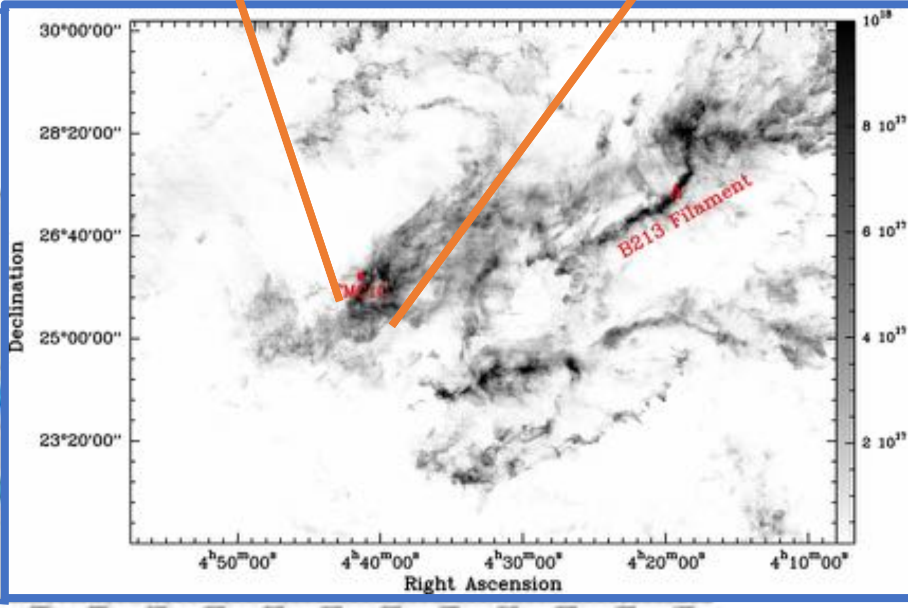

$M \sim 5 M_\odot$

In chapter 3, we saw that for a standard cloud of hydrogen, the Jeans Mass $\sim 1\ M_\odot$. This means that any more massive cloud (exceeding the Jeans Mass) is unstable.

**Why do we not produce a few very massive stars from the collapse of a giant molecular cloud ?**

**FRAGMENTATION**





# Cloud fragmentation

We consider a homogeneous spherical cloud of gas of density $\rho_0$, radius $R$, and mass $M_0$. We assume no internal pressure throughout the collapse.

Gas molecules initially at a radius $r_0$ will have a mass of gas $M_r$ within the radius, and during the collapse this mass remains constant.

This analysis will follow the same calculation as before for free-fall collapse. Then the equation of motion of the molecules is (*Newton's gravity*) :

$$\frac{\partial^2 r}{\partial t^2} = -\frac{GM_r}{r^2}$$

Integrating by $\dot{r} = \frac{dr}{dt}$ and integrating with respect to time gives :

$$\frac{1}{2}\dot{r}^2 = \left[\frac{GM}{r}\right]_{r_0}^{r} = \frac{4\pi}{3} r_0^2 \rho_0 G \left(\frac{r_0}{r} - 1\right)$$

Hence :

$$\frac{dr}{dt} = -\sqrt{\frac{8\pi G\rho r_0^2}{3}} \left(\frac{r_0}{r} - 1\right)^{1/2}$$

The negative sign is chosen to indicate the collapse

# Cloud fragmentation

From the previous equation, let's determine : the free-fall time and the density evolution.

We introduce the dimensionless length :

$$\xi = r/r_0$$

and the characteristic time :

$$t_0 = \sqrt{\frac{3}{8\pi G\rho_0}}$$

and the dimensionless time :

$$\tau = t/t_0$$

Hence :

$$\frac{dr}{dt} = -\sqrt{\frac{8\pi G\rho_{r_0^2}}{3}}\left(\frac{r_0}{r} - 1\right)^{1/2}$$

Becomes :

$$\frac{d\xi}{d\tau} = -\left(\frac{1}{\xi} - 1\right)^{1/2}$$

# Cloud fragmentation

We can now make the substitution :

$$\xi = \cos^2\alpha$$

Then :

$$\frac{d\xi}{d\tau} = -\left(\frac{1}{\xi} - 1\right)^{1/2}$$

becomes :

$$\frac{d\alpha}{d\tau} = \frac{1}{2\cos^2\alpha}$$

Separating the variables :

$$2\cos^2\alpha \;\; d\alpha = d\tau$$

Integrating :

$$\alpha + \frac{1}{2}\sin 2\alpha = \tau$$

Remember that :

$$\xi = r/r_0$$
$$\xi = \cos^2\alpha$$
$$t_0 = \sqrt{\frac{3}{8\pi G\rho_0}}$$
$$\tau = t/t_0$$

The end of the collapse occurs when $r \to 0$, then when $\xi \to 0$ which means that $\alpha = \pi/2$ :

$$t_{ff} = \left(\frac{\pi}{2} + \frac{1}{2}\sin\pi\right)t_0 = \frac{\pi}{2}t_0 = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2}$$

Like what we found at the stellar scale

# Cloud fragmentation

The mass within each collapsing shell is conserved, and since the density is initially uniform, it must remain uniform (because the free-fall time does not depend on $r_0$).

Because of the mass conservation :

$$r_0^3 \rho_0 = r^3 \rho(t)$$

Or

$$\frac{r}{r_0} = \cos^2 \alpha$$

Hence :

$$\frac{\rho(t)}{\rho_0} = \frac{r_0^3}{r^3} = \frac{1}{\cos^6 \alpha}$$

If we know assume a time $t$ close to the collapse :

$$\tau = \frac{t}{t_0} = \frac{t_{ff} - (t_{ff} - t)}{t_0}$$

because $t \rightarrow t_{ff}$, then $r \rightarrow 0$ and $\alpha \rightarrow \pi/2$.

Then

$$\tau = \alpha + \frac{1}{2}\sin 2\alpha = \frac{\pi}{2} - \epsilon$$

We can also say that :

$$r \sim 0 \iff \alpha = \frac{\pi}{2} - \beta$$

Then :

$$\tau = \alpha + \frac{1}{2}\sin 2\alpha = \frac{\pi}{2} - \beta + \frac{1}{2}\sin(\pi - 2\beta)$$

Or

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \dots$$

Hence

$$\tau = \frac{\pi}{2} - \beta - \frac{1}{2}\left(-2\beta + \frac{8\beta^3}{6}\right) = \frac{\pi}{2} - \epsilon$$

$$\frac{\pi}{2} - \epsilon = \frac{\pi}{2} - \frac{8\beta^3}{12} \iff \beta^3 = \frac{3\epsilon}{2}$$

# Cloud fragmentation

At the time close to the collapse, the density could be rewritten as :

$$\frac{\rho(t)}{\rho_0} = \frac{1}{\cos^6(\frac{\pi}{2} - \beta)} = \frac{1}{\sin^6(\beta)} \approx \left(\frac{2}{3\epsilon}\right)^2$$

Small angle approximation
$\sin\beta \approx \beta$

Remember that :

$$\tau = \frac{t}{t_0} = \frac{t_{ff} - (t_{ff} - t)}{t_0} = \frac{\pi}{2} - \epsilon$$

Hence :

$$\frac{\rho(t)}{\rho_0} = \left(\frac{2t_0}{3\left(t_{ff} - t\right)}\right)^2$$

Note that $\rho(t)/\rho_0$ depends only on $t_{ff}$ and not on the initial radius $r_0$

# Cloud fragmentation

Now consider that towards the centre of the initial sphere the density was <u>perturbated to have a slightly higher density</u>, an overdensity such as : $\rho' = \rho_0 + \delta_0$

$$t_{ff} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2}$$

The free-fall time of the overdensity is given by :

$$t'_{ff} = \left(\frac{3\pi}{32G\rho'}\right)^{1/2} = \left(\frac{3\pi}{32G}\right)^{\frac{1}{2}}\left(\frac{1}{\rho'}\right)^{1/2}$$

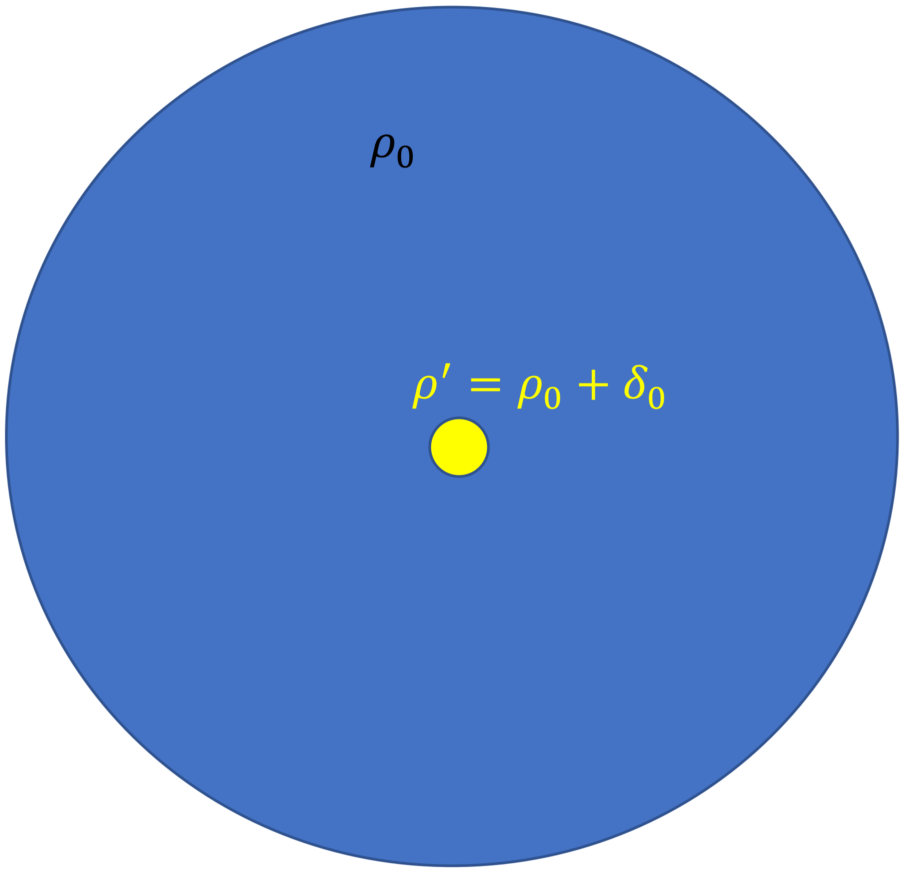
Or, according to the binomial theorem :

$$(\rho + \delta_0)^{-1/2} \approx \frac{1}{\rho_0^{1/2}}\left(1 - \frac{\delta_0}{2\rho_0}\right)$$

Hence :

$$t'_{ff} = t_{ff}\left(1 - \frac{\delta_0}{2\rho_0}\right)$$

In that case the free-fall time is slightly shorter.



# Cloud fragmentation

Towards the end of the collapse, this overdensity will have grown relative to the mean density of the cloud :

$$\frac{\rho'(t)}{\rho(t)} \approx \left(\frac{t_{ff}-t}{t'_{ff}-t}\right)^2 \approx 1+\frac{\delta_0 t_{ff}}{\rho_0\left(t_{ff}-t\right)}$$

$$\frac{\rho(t)}{\rho_0}=\left(\frac{2t_0}{3\left(t_{ff}-t\right)}\right)^2 \qquad t'_{ff}=t_{ff}\left(1-\frac{\delta_0}{2\rho_0}\right)$$

Therefore, the overdensity grows as :

$$\frac{\delta(t)}{\rho(t)} \approx \frac{\delta_0 t_{ff}}{\rho_0\left(t_{ff}-t\right)}$$

This becomes very large when $t \rightarrow t_{ff}$

All over-densities in the cloud grow at the same time. No dependence on mass or radius

Remember that from the Jeans analysis, we obtained :

- The dispersion relation :

$$\omega^2 = a_T^2\left(k^2-k_J^2\right)$$

- The mass associated with a perturbation :

$$M \sim \left(\frac{\lambda}{2}\right)^3 \rho \sim \left(\frac{k}{\pi}\right)^{-3} \rho$$

➔ In the Jeans analysis, the growth of overdensity depends on the wavevector $k$ and hence on the mass $M$.

Conclusions : In this analysis, a small inhomogeneity in the pressure-free case will grow algebraically with time, and all perturbations grow at the same rate

➔sub-clumps of different masses form stars of different masses at the same time

# Cloud fragmentation

Qualitatively therefore we expect the following :

- A cloud which is initially very large compared to the Jeans mass will start to undergo approximately pressure-free collapse.
- Many factors will break the symmetry :
  - Initial shape of the cloud
  - Large-scale rotation
  - Small scale velocity variations (i.e. turbulence)
- Any initial inhomogeneities will grow with time and they all grow on similar timescales
- Eventually we expect the densest of these to become self gravitating in their own right

# Cloud fragmentation

**HOW DE WE FORM THE INITIAL MASS FUNCTION ?**

Observationally, there is a good correspondence between the cloud-mass spectrum and the shape of the IMF.

But how does this mass spectrum come about ?

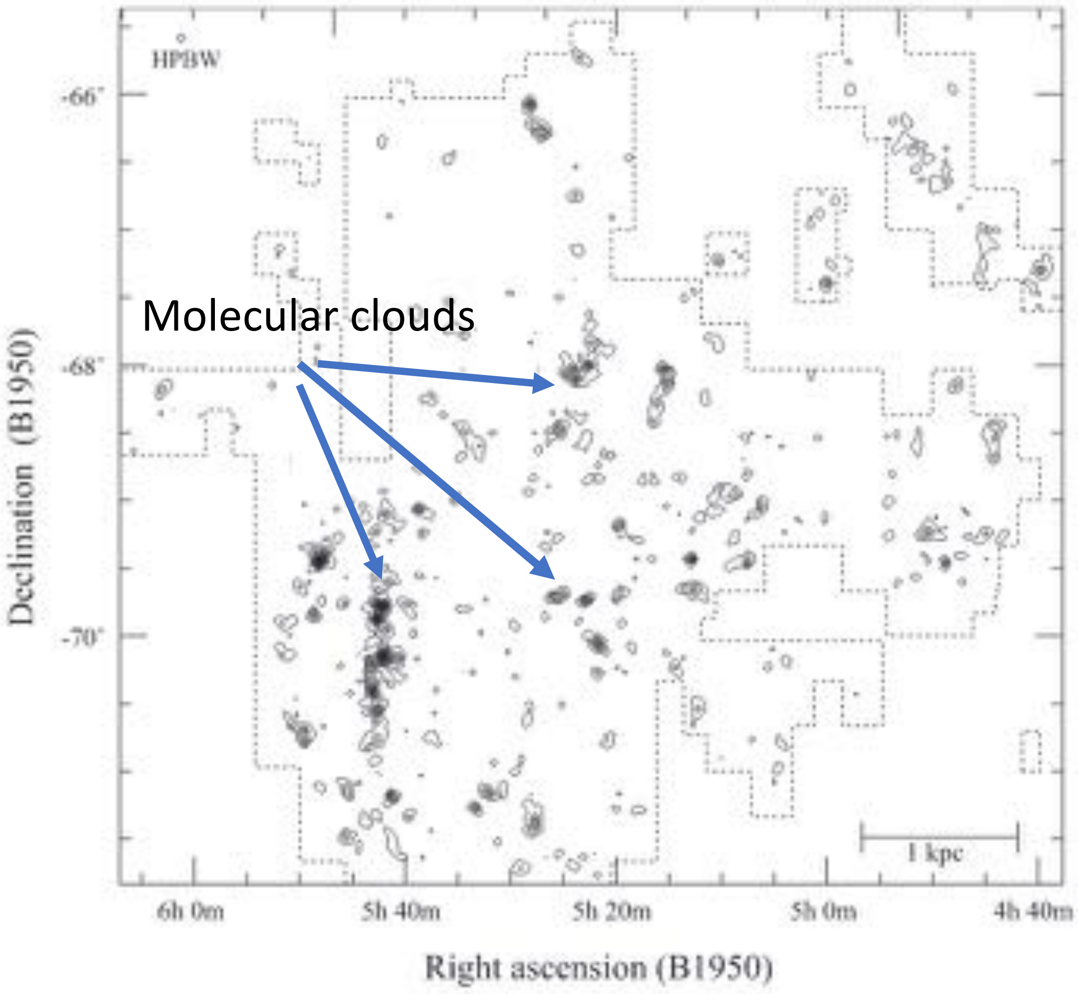
Input physics almost certainly includes :

- Turbulence-energy input drives random motions in the gas giving rise to a turbulent cascade. The standard result is that the spectrum of energy in turbulent motion satisfies :

$$E(k)dk \propto k^{-5/3}dk$$

- The most successful models invoke scale-free, or fractal, structures within the cloud
- Competitive accretion – for example, the denser cores grow by faster than the less dense cores by competing more strongly for the low-density gas

Further progress requires numerical simulations…




Fukui et al. 2001

UK Astrophysical
Fluids Facility

Matthew Bate

EXETER

# Cloud fragmentation


**t=0** : We begin with such a gas cloud, 2.6 light-years across, and containing 500 times the mass of the Sun. The images measure 1 pc (3.2 lightyears across).

**t=38kyr** : 38,000 yr: Clouds of interstellar gas are seen to be very turbulent with supersonic motions.

**t=76kyr** : As the calculation proceeds, the turbulent motions in the cloud form shock waves that slowly damp the supersonic motions.

**t=152kyr** : When enough energy has been lost in some regions of the simulation, gravity can pull the gas together to form dense "cores".

**t=190kyr** : The formation of stars and brown dwarfs begin in the dense cores

**t=209 kyr**: As the stars and brown dwarfs interact with each other, many are ejected from the cloud.

# Galactic-wide star formation

**WHAT FACTORS CONTROLS STAR-FORMATION ON A GALACTIC SCALE ?**

Some definitions first :

- $\psi$ : SFR per unit of volume of the galaxy
- $\Psi$ : SFR for the whole galaxy
- $\Sigma_{SFR}$ : SFR per unit projected area of a galaxy

Intuitively, we expect the SFR to depend on the amount of available fuel.

This has been demonstrated observationally: the Schmidt law

$$\Sigma_{SFR} \propto \sigma^n$$

where $\sigma$ is the surface gas density.

For a constant disc thickness, then :

$$\psi \propto \rho^n$$

# Galactic-wide star formation

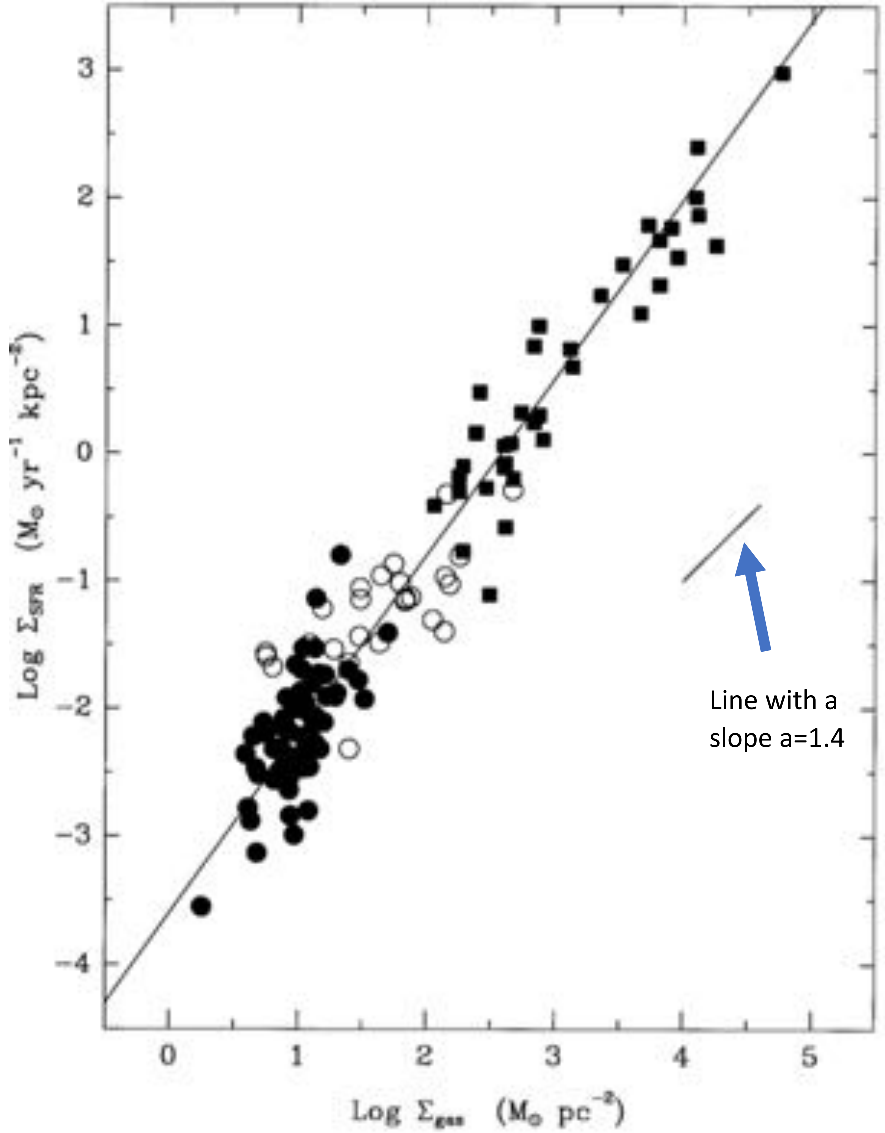
**WHAT FACTORS CONTROLS STAR-FORMATION ON A GALACTIC SCALE ?**

The best observational result is the ***Schmidt-Kennicutt*** law :

$$\frac{\Sigma_{SFR}}{M_{\odot} yr^{-1} kpc^{-2}} = 2.5\times10^{-4} \left(\frac{\sigma}{M_{\odot} pc^{-2}}\right)^{1.4}$$

This result has been obtained by observing 97 galaxies.

In the following, we will consider simple models to explain the observational S-K relation. We start our analysis by considering clouds which exceed the Jeans Mass, and then collapse.



# Galactic-wide star formation

**MODEL 1 : Collisional Assembly**

Here we assume that the ISM consists of many small clouds, each less massive than the Jeans Mass.

Larger clouds are constructed by collisions between the small clouds.

In this simple model, the collision rate will be :

$$C = n^2 v A$$

Density of clouds

The RMS mean velocity in the disc

Cross section of the cloud

We also assume the collision time is long compared to the free-fall time of the clouds once they exceed their Jeans Mass.

This suggests that :

$$\psi \propto \rho^2$$

which is not in agreement to the observed S-K law.

# Galactic-wide star formation

**MODEL 2 : Collapse-time limited**

In this model, we assume that we already have large clouds already in place, which exceeds the Jeans Mass by a large factor (e.g., Giant Molecular Clouds)

From the previous discussion, we know that these clouds will collapse and fragment on a free-fall time.

In that case, the SFR will be proportional to the gas density divided by the collapse time scale, such as :

$$\psi \propto \frac{\rho}{t_{ff}}$$

Remember that $t_{ff}$ is given by :

$$t_{ff} = \left(\frac{3\pi}{32G\rho}\right)^{1/2}$$

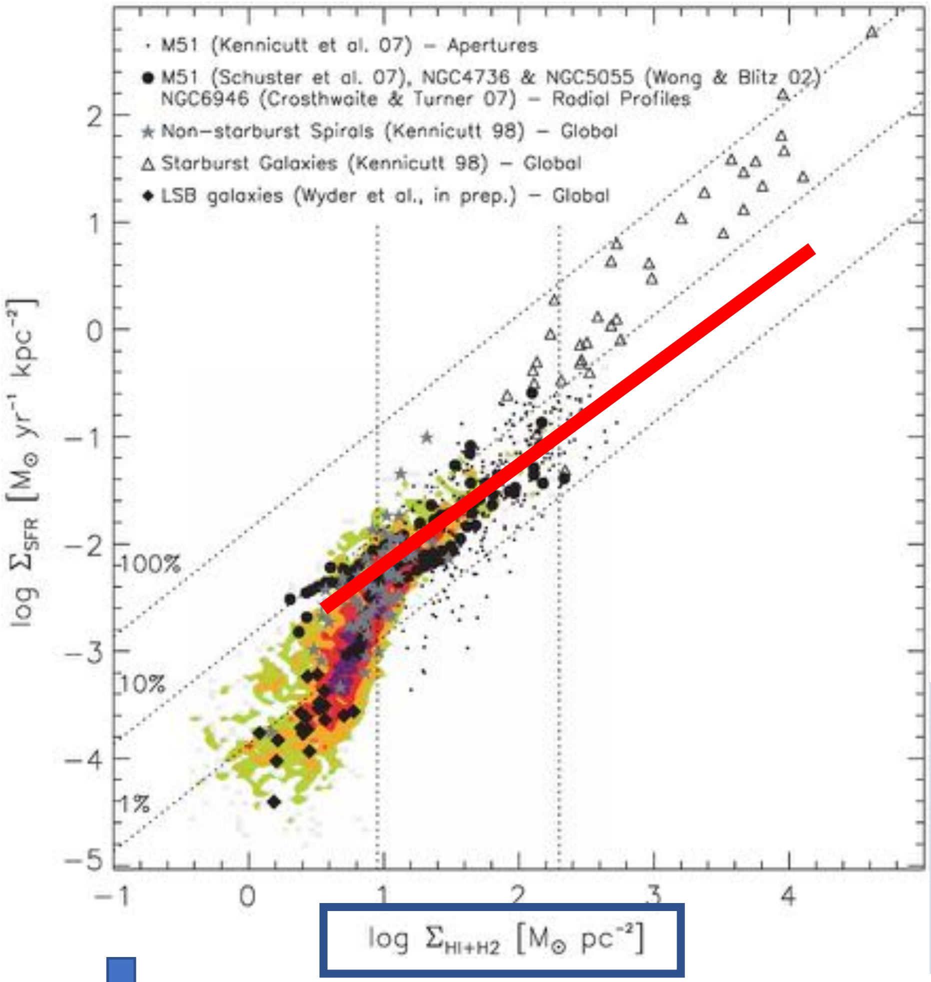
Therefore, we get :

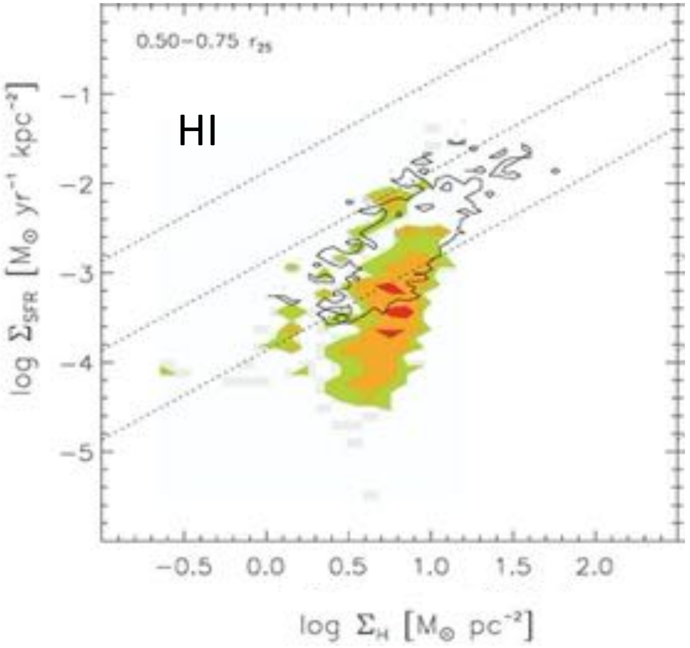
$$\psi \propto \rho^{3/2}$$

which is close to the S-K relation :

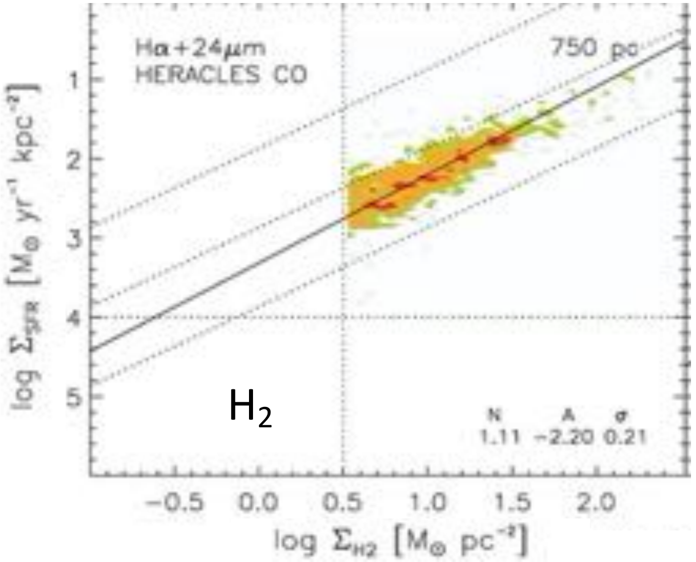
$$\frac{\Sigma_{SFR}}{M_{\odot} yr^{-1} kpc^{-2}} = 2.5\times10^{-4} \left(\frac{\sigma}{M_{\odot} pc^{-2}}\right)^{1.4}$$

# Galactic-wide star formation




The star-formation depends only on the molecular gas and not on the atomic component.




If we only consider the dense component of the molecular gas, we have a linear relation. The non-linearity at low-density is probably due to the diffuse molecular gas which does not participate to the Star Formation.

There is a limit in gas mass below which star formation is very inefficient (Bigiel et a. 2008)

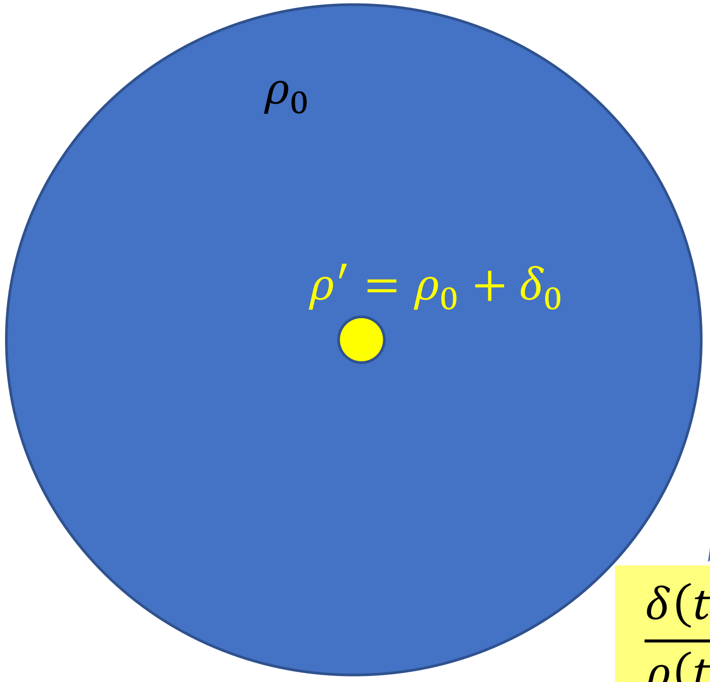
# Summary of Monday's lecture

$$t_{ff} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2}$$

Only depends on the initial density

$$\frac{\rho(t)}{\rho_0} = \left(\frac{2t_0}{3\left(t_{ff} - t\right)}\right)^2$$

Only depends on the free-fall time



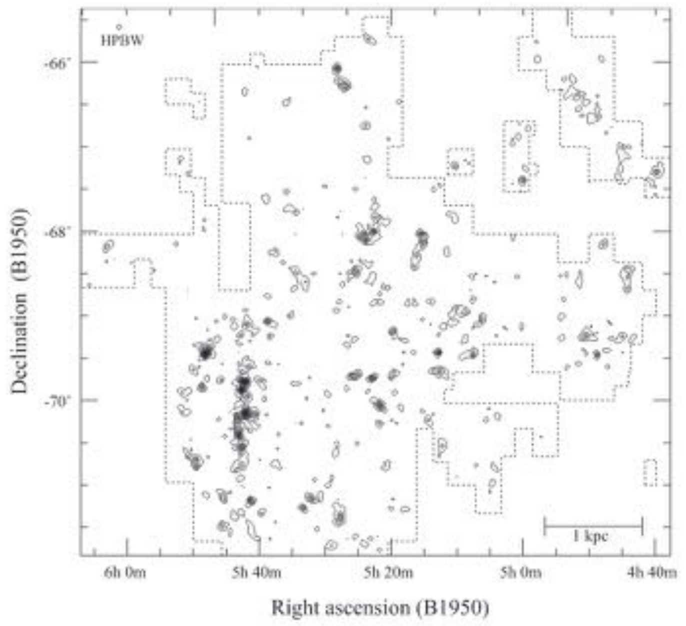

All overdensities grow at the same time, no dependance on mass or radius

$$\frac{\delta(t)}{\rho(t)} \approx \frac{\delta_0 t_{ff}}{\rho_0\left(t_{ff} - t\right)}$$

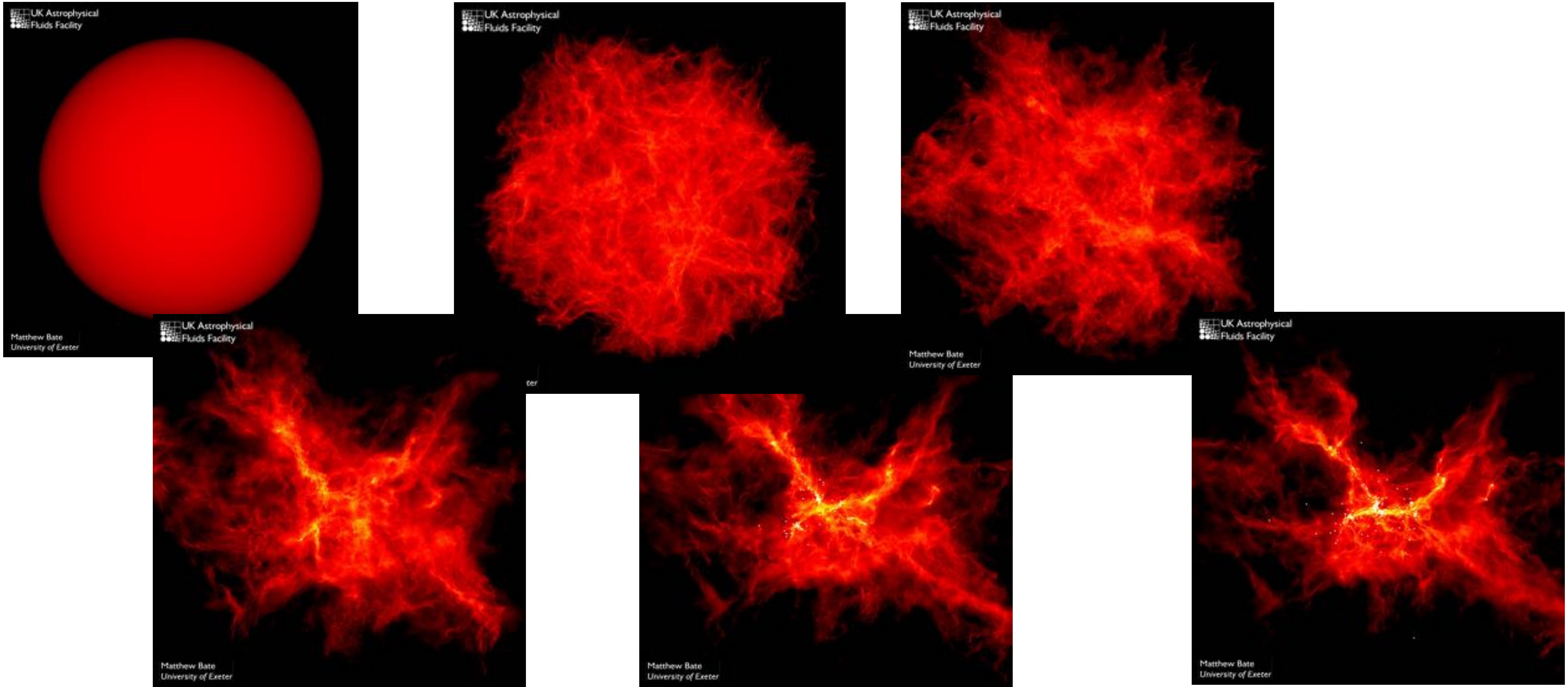
$$t'_{ff} = t_{ff}\left(1 - \frac{\delta_0}{2\rho_0}\right)$$

The free-fall time of the perturbed region is shorter than in its surrounding

fragmentation



# Summary of Monday's lecture

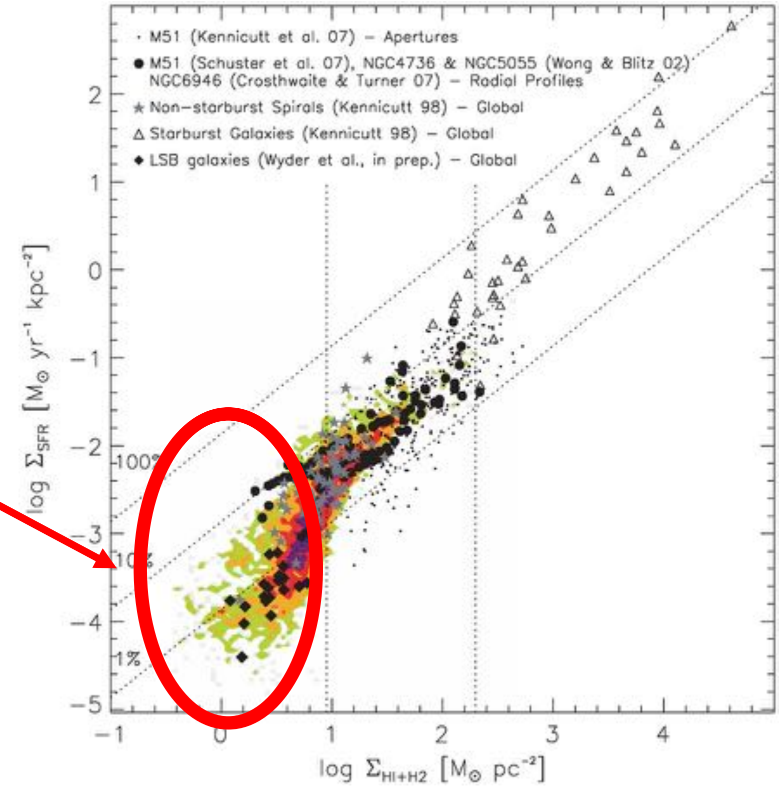


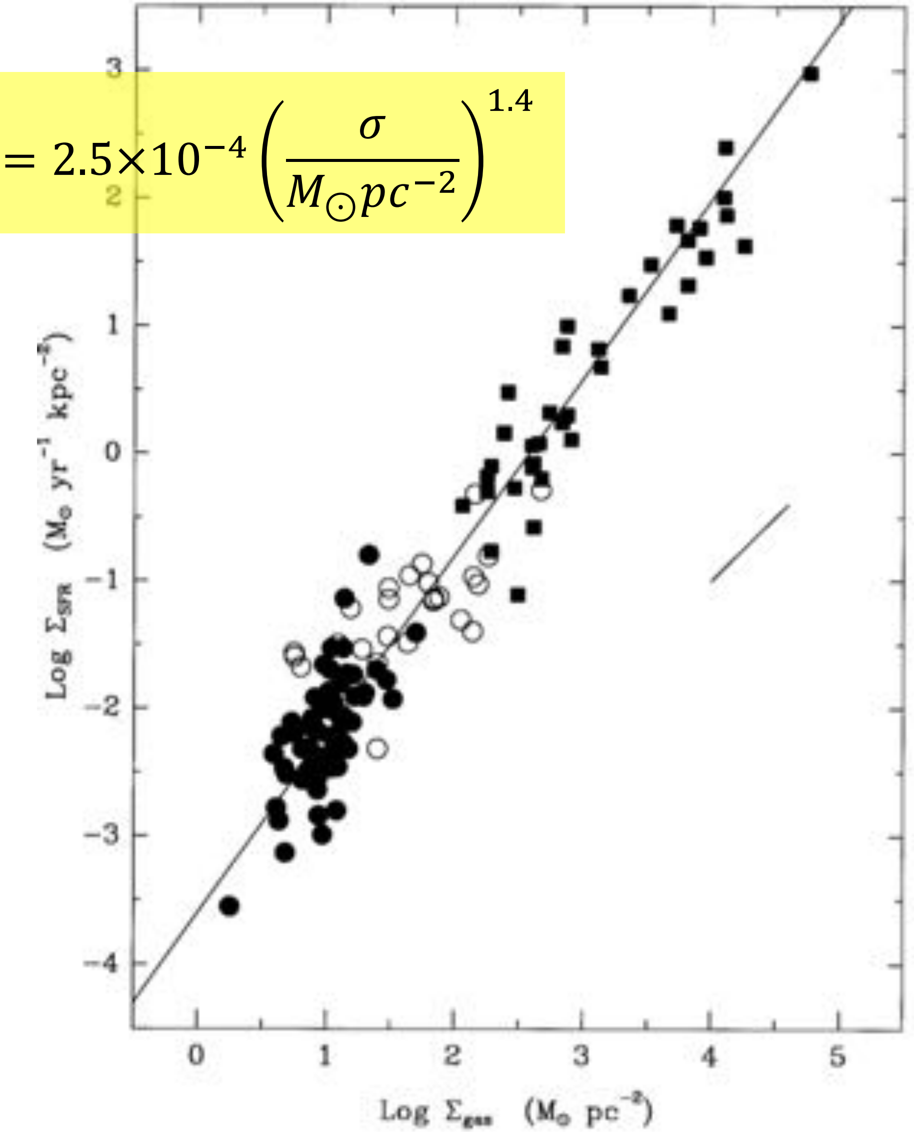
# Summary of Monday's lecture

Collisions/merging of small clouds can not explain the shape of the SK relation. The only way we can explain the slope of the SK relation is by assuming that Giant Molecular Clouds exist in the Universe.

$$\frac{\Sigma_{SFR}}{M_{\odot}yr^{-1}kpc^{-2}} = 2.5\times10^{-4}\left(\frac{\sigma}{M_{\odot}pc^{-2}}\right)^{1.4}$$

There is a minimum gas mass below which star formation is inefficient



# Simple models of gas and star-formation evolution in galaxies

In the following, we aim to trace the evolution of star-formation rate in galaxies.

We first need to make the following definitions :

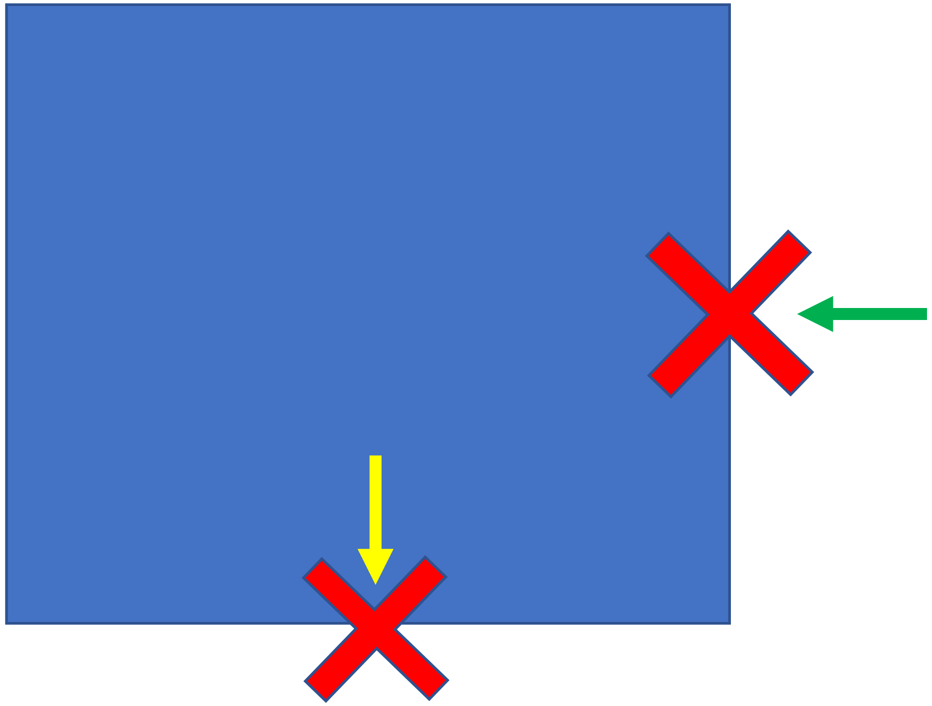
- The initial total gas mass is $M_0$
- The mass in the gas is a function of time : $g(t)$
- The mass in stars is given by $s(t)$
- The star formation rate is given by $\Psi(t)$

and the following assumptions :

- Gas is returned from the stars to the ISM via supernovae in an instantaneous process
- The fraction of mass locked up in old stars is $\alpha$

Gas is returned to the ISM from supernovae at a rate : $(1-\alpha)\Psi$. This phenomena is called ***feedback***.

**MODEL 1 : Closed-box**



In this model, there is no gas inflow and no gas outflow.

# Simple models of gas and star-formation evolution in galaxies

**MODEL 1 : Closed-box**

The mass of the gas evolves as :

$$\frac{dg}{dt} = -\Psi + (1-\alpha)\Psi = -\alpha\Psi$$

Amount of gas in new stars

The gas returned to the ISM by SNe

We can then assume a linear relation :

$$\Psi(t) = \epsilon g(t)$$

where $\epsilon$ is the *star formation efficiency* (the mass of stars formed per unit gas mass)

$\epsilon$ can also be seen as the *depletion time*, i.e. the time needs by a star-formation rate to completely use the available gas :

$$\tau_{depl} = \frac{1}{\epsilon} = \frac{\mathrm{g}}{\Psi}$$

We then get the following equation for the evolution of the gas mass :

$$\frac{dg}{dt} = -\alpha\epsilon\mathrm{g}$$

Hence :

$$\frac{dg}{g} = -\alpha\epsilon dt$$

Then :

$$[\ln g(t)]_{g(t=0)}^{g(t)} = -\alpha\epsilon t$$

Or $g(t=0) = M_0$ (by definition), then :

$$\ln g(t) - \ln M_0 = -\alpha\epsilon t$$

Finally :

$$g(t) = M_0 e^{-\alpha\epsilon t}$$

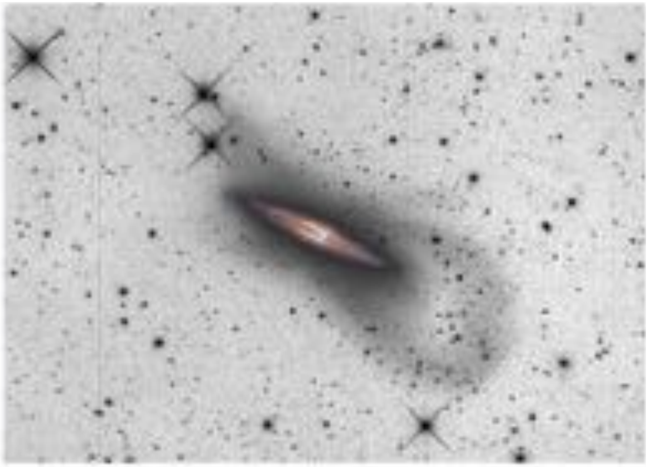
For a galaxy in a closed-box, the gas mass decreases exponentially with time.

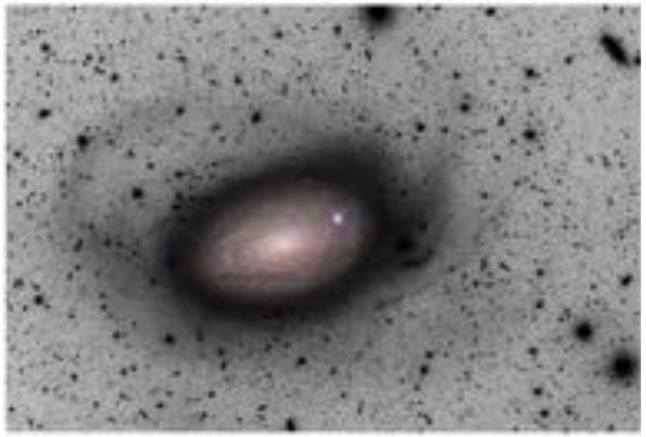
The stellar mass :

$$s(t) = M_0 - g(t) = M_0(1 - e^{-\alpha\epsilon t})$$

# Simple models of gas and star-formation evolution in galaxies

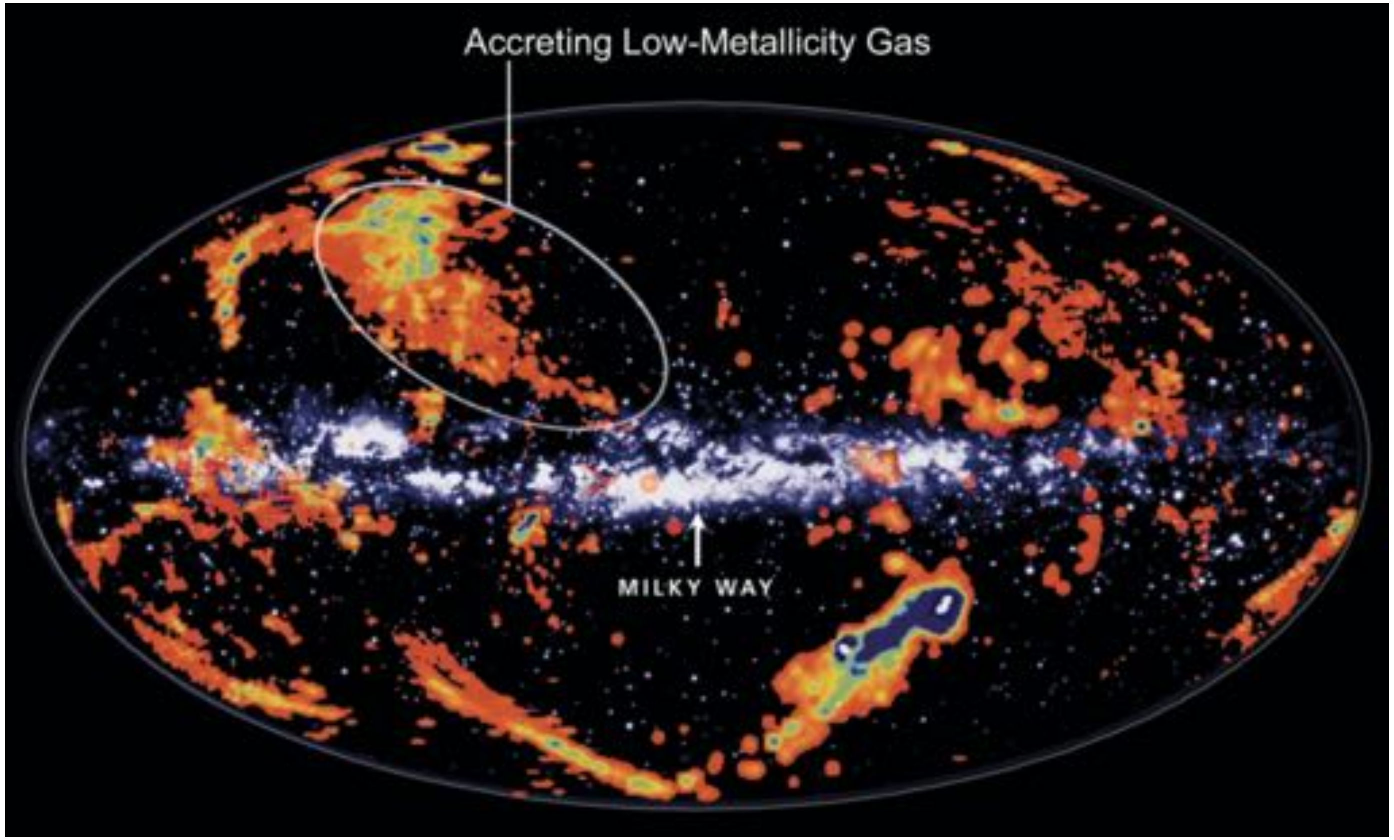
Galaxies in the Universe are not in closed-box : inflows and outflows of gas characterize the life of most galaxies.





Observing the atomic gas in and around the Milky Way reveals large gas clouds in the halo of our galaxy.

Similar accreting gas has also been observed in other galaxies.



# Simple models of gas and star-formation evolution in galaxies

**MODEL 2 : Bathtub**

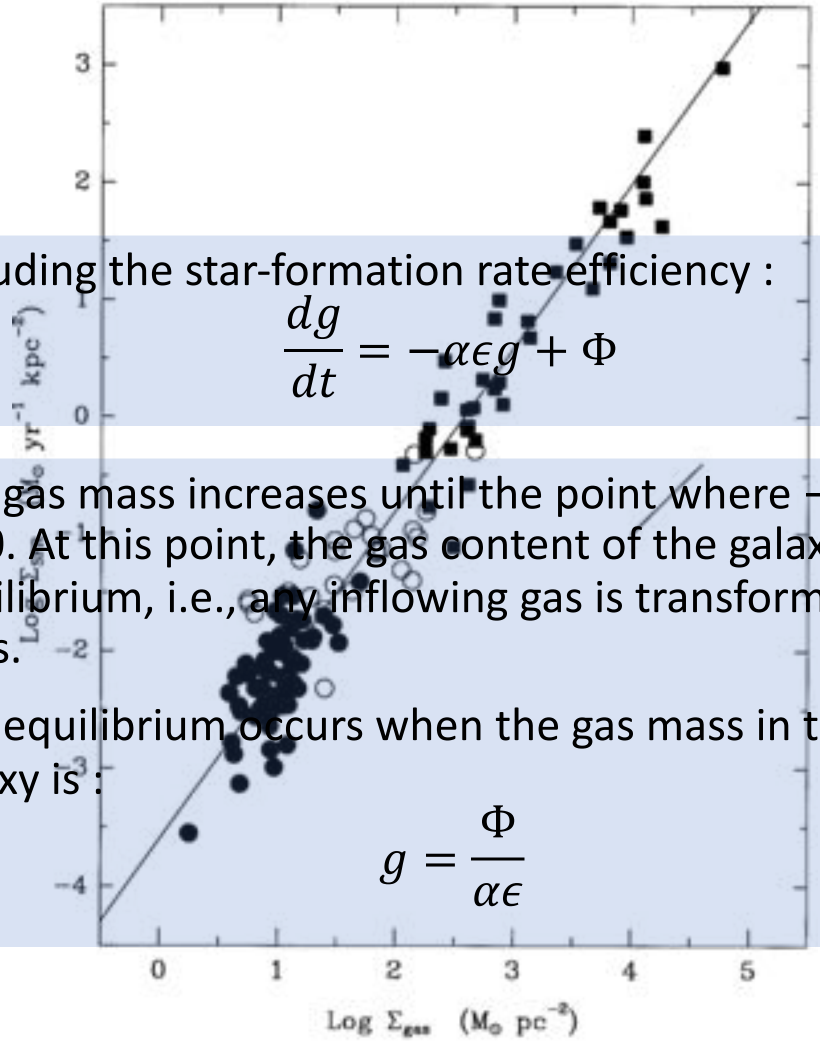
Suppose now a galaxy subject to a constant gas inflow rate $\Phi$, then the evolution of the gas mass becomes :

$$\frac{dg}{dt} = -\alpha\Psi + \Phi$$

One can naively expect that a very large inflow rate $\Phi$ may produce a galaxy extremely rich in gas, with a total mass completely dominated by the gas mass.

➔ *This is clearly not seen for the bulk of galaxies*

The simple reason is that the Star Formation Rate is linked to the total amount of gas through the Schmidt –Kennicutt relation, and it acts as a "valve" that regulates the total amount of gas in the galaxy by transforming the excess inflowing gas into stars.




Including the star-formation rate efficiency :

$$\frac{dg}{dt} = -\alpha\epsilon g + \Phi$$

The gas mass increases until the point where $-\alpha\epsilon g + \Phi = 0$. At this point, the gas content of the galaxy is in equilibrium, i.e., any inflowing gas is transformed into stars.

The equilibrium occurs when the gas mass in the galaxy is :

$$g = \frac{\Phi}{\alpha\epsilon}$$

# Simple models of gas and star-formation evolution in galaxies

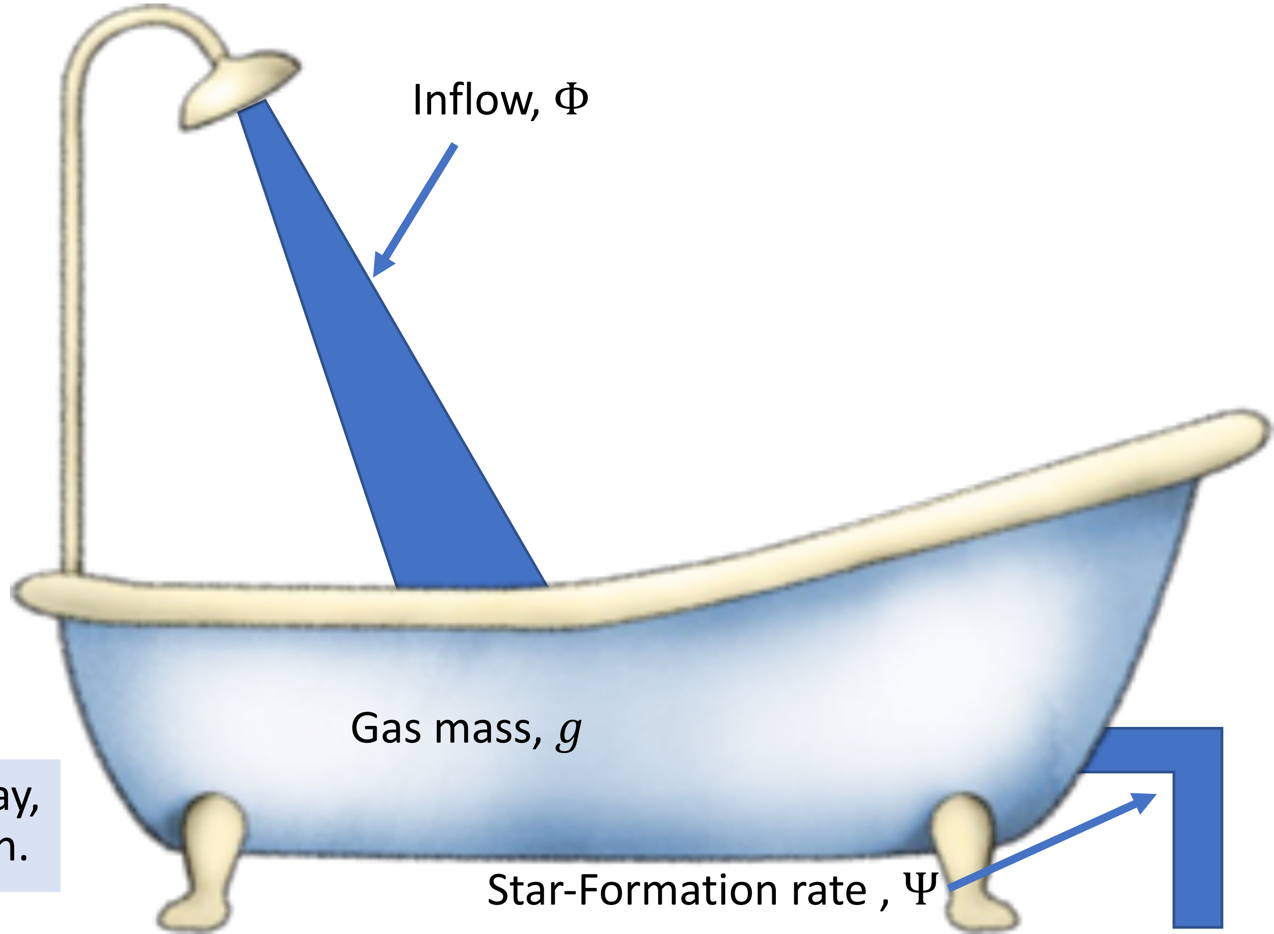
**MODEL 2 : Bathtub**

In the bathtub model, the gas inflow can be seen as the water flowing from the tap, the gas mass can be seen as the water in the bathtub and the star formation rate is the water flowing out of the drain.

The rate at which the water flows out of the drain is proportional to the water pressure, hence proportional to the amount of the water in the bathtub.

If the rate of inflowing water from the tap is increased or decreased the level of water in the bathtub increases or decreases to reach a new equilibrium point, where the associated pressure makes the outflow rate again in equilibrium with the inflow rate.

The amount of gas in a galaxy works in a similar way, where the water pressure is replaced by the S-K relation.



# Simple models of gas and star-formation evolution in galaxies

**MODEL 2 : Bathtub**

The gas fraction is the mass of the gas relative to the total baryonic content (i.e., gas and stars), which is often an indicator of evolutionary stage of a systems :

$$f_{gas} = \frac{M(gas)}{M(baryons)} = \frac{M(gas)}{M(gas) + M(star)} = \frac{g}{g+s}$$

At equilibrium, for a constant inflow rate, the gas mass is constant :

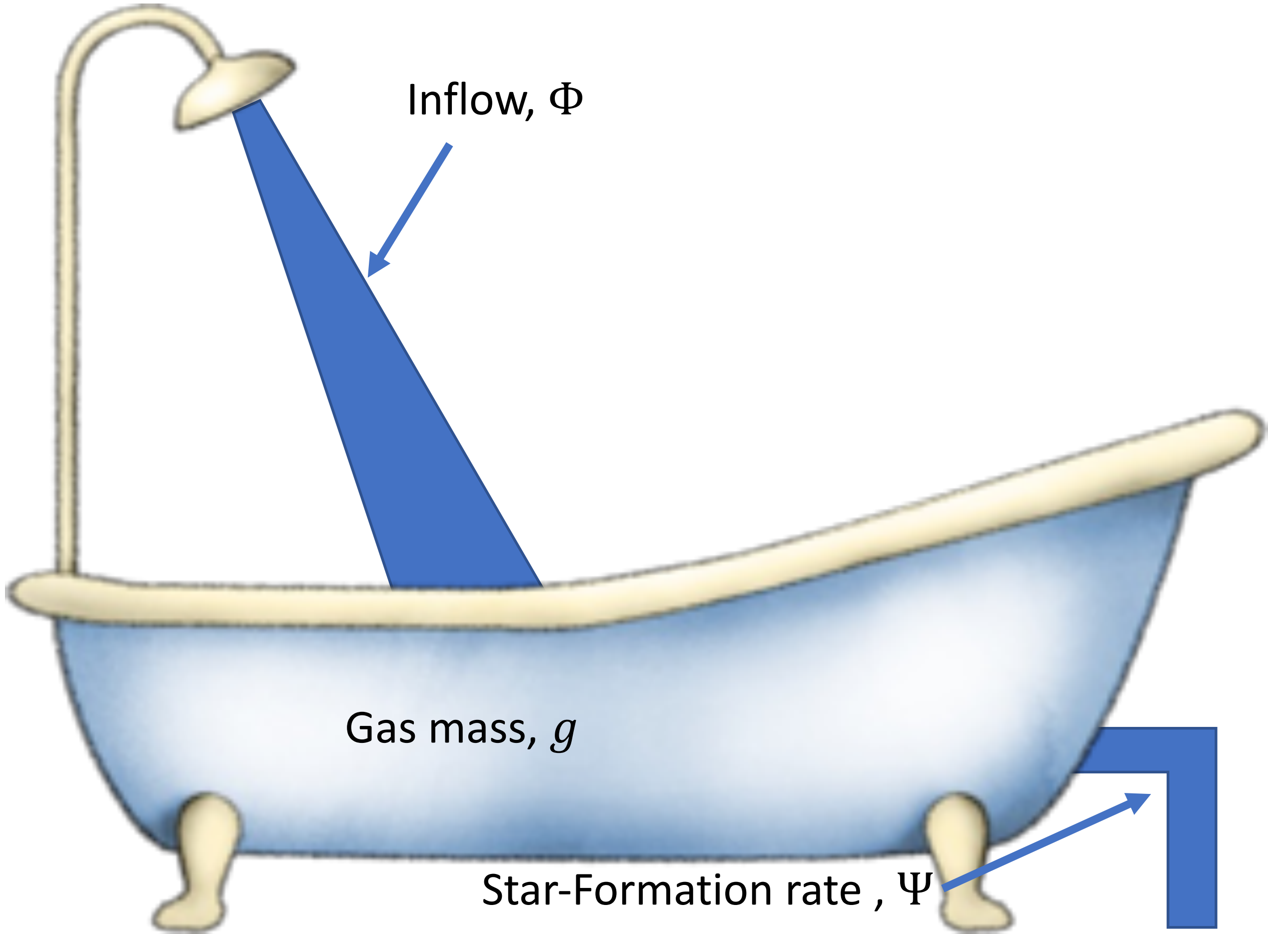
$$g = \frac{\Phi}{\alpha\epsilon}$$

The stellar mass keeps growing, and is given by :

$$s(t) = \Phi t - g$$

The gas fraction steadily decreases with time, hence making galaxies "gas poor". After the equilibrium has been reached :

$$f_{gas} = \frac{1}{\alpha\epsilon t}$$



# Simple models of gas and star-formation evolution in galaxies

**MODEL 2 : Bathtub**

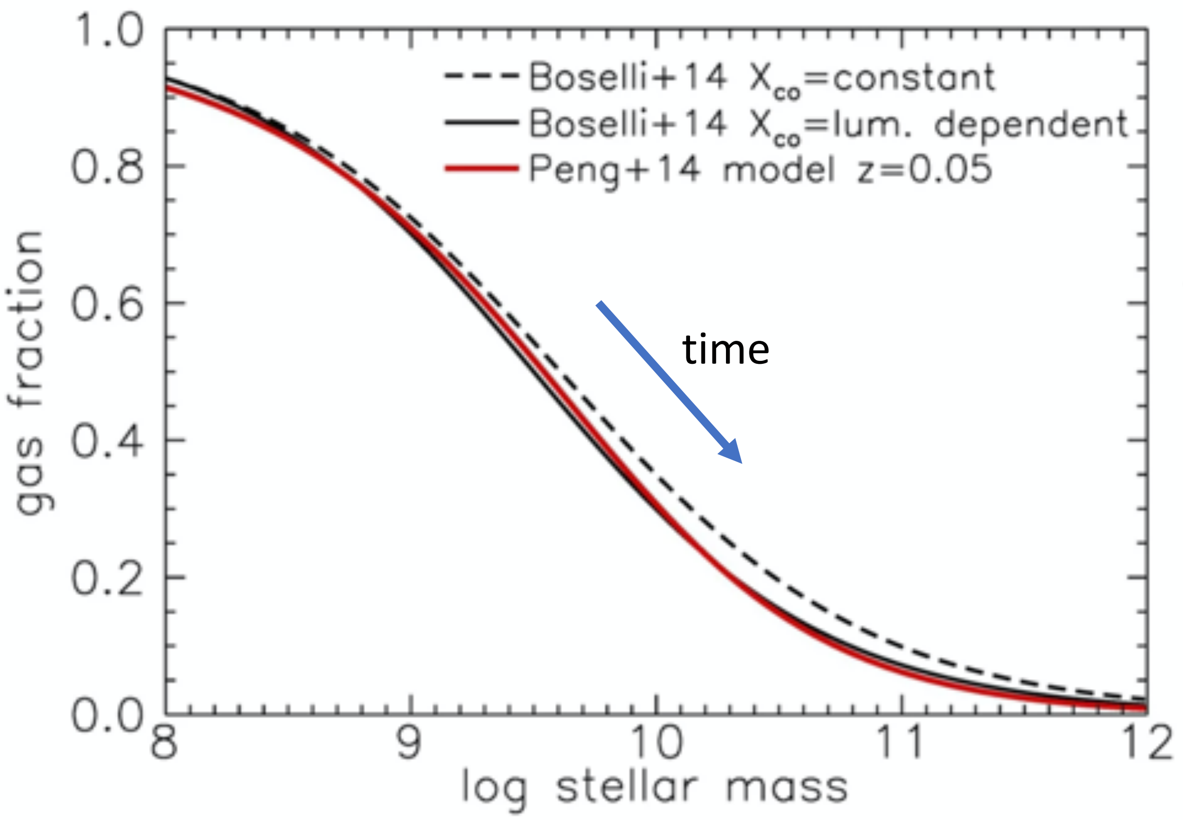
The gas fraction is also indirectly related to the stellar mass and can be expressed as

$$f_{gas}(t) = \frac{\alpha + 1}{\alpha \epsilon s(t)}$$

which highlights the relation between the stellar mass and the gas fraction.

It is expected that the gas fraction should decrease with the stellar mass, which is indeed observed in local galaxies

The gas fraction is therefore a good tracer of the galaxy evolutionary stage, i.e., galaxies with low gas fraction are typically more evolved than galaxies with high gas fraction.



# Simple models of gas and star-formation evolution in galaxies

**THE EFFECT OF OUTFLOWS**

One of the primary mechanisms responsible for driving outflows is associated to SNe explosions. Radiation pressure from the light emitted by young luminous stars is another possible mechanism.

SNe and radiation pressure from young stars are linked to the star formation rate, hence we can express the outflow rate $\Lambda$ as a function of the star-formation rate :

$$\Lambda = \lambda\Psi$$

where $\lambda$ is the outflow loading factor (observations give $\lambda \sim 1$ for actively star-forming galaxies).

Introducing the outflows effect, the gas evolution with time can be written as :

$$\frac{dg}{dt} = \underbrace{-\alpha\Psi}_{\text{Star formation}} + \underbrace{\Phi}_{\text{Inflow}} - \underbrace{\lambda\Psi}_{\text{Outflow}}$$

# Simple models of gas and star-formation evolution in galaxies

**THE EFFECT OF OUTFLOWS**

The gas evolution with time is given by :

$$\frac{dg}{dt} = -\alpha\Psi + \Phi - \lambda\Psi$$

Including the Schmidt-Kennicutt relation ($\Psi = \epsilon g$) gives :

$$\frac{dg}{dt} = -\alpha\epsilon g + \Phi - \lambda\epsilon g$$

which gives an equilibrium gas mass ($dg/dt$=0) :

$$g = \frac{\Phi}{(\alpha + \lambda)\epsilon}$$

The effects of outflows is to greatly increase the effective value of $\alpha$, i.e., the amount of gas lost.

At equilibrium, the stellar mass is given by $s \approx \alpha\Psi t = \alpha\epsilon g t$.

Therefore, at equilibrium, the gas fraction is still given by :

$$f_{gas} \approx \frac{1}{\alpha\epsilon t + 1} \approx \frac{1}{\alpha\epsilon t}$$

Independent of outflow rate

Gas fraction seems not to have an effect in explaining the lower gas fraction in massive galaxies.

In massive galaxies, Active Galactic Nuclei can greatly contribute to enhance the outflow rate, hence effectively increasing the value of $\lambda$, even by a factor of several, hence contributing to greatly reduce the gas content in massive galaxies

# Metallicity evolution of galaxies

The definition of metals for astronomer is different from the well-admitted definition of metals in Physics : “all elements heavier than Helium”.

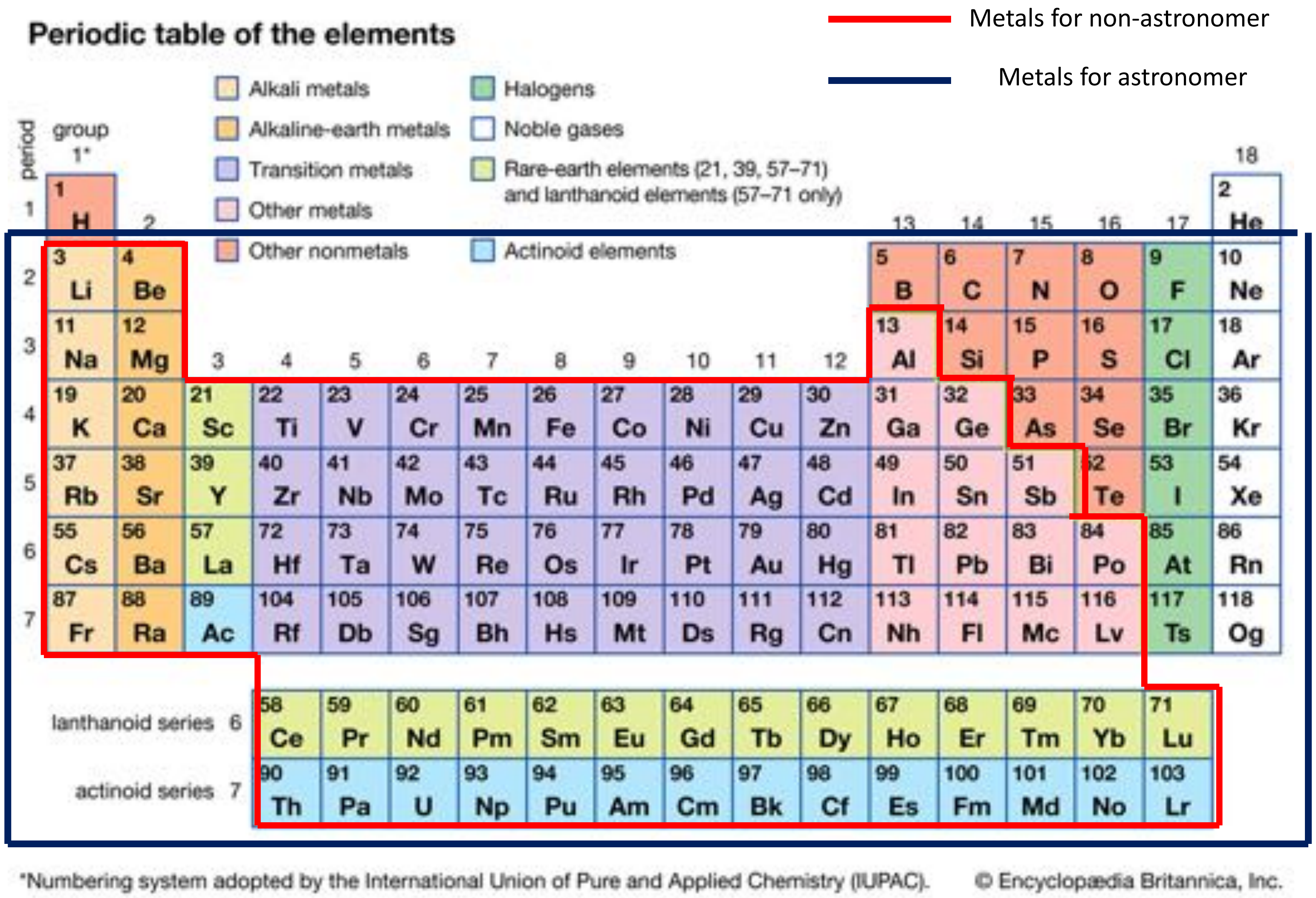
The metallicity is the mass fraction of heavy elements defined as :

$$Z = \frac{M_{metals}}{M_{tot}} \approx \frac{M_{metals}}{M_{gas}}$$

We differentiate :

- Stellar metallicity : mass fraction of metals in the stellar atmosphere
- Gas metallicity : mass fraction of metals in the Inter Stellar Medium (ISM)

The solar metallicity is $Z_{\odot}$=0.014



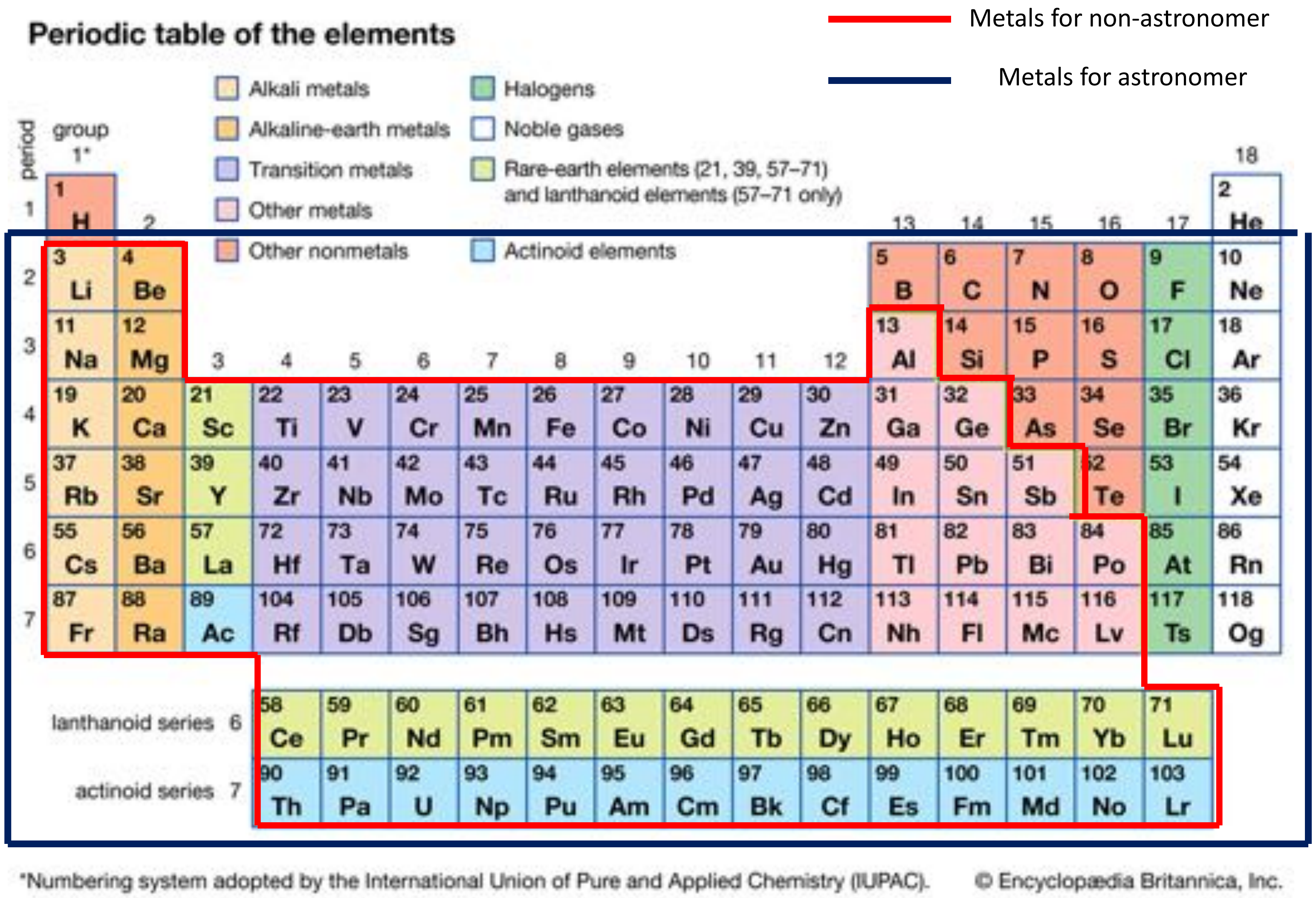
# Metallicity evolution of galaxies

We can also use the relative numeric abundances of elements when discussing the metallicity :

$$[A/B] = \log_{10}\left(\frac{N_A}{N_B}\right) - \log_{10}\left(\frac{N_A}{N_B}\right)_{\odot}$$

with an alternative notation of the numeric abundance :

$$12 + \log A/H = 12 + \log_{10}\left(\frac{N_A}{N_H}\right)$$



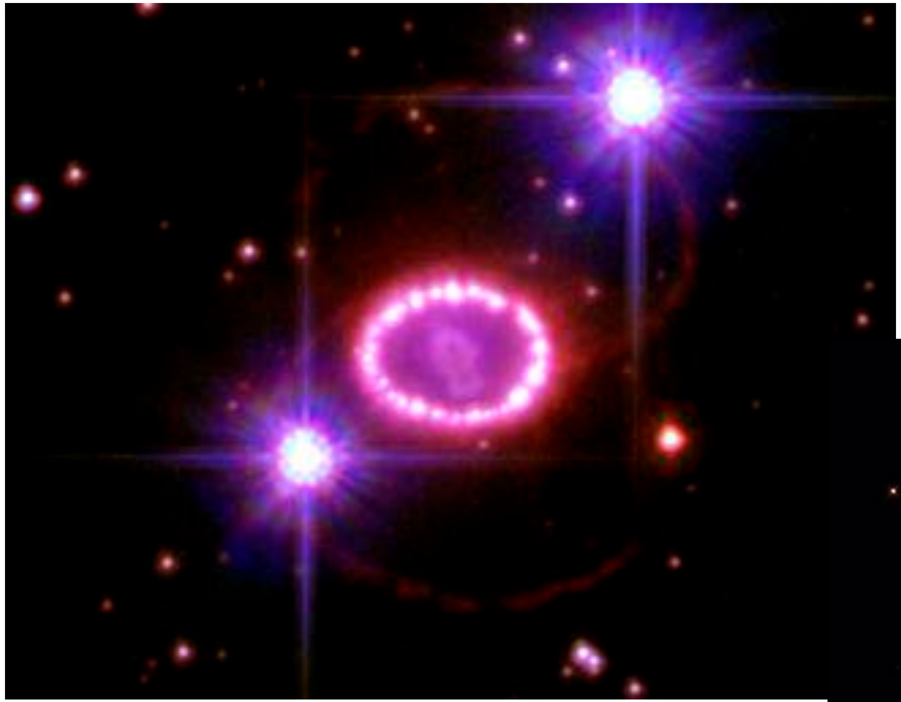
# Metallicity evolution of galaxies

Excepts for Lithium and Beryllium, metals are produced by stellar nucleosynthesis and released into the ISM by SNe and stellar winds.

The metals injected into the ISM is then use to produce new stars.

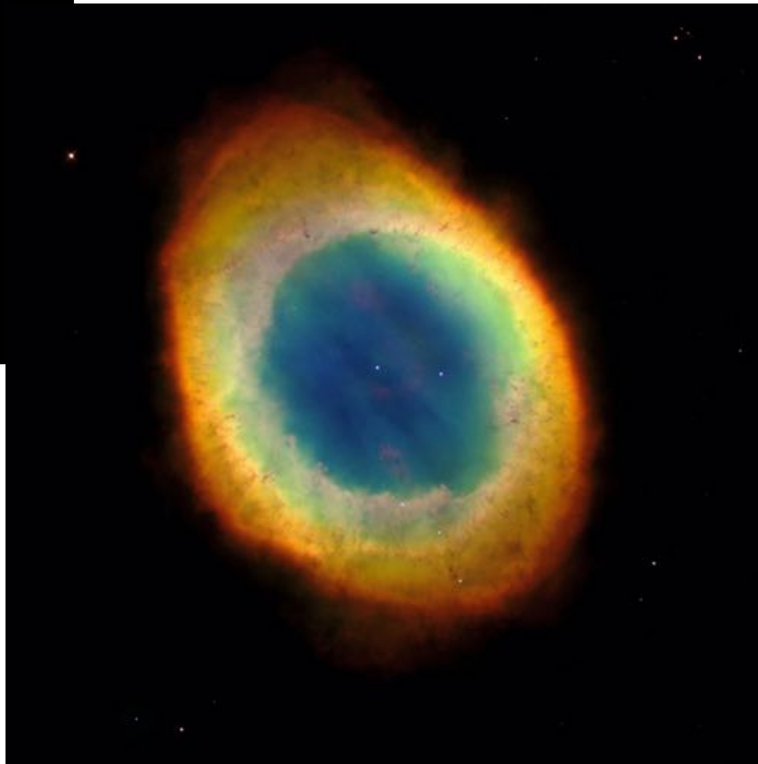
There are two types of SNe :

- **Type II** (core-collapse) : stars with mass larger than $8M_\odot$ leave the Main Sequence in less than 30Myr, explode in type II SNe, and enrich the ISM in $\alpha$ elements (O, Ne, Mg, Si, S, Ca, …)
- **Type I** : stars with mass less than $8M_\odot$ take much longer to leave the MS, they evolve as Asymptotic Giant Branch stars (AGB) and then planetary nebulae and enrich the ISM mostly with C and N

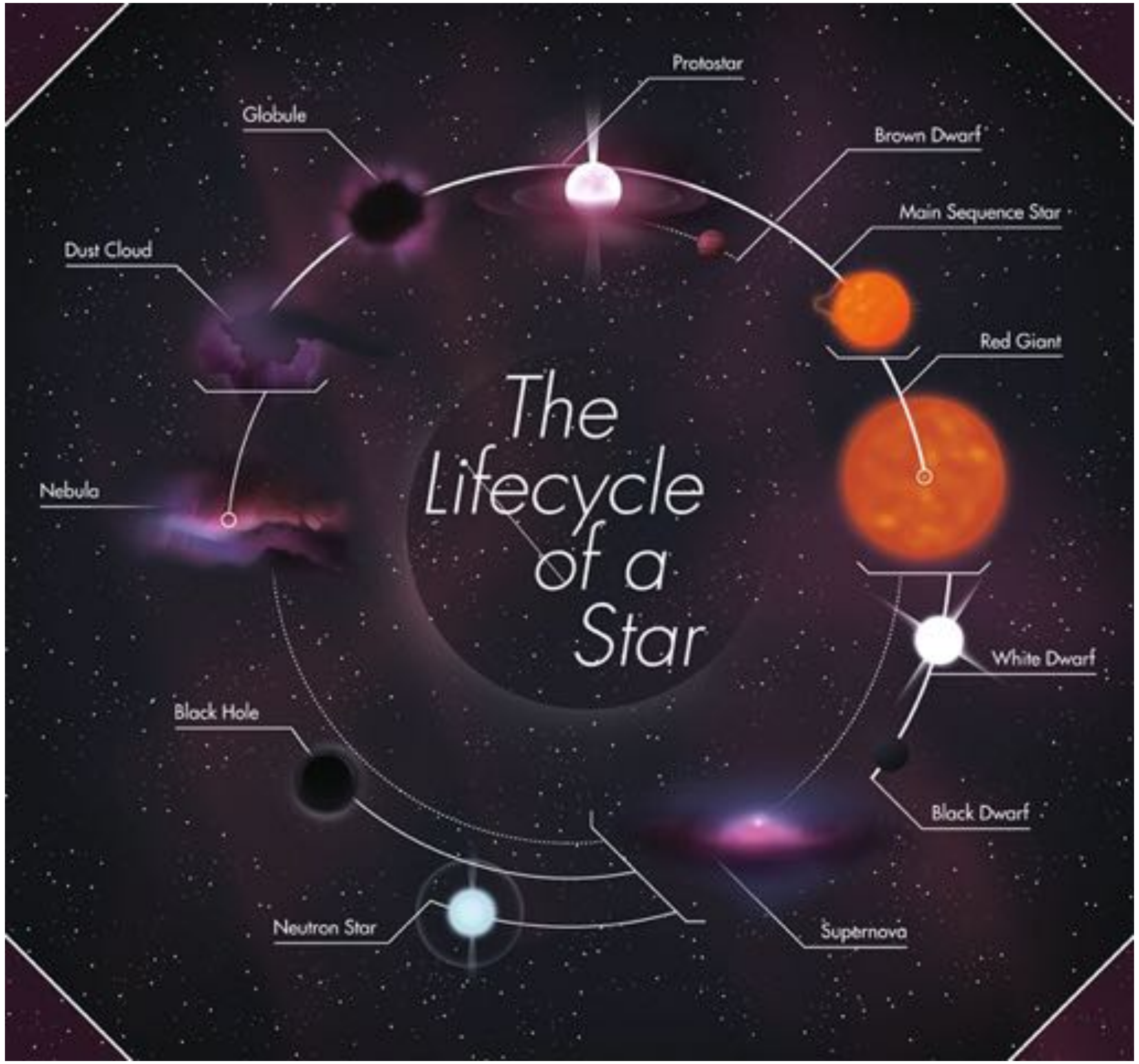


Remnant of type II SNe
SN1987 in LMC

Planetary Nebulae



# Metallicity evolution of galaxies




Each time a star explode, it enriches the ISM in metals. Then, the metallicity of the ISM (and therefore of the new born stars) increases with time.

We can then expect that the first generation of stars (pop III stars) had a smaller metallicity than stars currently form in our Milky Way.

# Metallicity evolution of galaxies

**IN A CLOSED-BOX SYSTEM**

This is the simplest model we can use to study the evolution of metallicity in a galaxy.

We recall the definitions and assumptions we previously made :

- The initial total gas mass is $M_0$
- The mass in the gas is a function of time : $g(t)$
- The mass in stars is given by $s(t)$
- The star formation rate is given by $\Psi(t)$
- Gas is returned from the stars to the ISM via supernovae in an instantaneous process at a rate $(1-\alpha)\Psi$
- The fraction of mass locked up in old stars is $\alpha$

Because we want to follow the evolution of metallicity, we need to define new variables :

- The production of new metals per mass of stars is $p$
- The rate of mass of <u>new</u> metals returned to the ISM via SNe is therefore : $p(1-\alpha)\Psi$
- The total mass of metals returned to the ISM via supernovae is :

$$(p+Z)(1-\alpha)\Psi$$

Additional metals produced by SNe

Pre-existing metals, recycled and returned to the ISM by SNe

# Metallicity evolution of galaxies

**IN A CLOSED-BOX SYSTEM**

From our previous study of the closed-box system, we have demonstrated that :

$$\frac{dg}{dt} = -\Psi + (1-\alpha)\Psi = -\alpha\Psi$$

$$Z = \frac{M_{metals}}{M_{tot}} \approx \frac{M_{metals}}{M_{gas}}$$

The evolution of mass of metals in the ISM is given by

$$\frac{d(gZ)}{dt} = (p+Z)(1-\alpha)\Psi - Z\Psi = p(1-\alpha)\Psi - \alpha Z\Psi$$

Metals produced by star formation

Metals “eaten” by star formation

Combining the previous equation gives :

$$g\frac{dZ}{dt} = p(1-\alpha)\Psi = -p\frac{1-\alpha}{\alpha}\frac{dg}{dt} = -P\frac{dg}{dt}$$

where P is called the “yield”, with $P \sim 0.5\, M_\odot$

Hence :

$$dZ = -P\frac{dg}{g}$$

This equation can be integrated easily using the following boundary conditions :

-At $t = 0$ : $Z = 0$ and $g = M_0$

Then :

$$Z(t) = -P\ln\frac{g}{M_0}$$

which gives :

$$g(t) = M_0 \exp\left(-\frac{Z}{P}\right)$$

$$s(t) = M_0 - g(t) = M_0(1-\exp\left(-\frac{Z}{P}\right))$$

All these expressions are independent of the SFR, star formation history and star formation efficiency. They are simply a consequence of recycling the metals in the new stellar generation

# Metallicity evolution of galaxies

**IN A CLOSED-BOX SYSTEM**

Stars formed at a time $t < t_1$ must have a metallicity less than $Z(t_1)$ since they formed out of gas which was less enriched in the past.

For example, the fraction of stars with metallicities les than $0.1Z_\odot$ is given by :

$$s(t) = M_0 - g(t) = M_0(1 - \exp\left(-\frac{Z}{P}\right))$$

$$\frac{s\left(< \frac{Z_\odot}{10}\right)}{M_0} = 1 - \exp\left(-\frac{Z_\odot}{10P}\right) \approx 1 - \exp\left(-\frac{1}{5}\right) \approx 0.2$$

But observationally, the number of old low-metallicity stars in the disc of the Milky Way is much smaller (<0.01)

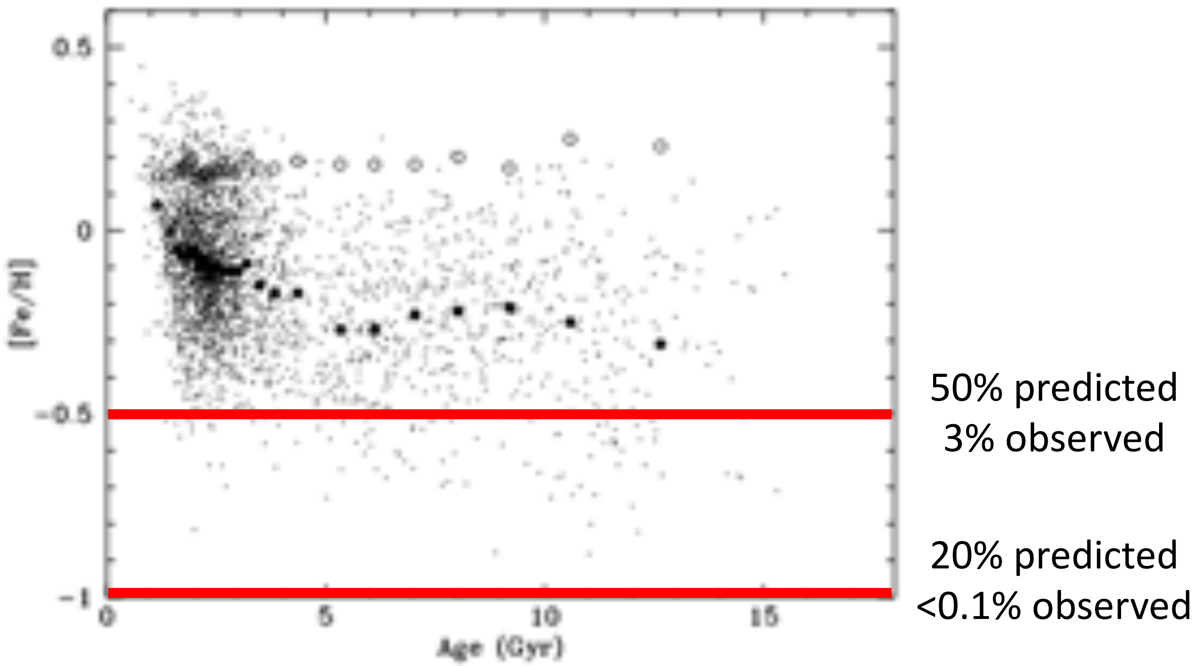
The "G-dwarfs problem"

Our Sun is itself an old G-dwarf star with a relatively high metallicity.

Applying the same equation to determine the fraction of stars with $Z = \frac{1}{3}Z_\odot$ gives :

$$\frac{s\left(< \frac{Z_\odot}{3}\right)}{M_0} = 1 - \exp\left(-\frac{Z_\odot}{3P}\right) \approx 1 - \exp\left(-\frac{1}{1.5}\right) \approx 0.5$$

The closed-box model predicts that half of the stars should have a metallicity $Z = \frac{1}{3}Z_\odot$



# Metallicity evolution of galaxies

A solution to the G-dwarfs problem

Galaxies have not evolved as “closed boxes”

**Gas inflows and outflows, during galaxy evolution, play a major role**

Inflows are the key to solve the “G-dwarfs problem” : by providing additional gas they can prolong star formation, hence enabling a larger number of stars to form out of pre-enriched gas.

# Stellar orbits and spiral structure

We have seen that star-formation should occur in regions of overdensities, i.e., where it is more likely that gas clouds are compressed, perturbated and collapse to form stars.

Observationnally, we clearly see that star formation happens in the spiral arms of disk galaxies.

# Stellar orbits and spiral structure

**Rotation Curves in galaxy disks**

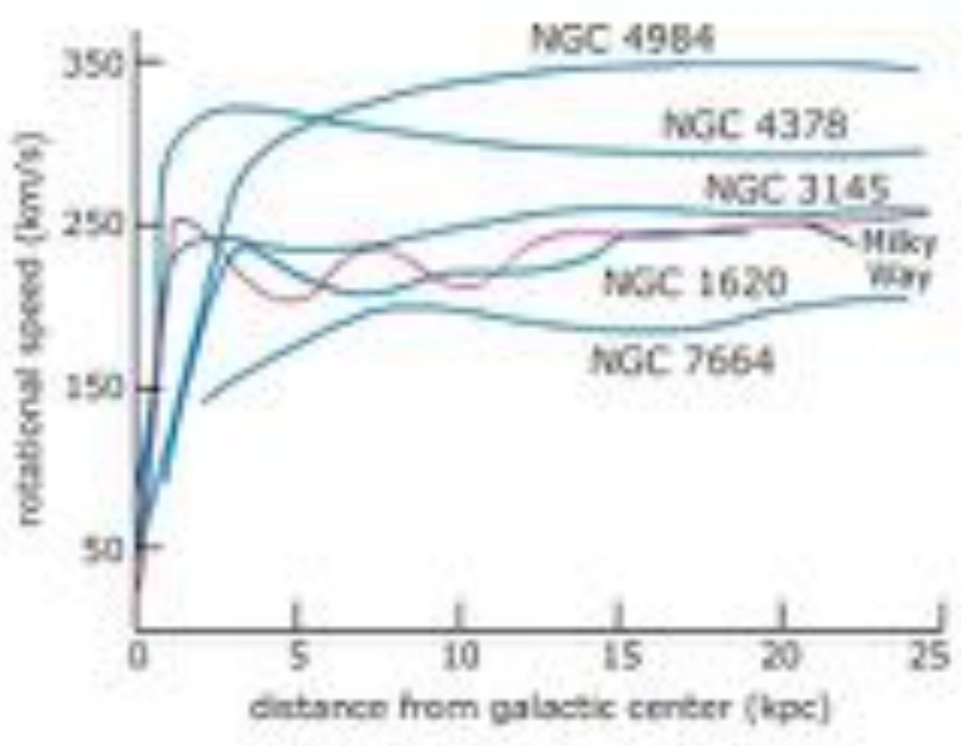
Rotation of stars in a disk is probed by Doppler shift of spectral lines such as the neutral hydrogen (HI at 21cm) or nebular optical lines (e.g., H$\alpha$)

Observationally, we know since the 1970s that the Rotation curves are relatively flat with radius

Assuming a spherical model, the mass within the radius $r$ is obtained by applying the Gauss's theorem :

$$\frac{v^2}{r} = \frac{GM(r)}{r^2}$$

Assuming that the velocity is independent of radius, we get : $M(r) \propto r$, and then $\rho(r) \propto 1/r^2$




Inconsistency

➔ DARK MATTER

But we have seen that the density profile in the disc is given by :

$$\rho(r,z) = \rho_0 \exp\left(-\frac{r}{a}\right) \exp\left(-\frac{|z|}{h}\right)$$

# Stellar orbits and spiral structure

**Stellar Orbits**

To simplify our analysis, we assume a cylindrically symmetric model in which the potential is given by $\Phi(r,z)$ and we examine orbits initially in the $z=0$ plane.

We make the following assumptions :

- The angular momentum per unit mass for each star is conserved : $l = r^2\frac{d\phi}{dt} = constant$ ; where $\phi$ is the azimuthal angle
- The energy per unit mass is also conserved :

$$E = \frac{1}{2}\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}\left(r\frac{d\phi}{dt}\right)^2 + \Phi(r) = \frac{1}{2}\left(\frac{dr}{dt}\right)^2 + \frac{l^2}{2r^2} + \Phi(r)$$

- The equation of motion in the radial direction is just :

$$\frac{\partial^2 r}{\partial t^2} - r\left(\frac{\partial\Phi}{dt}\right)^2 = -\frac{\partial\Phi}{\partial r}$$

It is also useful to introduce the effective potential :

$$\Phi_e = \Phi + \frac{l^2}{2r^2}$$

Hence the radial equation of motion becomes :

$$\frac{\partial^2 r}{\partial t^2} = -\frac{\partial\Phi_e}{\partial r}$$

In a uniform circular motion (no acceleration and r=cste) then :

$$\frac{\partial^2 r}{\partial t^2} = 0$$

Hence :

$$\frac{\partial\Phi_e}{\partial r} = 0 = \frac{\partial\Phi}{\partial r} - \frac{l^2}{r_0^3}$$

Given a $\Phi(r)$, or equivalently a mass distribution, we can calculate the properties of the circular stellar orbit at any $r$

# Stellar orbits and spiral structure

At a radius $r$, the angular velocity of the circular orbit is given by :

$$\Omega(r)^2 = \frac{l^2}{r^4} = \frac{1}{r}\frac{\partial \Phi}{\partial r}$$

A star when perturbed will undergo small motions about this circular orbit.

Write $x = r - r_0$ and expand the effective potential about $r_0$:

$$\Phi_e(x) = \Phi_e(r_0) + x \underbrace{\left(\frac{\partial \Phi_e}{\partial r}\right)_{r_0}}_{\substack{=0 \\ \text{circular orbit}}} + \frac{1}{2}x^2\left(\frac{\partial^2 \Phi_e}{\partial^2 r}\right)_{r_0} + O(x^3)$$

The radial equation ($\frac{\partial^2 r}{\partial t^2} = -\frac{\partial \Phi_e}{\partial r}$) is then

$$\frac{\partial^2 x}{\partial t^2} = -\frac{\partial \Phi_e}{\partial x} = -x\left(\frac{\partial^2 \Phi_e}{\partial r^2}\right)_{r_0}$$

The previous equation has the form of the *Simple Harmonic Motion* (SHM) :

$$\frac{\partial^2 x}{\partial t^2} = -x\kappa^2$$

$$\Phi_e = \Phi + \frac{l^2}{2r^2}$$

with $\kappa$ the *epicyclic frequency*, defined as :

$$\kappa^2 = \left(\frac{\partial^2 \Phi_e}{\partial r^2}\right)_{r_0} = \left[\frac{\partial^2 \Phi}{\partial r^2} + \frac{3l^2}{r^4}\right]_{r_0}$$

or $\Omega(r)^2 = \frac{l^2}{r^4} = \frac{1}{r}\frac{\partial \Phi}{\partial r}$, then :

$$\kappa^2 = \left[r\frac{\partial \Omega^2}{\partial r} + 4\Omega^2\right]_{r_0} = 4\Omega^2\left[1 + \frac{r}{2\Omega}\frac{\partial \Omega}{\partial r}\right]_{r_0}$$

Similarly, we can show that for small amplitude motion out of the plane of the disc, **the star undergoes SMH** with :

$$v^2 = \left(\frac{\partial^2 \Phi_e}{\partial z^2}\right)_{r=r_0, z=z_0}$$

# Stellar orbits and spiral structure

**Resonant Orbits**

In general, spiral structure is complicated, but one important physical idea is the concept of resonant orbits.

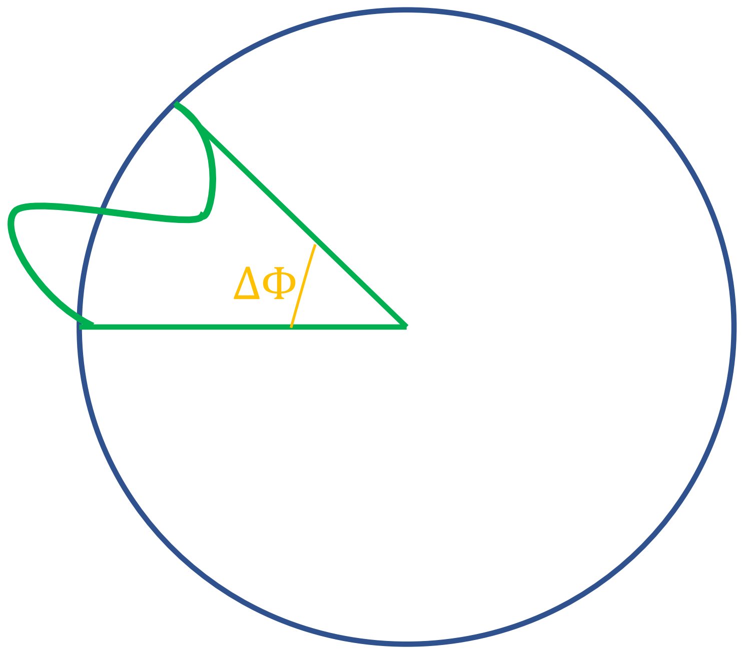
Near circular orbits is superposition of pure circular orbit plus radial motion.

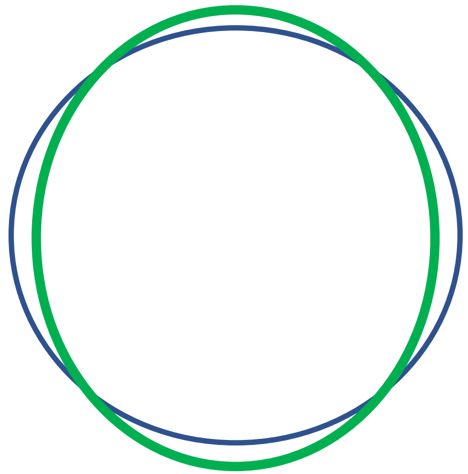
In the lab frame, after one radial oscillation of period $T_r = 2\pi/\kappa$ the orbit will have <u>precessed</u> by $\Delta\Phi \approx \Omega T_r$.

The orbit closes if $\Delta\Phi = 2\pi$

In general, the orbit will not close if $\Delta\Phi \neq 2\pi$, but consider the situation in a frame rotating at $\Omega_p$- the pattern speed, in this system the orbit precession is :

$$\Delta\Phi_p = \Delta\Phi - \Omega_p T_r$$



# Stellar orbits and spiral structure

**Resonant Orbits**

For the orbit to close after $m$ radial oscillations, we require :

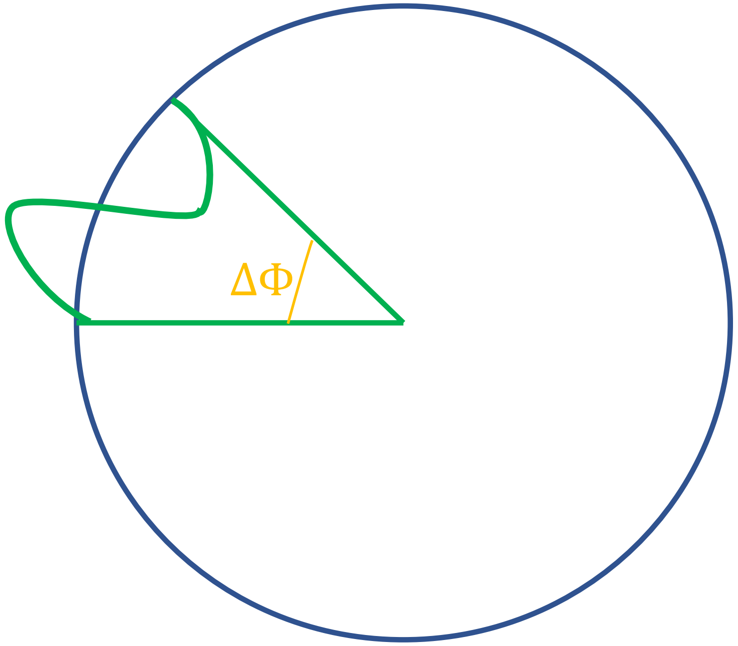
$$n2\pi = \Omega m T_r - \Omega_p m T_r$$

Hence :

$$\Omega_p = \Omega - \frac{n}{m}\kappa$$

Interestingly, in many systems (e.g., flat rotation curve in disc galaxies) the form of $\Omega(r)$ is such that for $n = 1, m = 2$, $\Omega_p$ is nearly constant across the disk, i.e., all epicyclic orbits close at all radii.

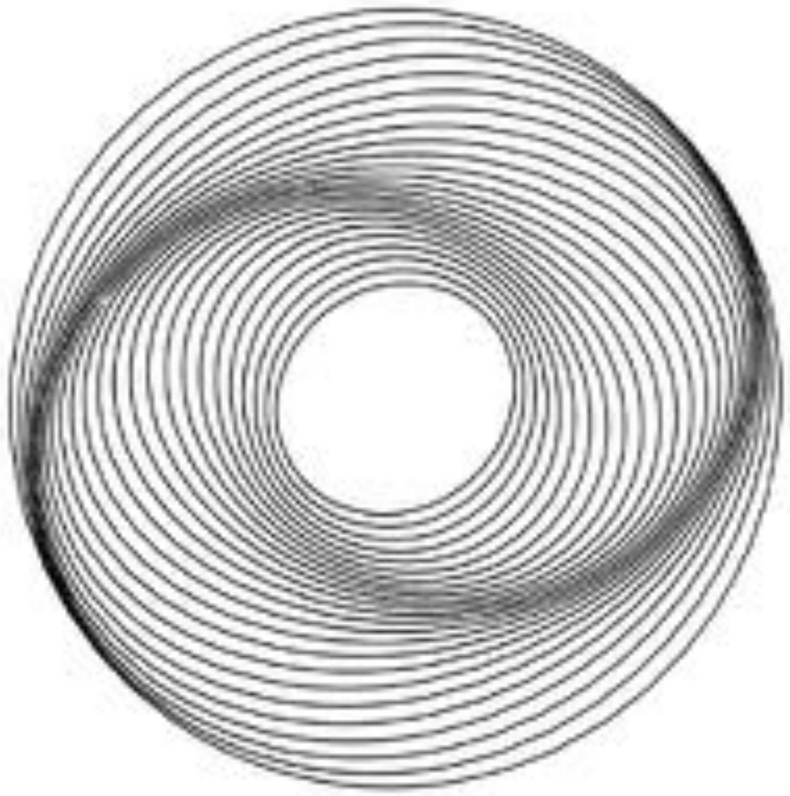
In this case, we can arrange the phase of the orbits so that adjacent stars in certain regions of the disk have a higher density, and put them on the $n = 1, m = 2$ perturbed orbits : <u>these will then be long lived</u>



# Stellar orbits and spiral structure

**Resonant Orbits**

We can achieve this, for example, if there is an external perturbation which causes a rotating potential which is resonant with these orbits



# Summary of Friday's lecture

$$\frac{dg}{dt} = -\alpha\Psi + \Phi - \lambda\Psi$$

Star formation Inflow Outflow

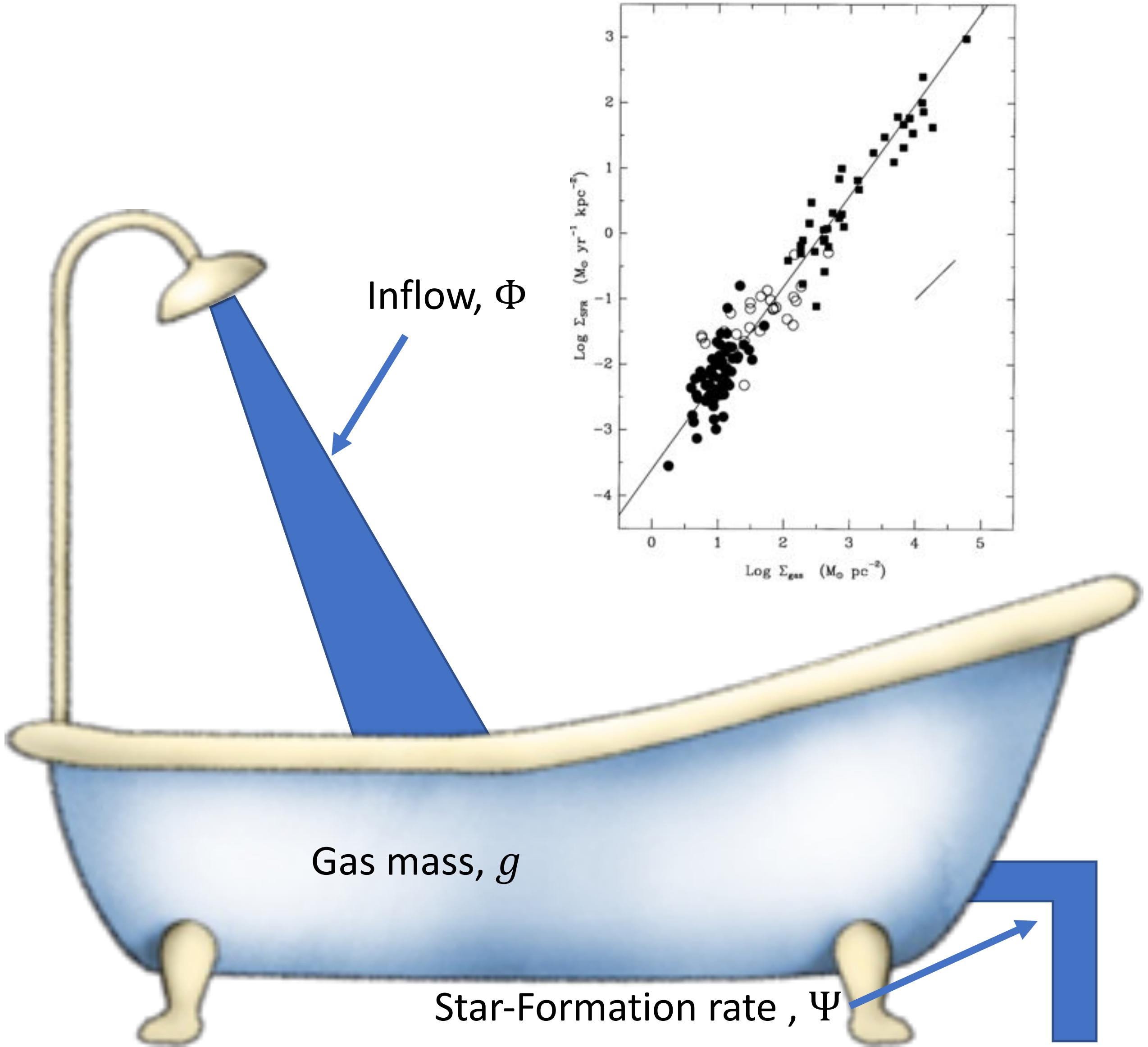
SK relation :
$SFR \propto M_{gas}$

$$\frac{dg}{dt} = -\alpha\epsilon g + \Phi - \lambda\epsilon g$$

At equilibrium

$$f_{gas} = \frac{M(gas)}{M(gas) + M(star)} \approx \frac{1}{\alpha\epsilon t}$$

Independent of inflow and outflow rate

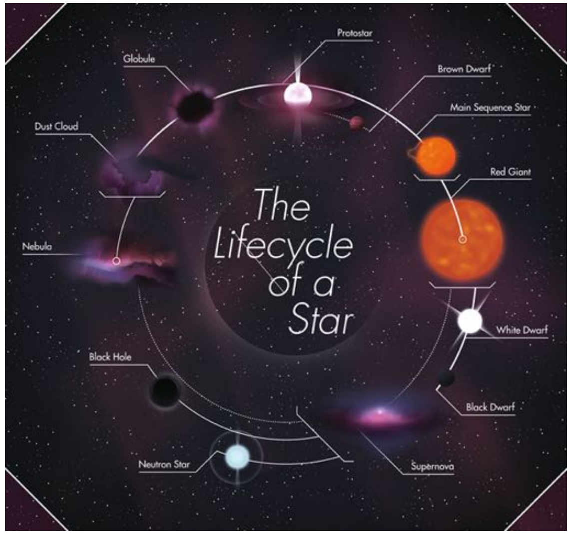


# Summary of Friday's lecture

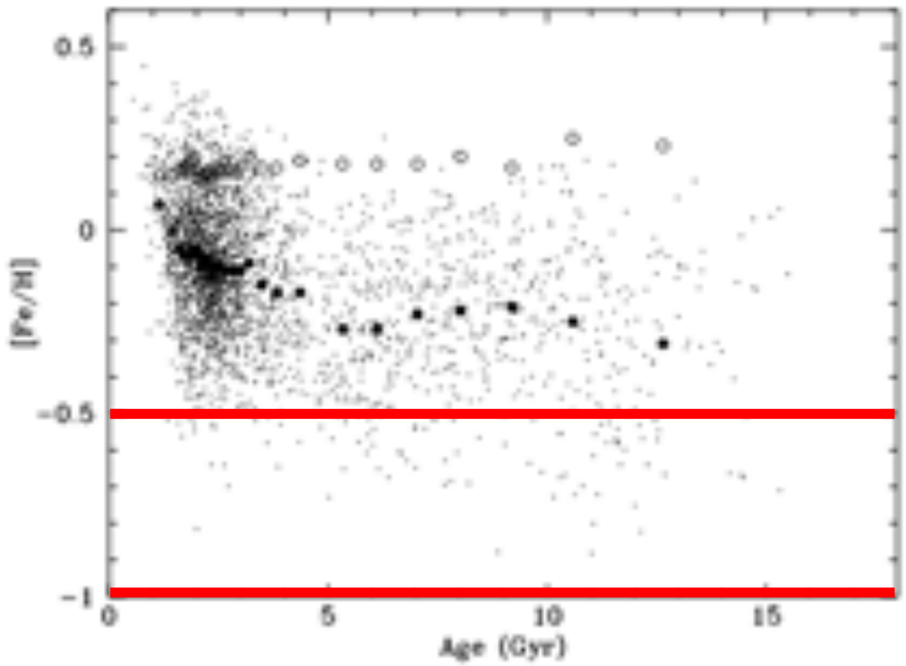
$$Z = \frac{M_{metals}}{M_{tot}} \approx \frac{M_{metals}}{M_{gas}}$$

$$12 + \log A/H = 12 + \log_{10}\left(\frac{N_A}{N_H}\right)$$

Abundance ratio with respect to hydrogen




The G-dwarfs problem

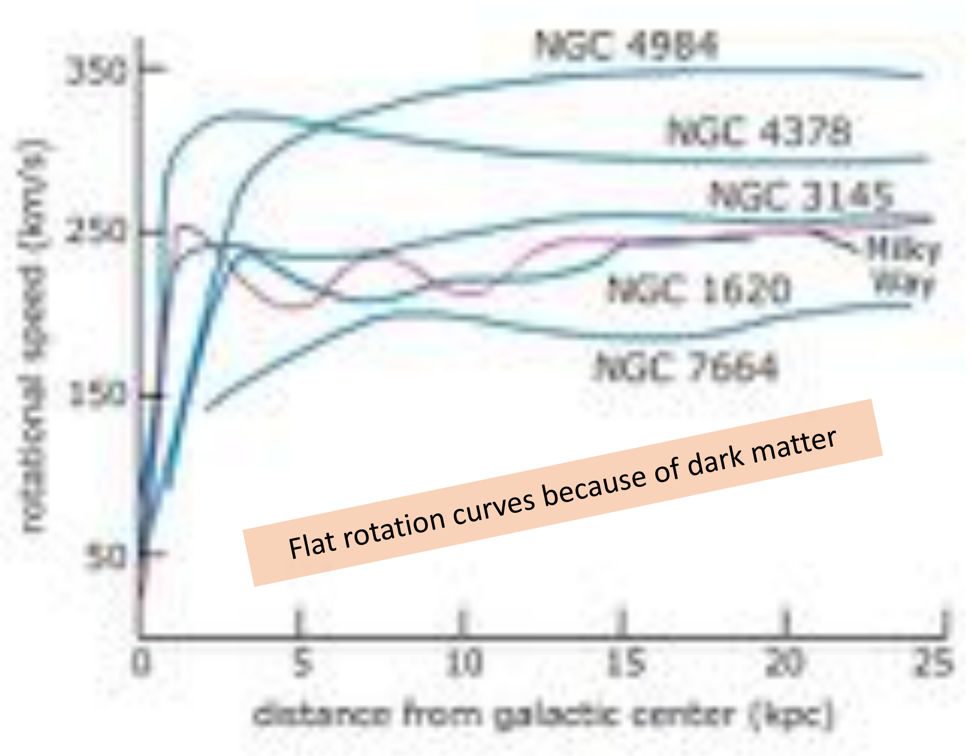



50% predicted
3% observed

20% predicted
<0.1% observed

Galaxies are not evolving like closed boxes : outflows and inflows are taking place !

# Summary of Friday's lecture

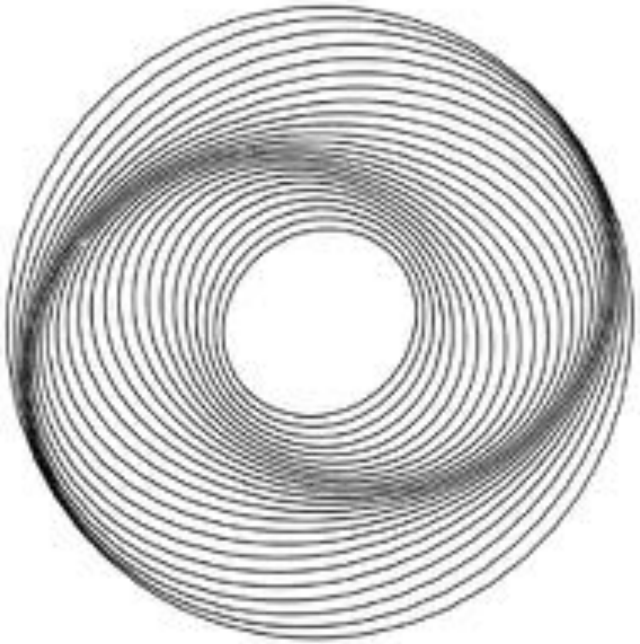



Star undergoes *Simple Harmonic Motion* in a galaxy

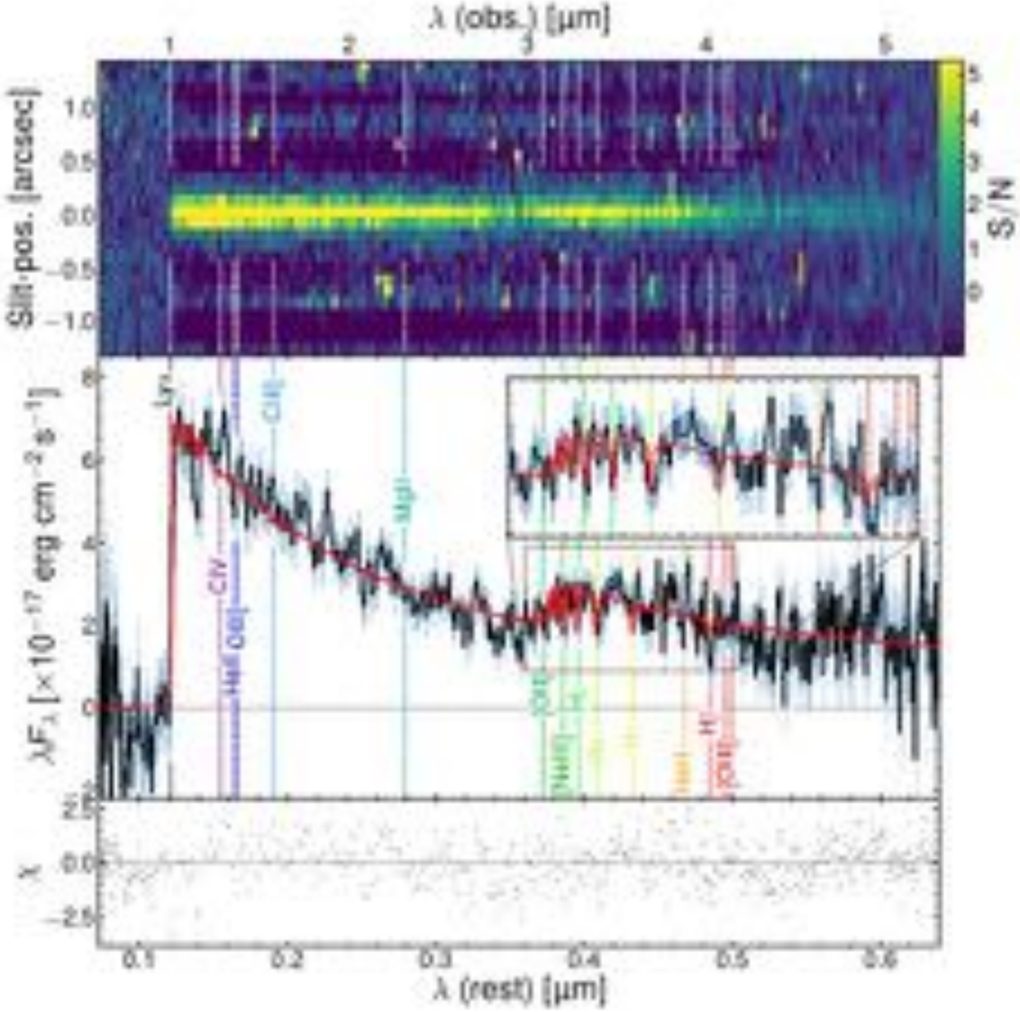
$$\frac{\partial^2 x}{\partial t^2} = -x\kappa^2$$

$$v^2 = \left(\frac{\partial^2 \Phi_e}{\partial z^2}\right)_{r=r_0, z=z_0}$$

Stellar orbits in galaxy could be resonant to a rotating potential caused by an external perturbation



# An interesting "JWST-discovery" from last week !




Discovery of a quiescent galaxy at $z$=7.3

Tobias J. Looser[1,2*], Francesco D'Eugenio[1,2], Roberto Maiolino[1,2,3], Joris Witstok[1,2], Lester Sandles[1,2], Emma

| Key inferred properties | PROSPECTOR |
|---|---|
| $\log_{10}(M_*/M_\odot)$ | $8.7^{+0.1}_{-0.1}$ |
| $\log_{10}(\mathrm{SFR}\ [M_\odot/\mathrm{yr}])$ | $-2.6^{+1.5}_{-2.7}$ |
| $\log_{10}(\mathrm{Z}/\mathrm{Z}_\odot)$ | $-1.7^{+0.2}_{-0.2}$ |
| $t_{\mathrm{quench}}$ [Myr] | $38^{+9}_{-10}$ |
| $t_{\mathrm{form}}$[Myr] | $116^{+85}_{-45}$ |
| $A_V$ [mag] | $0.1^{+0.1}_{-0.0}$ |

# Stellar orbits and spiral structure

**NOT EXAMINABLE**

**Stability of a rotating disk – spiral density waves**

One manifestation of unstable discs are spiral density waves.

The Euler's equations for an ideal fluid in cylindrical coordinate system are :

$$\rho \frac{d\vec{v}}{dt} + \rho(\overrightarrow{v} \cdot \nabla)\vec{v} = -\nabla P - \rho \nabla \Phi_g$$

-radial equation :

$$\rho \frac{\partial v_r}{\partial t} + \rho v_r \frac{\partial v_r}{\partial r} + \rho \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{\rho}{r} v_\theta^2 = -\rho \frac{\partial \phi_g}{\partial r} - \frac{\partial \rho}{\partial r}$$

- $\theta$ equation :

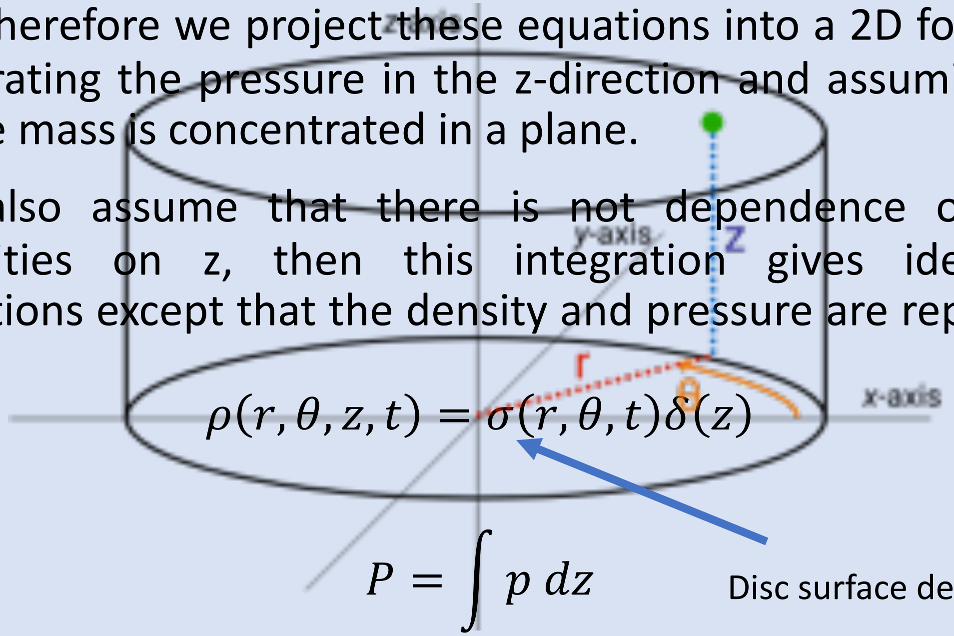
$$\rho \frac{\partial v_\theta}{\partial t} + \rho v_r \frac{\partial v_\theta}{\partial r} + \rho \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\rho}{r} v_\theta \, v_r = -\rho \frac{1}{r} \frac{\partial \phi_g}{\partial \theta} - \frac{1}{r} \frac{\partial \rho}{\partial \theta}$$

And the equation of continuity :

$$\frac{d\rho}{dt} + \nabla . (\rho v) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho v_\theta) = 0$$

We are not interested in the vertical structure in the disc, and therefore we project these equations into a 2D form by integrating the pressure in the z-direction and assuming all of the mass is concentrated in a plane.




We also assume that there is not dependence of the velocities on z, then this integration gives identical equations except that the density and pressure are replaced by :

$$\rho(r, \theta, z, t) = \sigma(r, \theta, t)\delta(z)$$

And

$$P = \int p \, dz$$

Disc surface density

# Stellar orbits and spiral structure

**Stability of a rotating disk – spiral density waves**

The unperturbed solution is an axially symmetric rotating mass distribution, $\sigma_0(r)$ with $v_0(r,\theta) = (0, r\Omega(r))$ and an unperturbed potential $\phi_{g0}$

**Perturbation Analysis**

Adding small perturbations which are all functions of $r, \theta$ and $t$ :

$$v = (u, v + r\Omega)$$
$$\sigma = \sigma_0(r) + \sigma'(r,\theta,t)$$
$$\phi_g = \phi_{g0} + \phi_g'(r,\theta,z,t)$$

where $\sigma_0(r)$ and $\sigma'(r,\theta,t)$ satisfy the Poisson's equation :

$$\nabla^2 \phi_{g0} = 4\pi G \sigma_0 \delta(z)$$
$$\nabla^2 \phi_g' = 4\pi G \sigma'(r,\theta,t)\delta(z)$$

$P$ is a force per unit length and is the equivalent of a pressure in 2D. We assume an isothermal-like equation of state and write :

$$P = a_0^2 \sigma$$

Stellar velocity dispersion

Disc surface density

If we keep only terms to first order in small quantities :

- Radial $u$ :

$$\frac{\partial u}{\partial t} + \Omega \frac{\partial u}{\partial \theta} - 2v\Omega = -\frac{a_0^2}{\sigma_0}\frac{\partial \sigma'}{\partial r} - \frac{\partial \phi_g'}{\partial r}$$

- Angular $v$

Epicyclic frequency

$$\frac{\partial v}{\partial t} + \Omega \frac{\partial v}{\partial \theta} - \frac{\kappa^2 u}{2\Omega} = -\frac{a_0^2}{r\sigma_0}\frac{\partial \sigma'}{\partial \theta} - \frac{1}{r}\frac{\partial \phi_g'}{\partial \theta}$$

- Continuity :

$$\frac{\partial \sigma'}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\sigma_0 u) + \frac{\sigma_0}{r}\frac{\partial v}{\partial \theta} + \Omega\frac{\partial \sigma'}{\partial \theta} = 0$$

# Stellar orbits and spiral structure

**Stability of a rotating disk – spiral density waves**

**Perturbation Analysis**

We now look for spiral-like solutions writing :

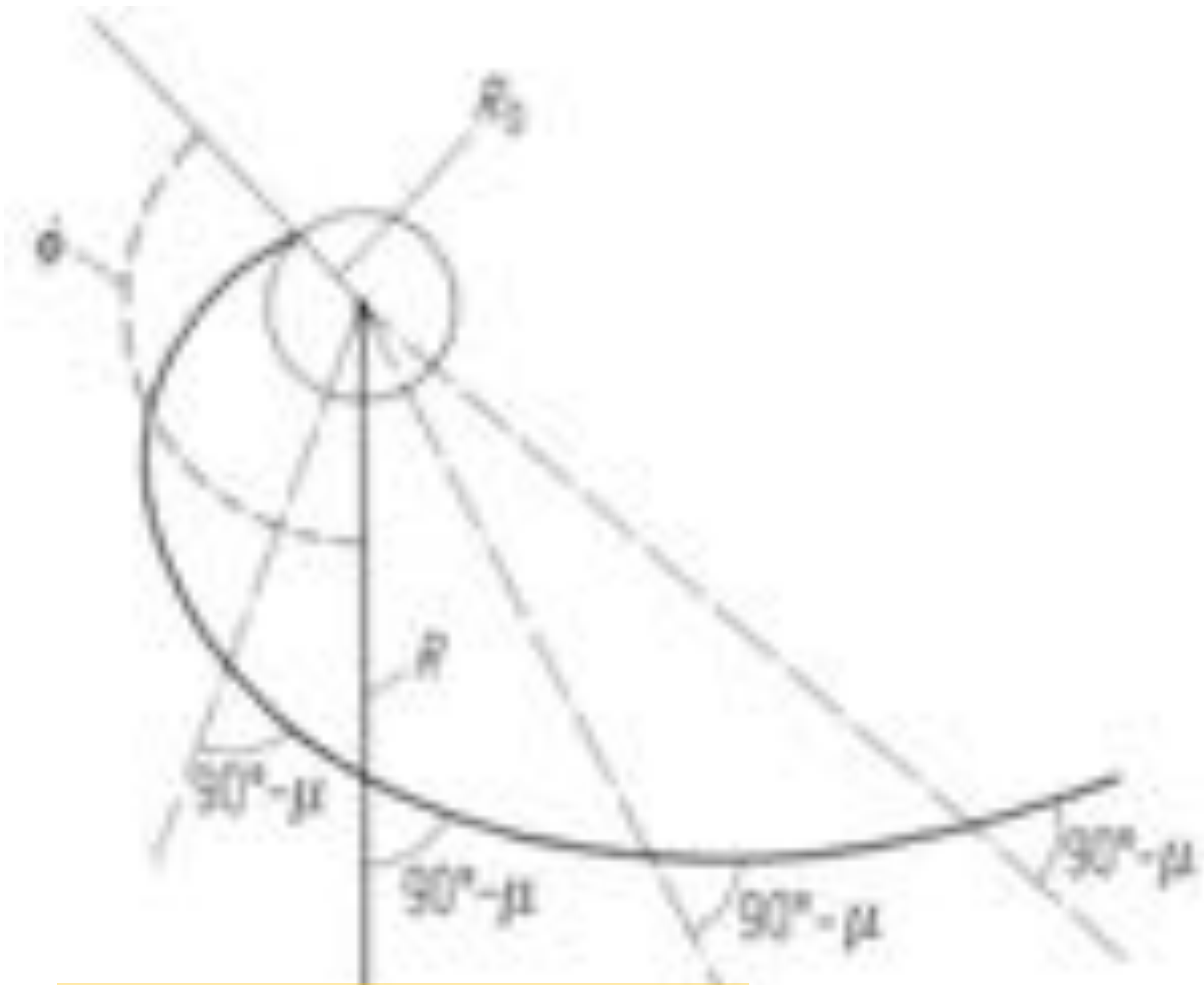
$$\sigma' = \hat{\sigma}\exp\left(i\left(\omega t - n\theta + \Psi(r)\right)\right)$$

with similar expression for $u, v$ and $\phi_g'$

Basic properties of solutions :

The maximum in the density occurs for $\omega t - n\theta + \Psi(r) = 0$. To understand the implications of this form, consider $t = 0$, then the locus of the maximum density has :

- $n\theta = \Psi(r)$ which is a spiral pattern which represents an n-armed spiral
- The pattern makes an angle to the $\theta$ direction, the *pitch angle*, $\tan\alpha = \frac{n}{r\frac{d\Psi}{dr}} = \frac{n}{kr}$ where we define $k = \frac{d\Psi}{dr}$




The pitch angle is the angle between the tangents to a spiral arm and a perfect circle

# Stellar orbits and spiral structure

## Stability of a rotating disk – spiral density waves

### Perturbation Analysis

The spiral-like solution make Euler's and Continuity equation simple, however solving Poisson's equation is more difficult.

The solution can be found is what is called the *tight-winding approximation* when $\frac{n}{kr} \ll 1$

The resulting dispersion relation is :

$$\kappa^2 - n^2\left(\Omega_p - \Omega\right)^2 + k^2 a_0^2 = 2\pi G|k|\sigma_0$$

Epicyclic frequency

Pattern speed

wavenumber

Velocity dispersion

Surface density

Re-arranging :

$$n^2\left(\Omega_p - \Omega\right)^2 = (\omega - n\Omega)^2 = \kappa^2 + k^2 a_0^2 - 2\pi G|k|\sigma_0$$

In the special case of rotating axisymmetric disk (n=0):

$$\omega^2 = \kappa^2 + k^2 a_0^2 - 2\pi G|k|\sigma_0$$

rotation

Velocity dispersion

gravity

# Stellar orbits and spiral structure

## Stability of a rotating disk – spiral density waves

### Perturbation Analysis

The solution dispersion can be re-written as :

$$\frac{\omega^2}{\kappa^2} = 1 + \frac{k^2 a_0^2}{\kappa^2} - \frac{2\pi G|k|\sigma_0}{\kappa^2}$$

Or

$$\frac{\omega^2}{\kappa^2} = 1 + \frac{Q^2}{4}\frac{k^2}{k_T^2} - \frac{|k|}{k_T}$$

where :

- $Q$ is the disc *stability parameter* :

$$Q = \frac{2k_T a_0}{\kappa} = \frac{\kappa a_0}{\pi G \sigma_0}$$

- $k_T$ is the **Toomre wave number** :

$$k_T = \frac{\kappa^2}{2\pi G \sigma_0}$$

The division between stable and unstable solutions occurs when $\omega^2 = 0$ or :

$$\frac{|k|}{k_T} = \frac{2}{Q^2}\left(1 \pm (1 - Q^2)^{\frac{1}{2}}\right)$$

- This only has a solution for $|k|$ when $Q < 1$. In this case there are regions where $\omega^2 < 0$ , hence spiral perturbations grow exponentially, yielding the collapse of clouds, likely resulting into **star formation**.
- When $Q > 1 \rightarrow \omega^2 > 0\ \forall\ |k|$ and the disc is always stable. The latter condition can be expressed in terms of a minimum velocity dispersion that makes the disk stable :

$$Q = \frac{\kappa a_0}{\pi G \sigma_0} > 1 \rightarrow a_0 \geq a_{0,min} = \frac{\pi G \sigma_0}{\kappa}$$

Or that the maximum surface density for stability is :

$$\sigma_0 < \sigma_{0,max} = \frac{\kappa a_0}{\pi G}$$

# Stellar orbits and spiral structure

The higher gas density on the spiral waves can make
$\sigma_0 > \sigma_{0,max}$

$$Q = \frac{2k_T a_0}{\kappa} = \frac{\kappa a_0}{\pi G \sigma_0}$$

$Q < 1$

Gravitational instability

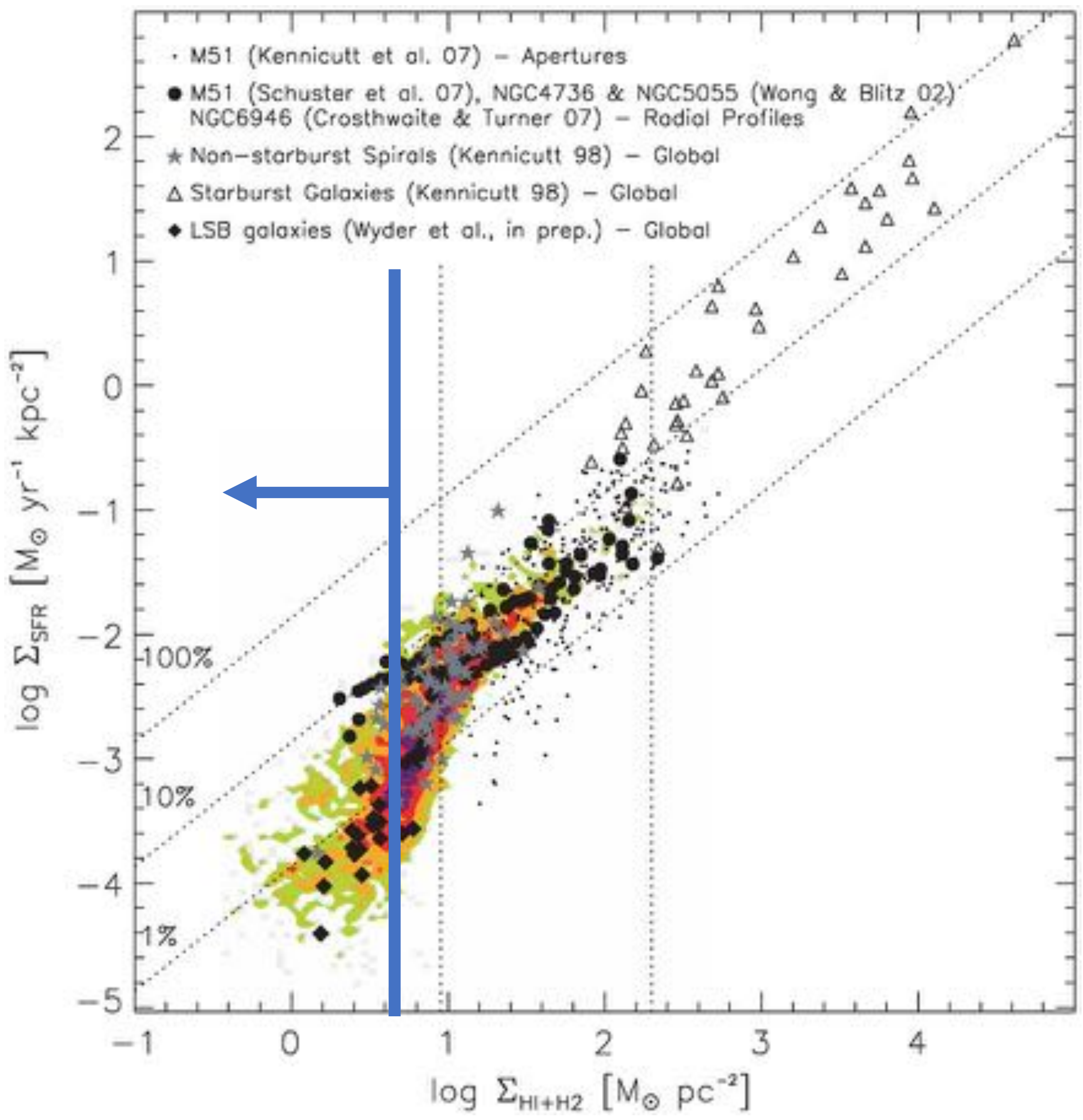
**Star Formation**

# Stellar orbits and spiral structure

We have seen previously in the Schmidt-Kennicutt relation that there is a surface gas density threshold for star formation.

This threshold can be explained in terms of gas surface density below the critical surface density in the outer parts of disks, making the gas stable and therefore avoiding star formation.

$$Q = \frac{2k_T a_0}{\kappa} = \frac{\kappa a_0}{\pi G \sigma_0}$$



# Stellar orbits and spiral structure

**Lindblad Resonances**

The full dispersion relation is given by :

$$n^2\left(\Omega_p - \Omega\right)^2 = (\omega - n\Omega)^2 = \kappa^2 + k^2 a_0^2 - 2\pi G|k|\sigma_0$$

and can be re-written as :

$$k^2 a_0^2 - \frac{|k|}{k_T}\kappa^2 + \left(\kappa^2 - n^2\left(\Omega_p - \Omega\right)^2\right) = 0$$

which is now a quadratic equation in |k|

Assuming $a_0 = a_{0,min}$, to have a real (and positive) solution for |k| (*wavenumber of spiral waves)* it is necessary that :

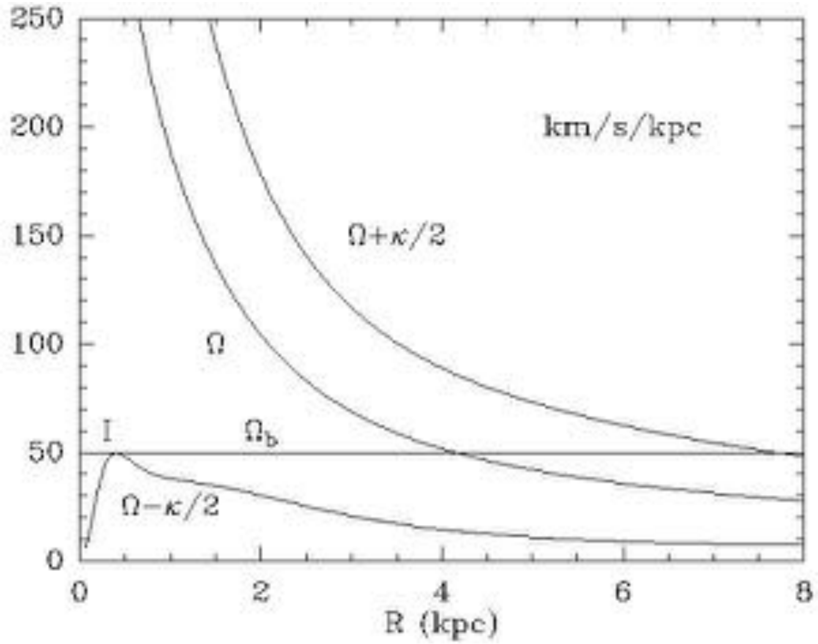
$$1 \pm \frac{n}{\kappa}\left(\Omega_p - \Omega\right) \geq 0$$

Therefore :

$$\Omega - \frac{\kappa}{n} \leq \Omega_p \leq \Omega + \frac{\kappa}{n}$$

Then :

- $\Omega_p = \Omega - \frac{\kappa}{n}$: inner Lindblad resonance
- $\Omega_p = \Omega + \frac{\kappa}{n}$: outer Lindblad resonance
- $\Omega_p = \Omega$ is called corotation

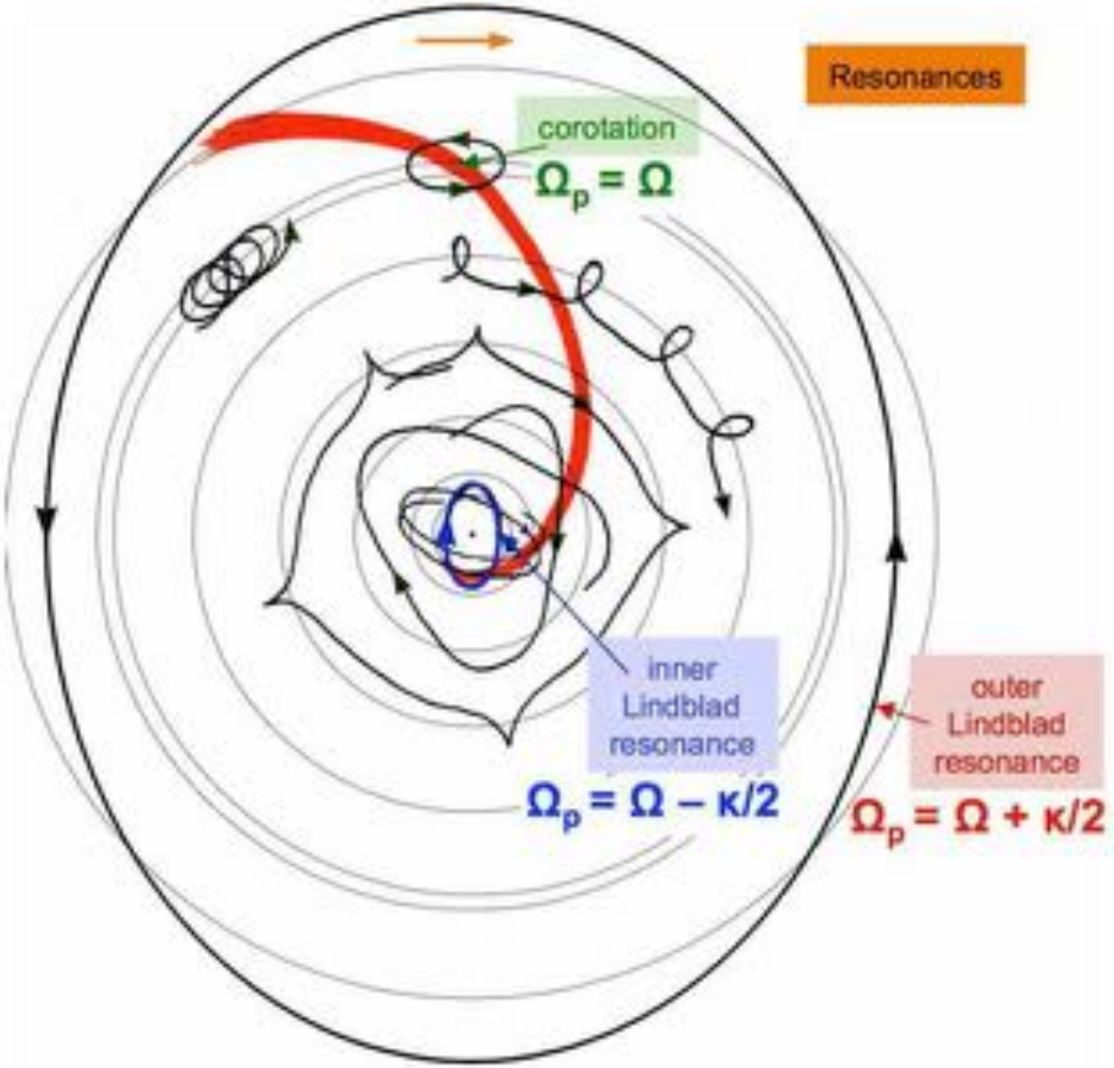


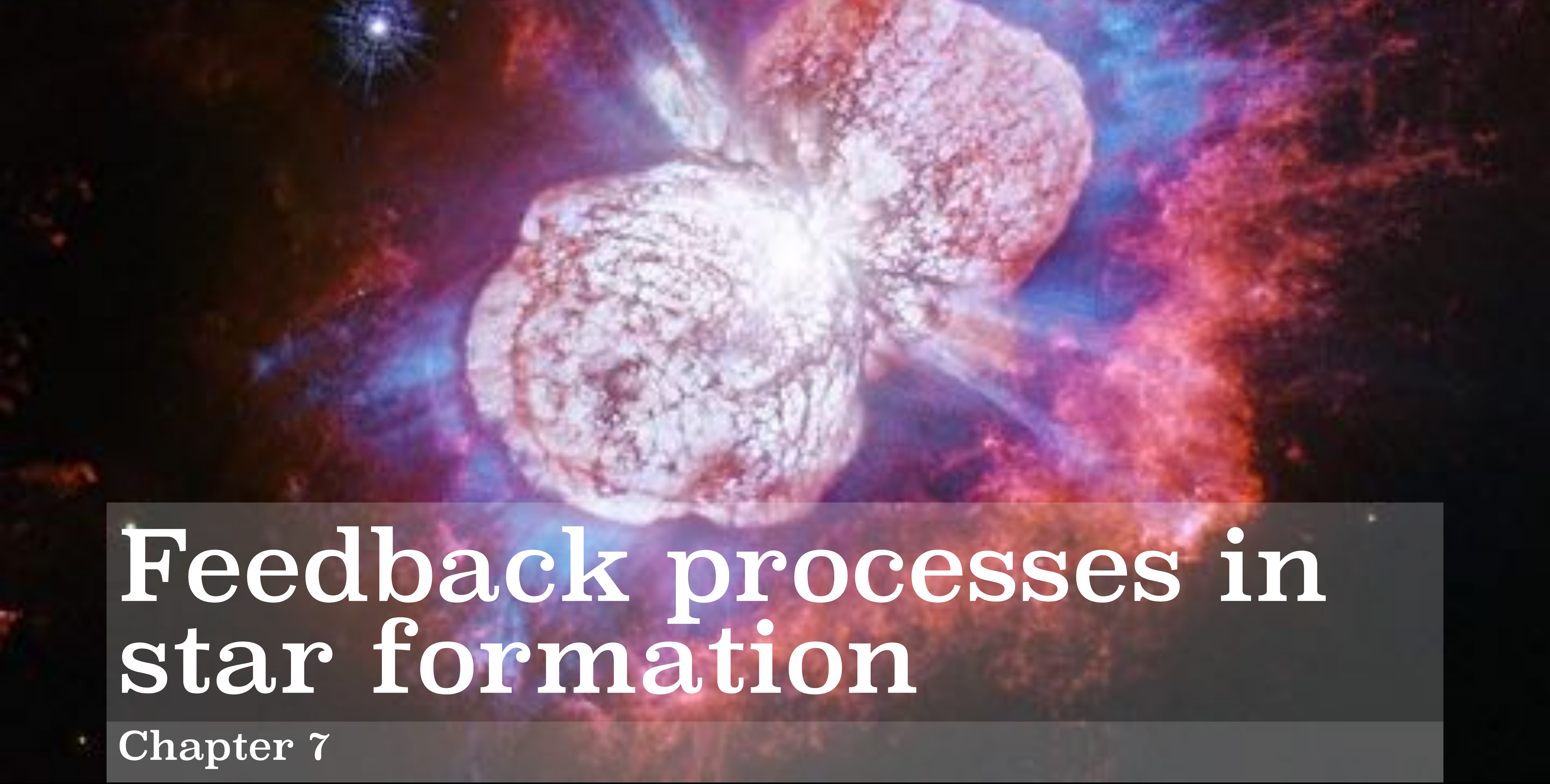
# Stellar orbits and spiral structure

**Lindblad Resonances**

- $\Omega_p = \Omega - \frac{\kappa}{n}$: inner Lindblad resonance
- $\Omega_p = \Omega + \frac{\kappa}{n}$: outer Lindblad resonance
- $\Omega_p = \Omega$ is called corotation

We only have spiral density wave solution between the inner and outer Lindblad resonance (where we have spiral arms in a galaxy)





# Feedback processes in star formation

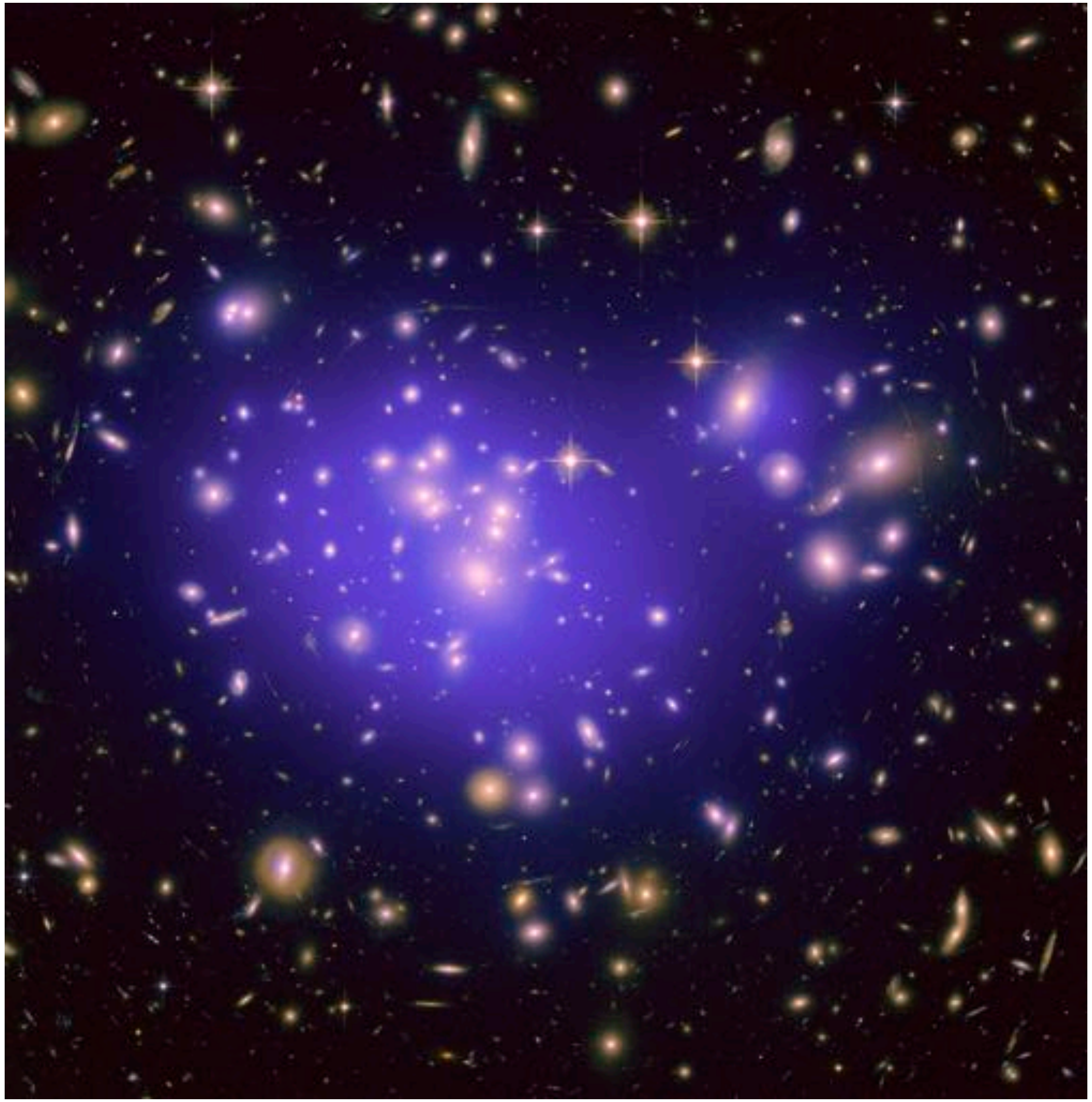
Chapter 7

In the local Universe, only 4% of the baryons have been converted into stars.

But gas cooling and gravitational collapse theories predict that over the entire life of the Universe 80% of the baryons should have been converted into stars.

Some mechanisms must have been responsible for suppressing the formation of stars and/or removing gas from galaxies

**Negative Feedback**

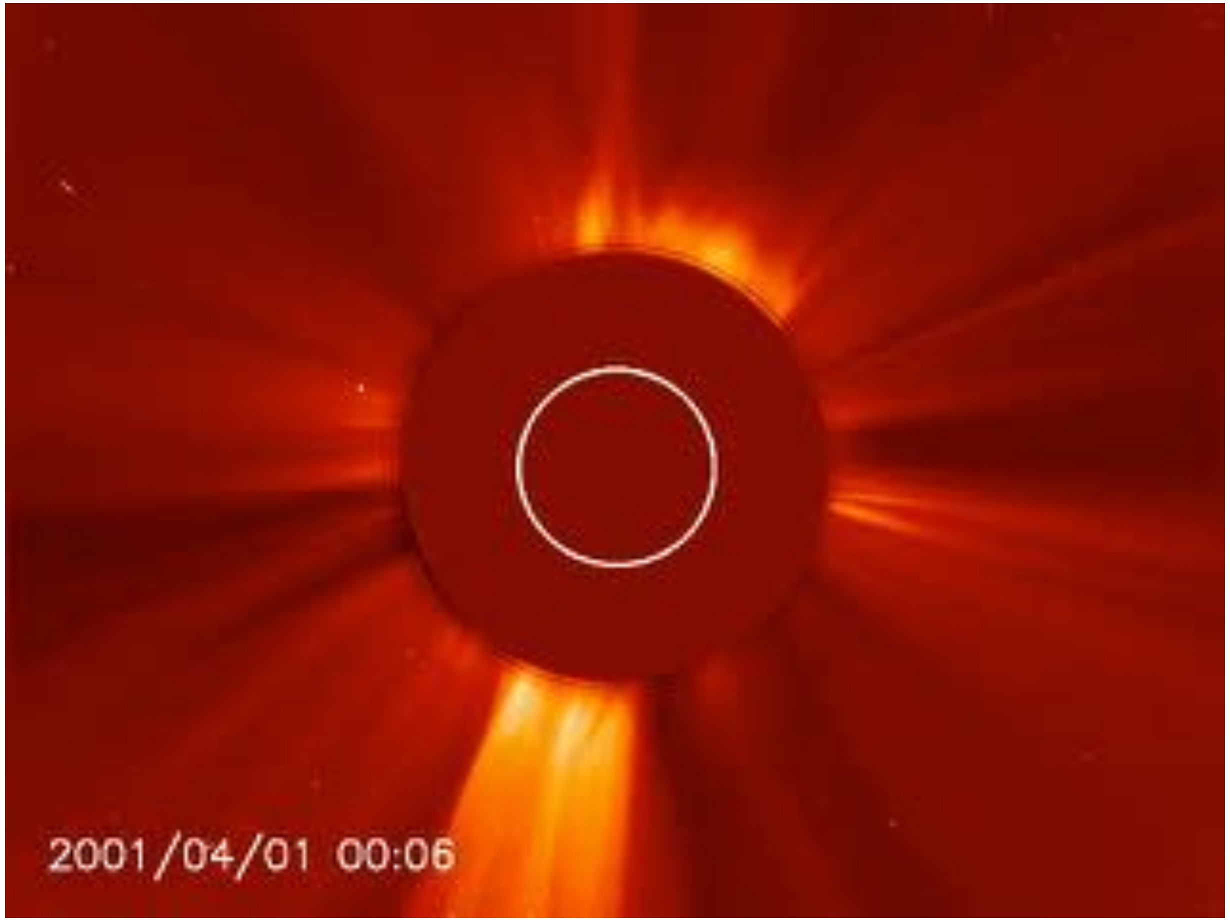


# Stellar Winds

Mass loss from massive and evolved stars occurs in the form of stellar wind.

The properties of the stellar wind depend on the star :

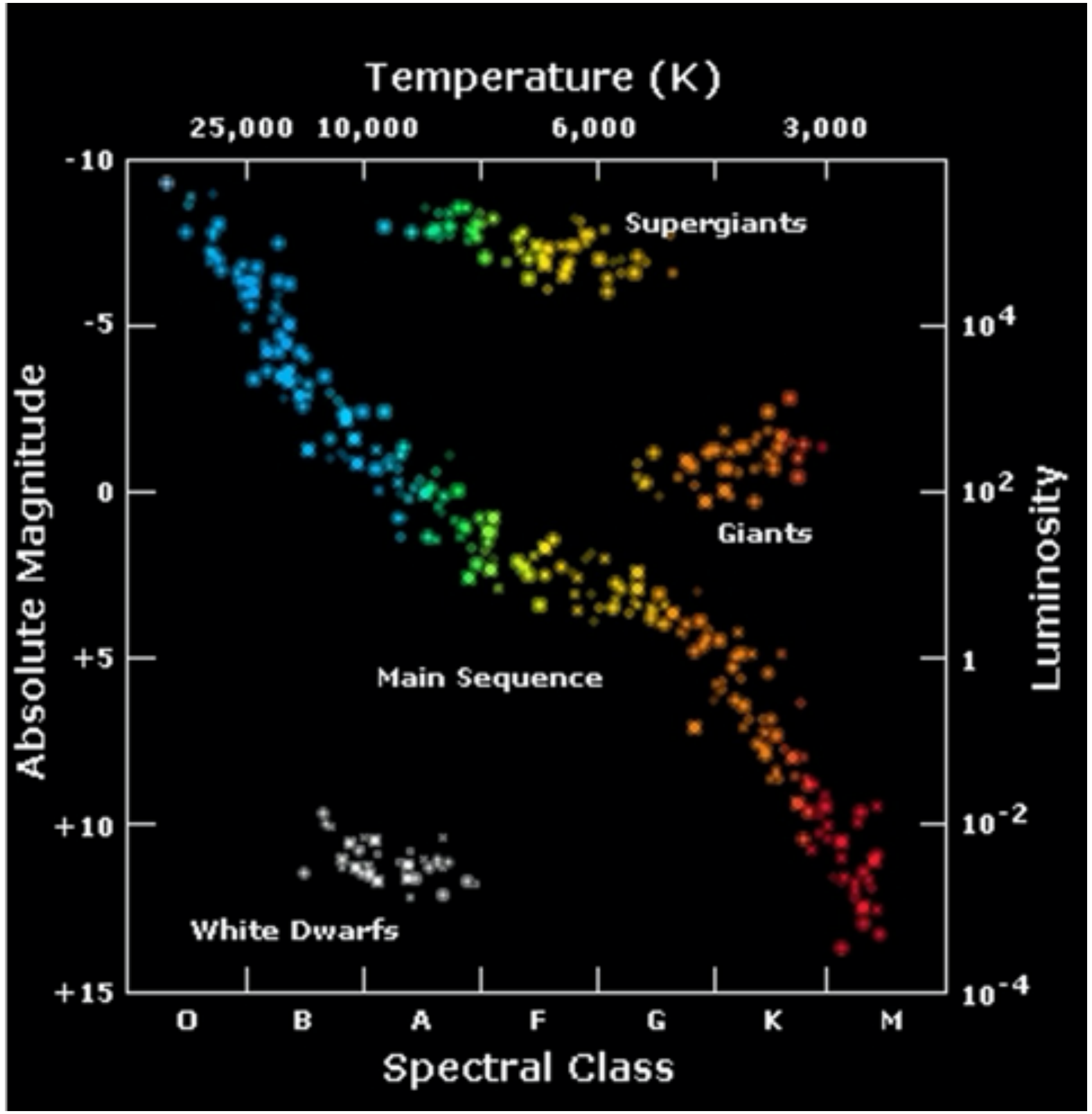
- Post-main sequence stars nearing the ends of their lives often eject large quantities of mass in massive ($\dot{M} > 10^{-3} M_{\odot} yr^{-1}$ ) but relatively slow winds ($v < 10 km\ s^{-1}$)



# Stellar Winds

Winds from Massive O and B stars

- lower mass loss rates ($\dot{M} < 10^{-6} M_{\odot} yr^{-1}$ ) but very high velocity ($v > 1000 - 2000\ km\ s^{-1}$).
- Winds are driven by radiation pressure on the resonance absorption lines of heavy elements such as carbon and nitrogen.
- These high-energy stellar winds provide significant feedback
- The winds drive shocks into the ISM – two-shock structure :
    - The freely-expanding stellar wind hits an inner termination shock, where its kinetic energy is thermalized, producing $10^6$K, X-ray emitting plasma.
    - The hot, high-pressure, shocked wind expands, driving a shock into the surrounding interstellar gas
    - If the surrounding stellar gas is dense enough, the swept-up gas radiatively cools far faster than the hot interior, forming a thin dense shell around the hot shocked wind.



# Stellar Winds

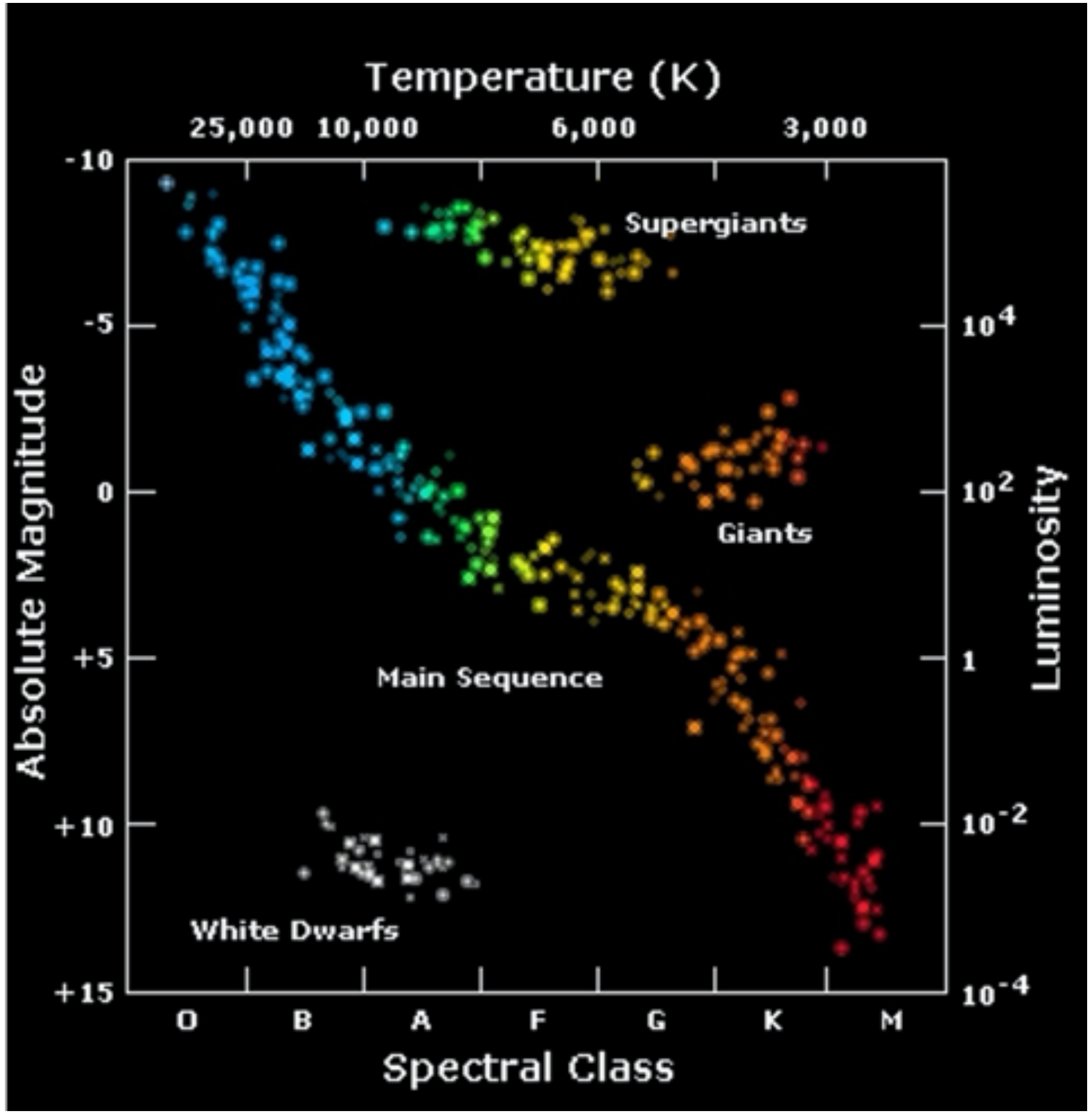
Winds from Massive O and B stars

About 20% of the energy of the stellar wind is converted into kinetic energy of the surrounding ISM.

Additional feedback from O and B stars

These are massive stars, with high luminosities and high temperature, producing a large number of ionising photons :

- Photoionisation, and hence heating of the ISM
- Strong radiation pressure onto the dust in the ISM clouds can drive powerful winds, which eject gas out of the galaxy



# SuperNovae

Stellar winds represent a minor fraction of feedback processes in galaxies which are actively producing very massive stars.

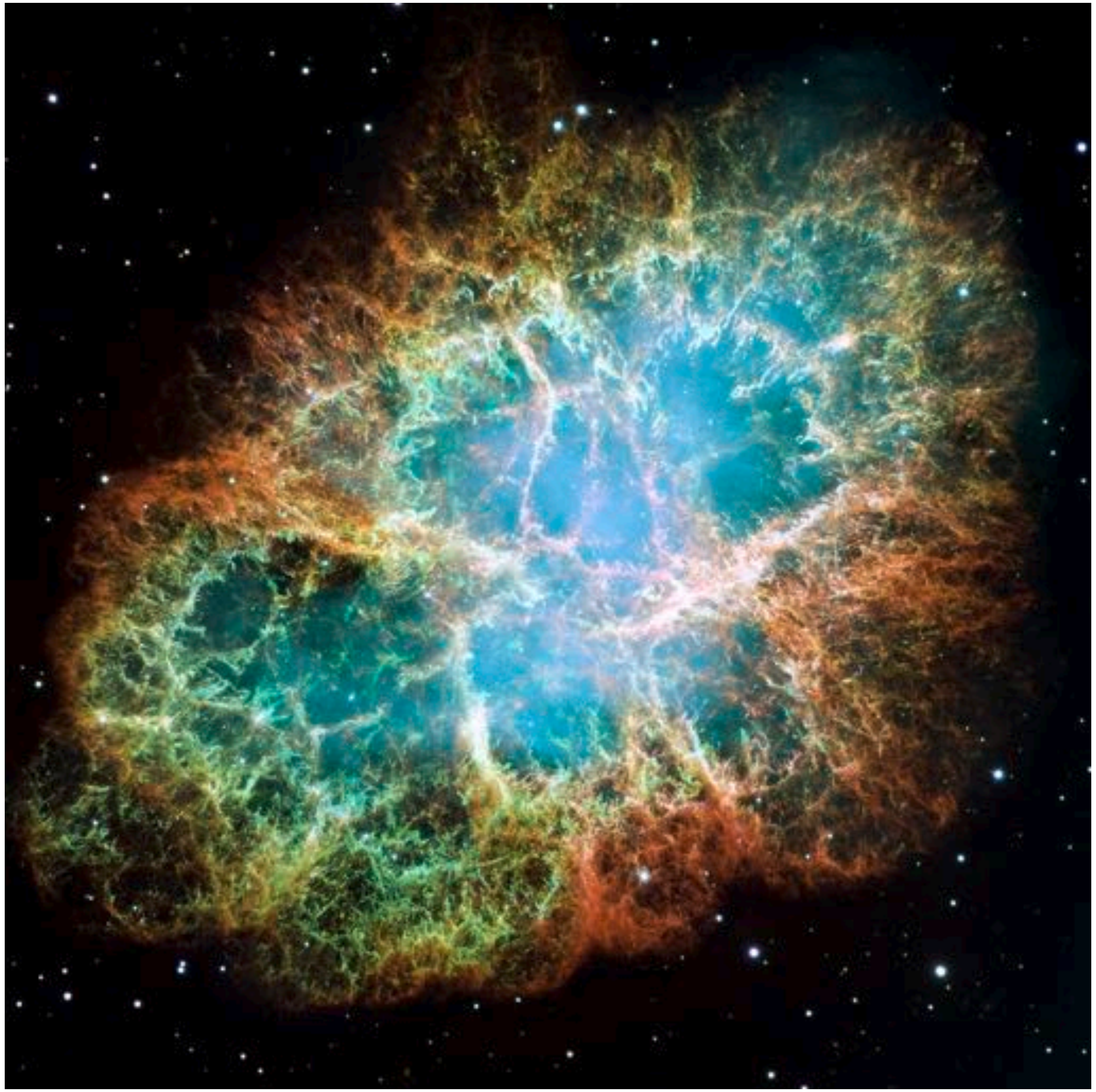
Stars with mass $M > 8M_{\odot}$ will end their lives as supernovae which have a dramatic effect on the galaxy.

The total energy input to the ISM from each SNe explosion is of order $10^{43} - 10^{44}$J.

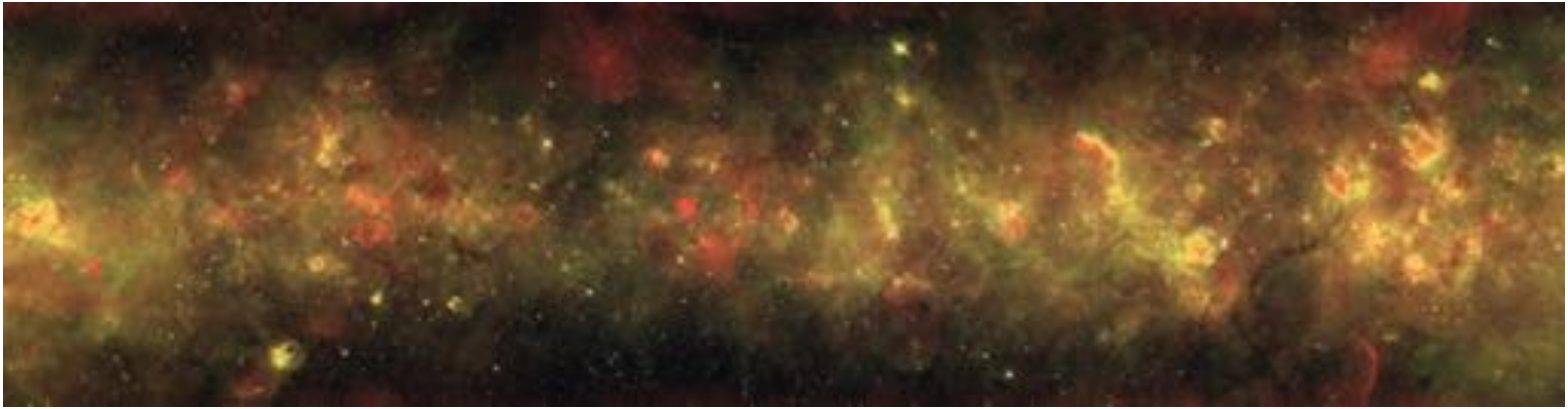
Moreover, SNe also release into the SNe processed stellar material, and are therefore responsible for an increase of metallicity.

Typically, one solar mass of stellar material is ejected back to the ISM

The effect of SNe explosions is both to heat the ISM through shocks and to drive powerful winds .



# SuperNovae



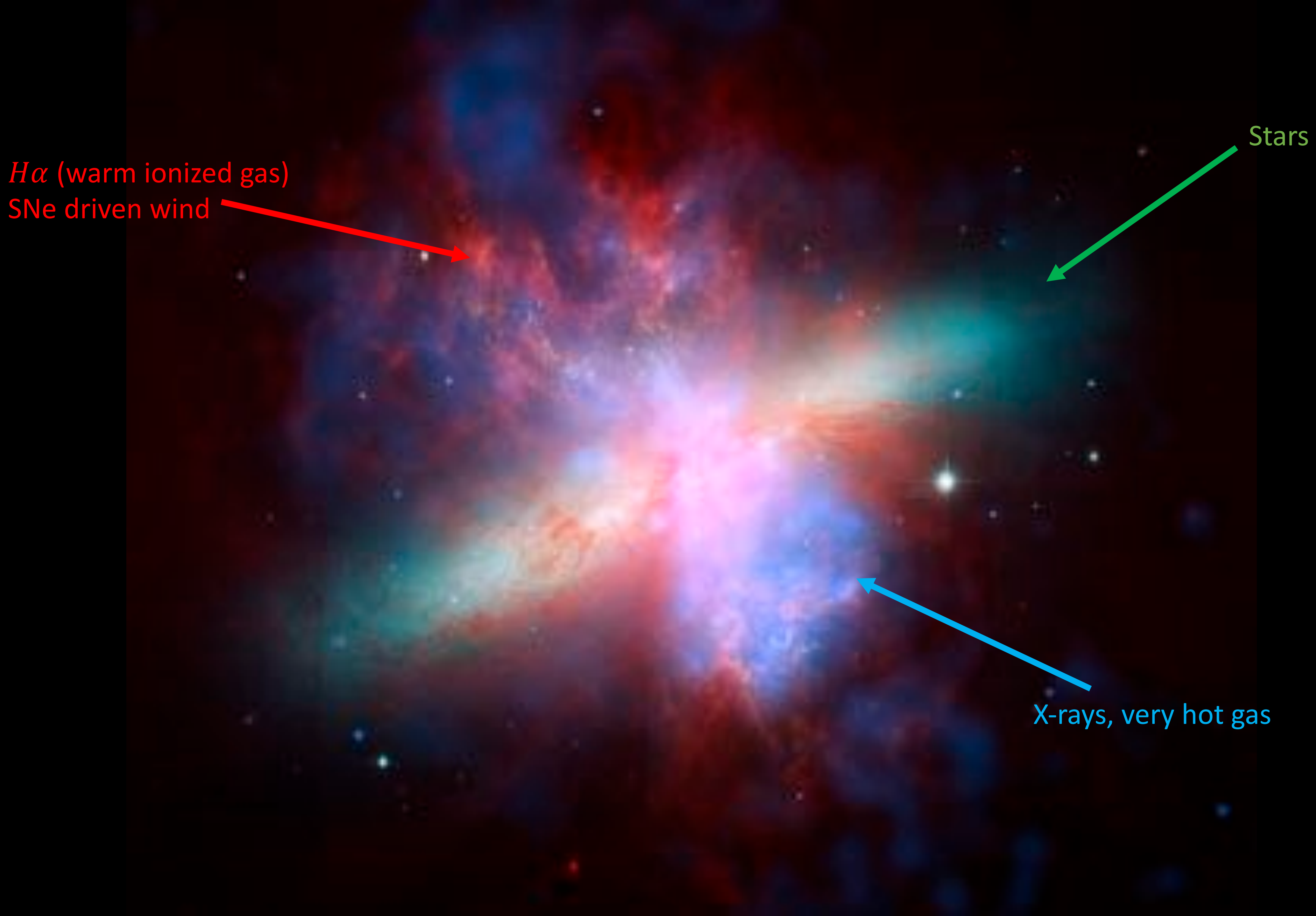
VLA image showing SNe remnants in the Milky Way

# Starburst and ultraluminous galaxies

In **a starburst** the measured rate of star formation is so large that the available gaseous fuel would be used in much less than the Hubble time (often within $\approx 10^8$ yr).

**Ultraluminous Infrared Galaxies** (ULIRGs) are the most extreme cases of starbursts: they have far infrared luminosities greater than $10^{12} L_\odot$ and inferred star formation rates of order 50 – 1000 $M_\odot\ yr^{-1}$

The Star Formation is concentrated in a nuclear region (a nuclear starburst). The associated SNe drive a galactic-scale outflow of hot gas which is clearly visible in the X-Ray, in the ionised gas, but also in molecular gas.


$H\alpha$ (warm ionized gas)
SNe driven wind
Stars
X-rays, very hot gas

# Global stellar feedback

Stellar winds, photoionization and supernova explosions have a cumulative effect of suppressing star formation, i.e. have a ***negative feedback*** effect onto star formation.

Heating by shocks and photoionization makes the collapse and fragmentation of molecular clouds more difficult (because efficient cooling requires radiation from molecules and dust which are often destroyed in these environments)

Galactic winds (mostly produced by the cumulative effect of SN explosions and radiation from young O-B stars) expel gas out of the galaxy, hence remove fuel available for star formation

The latter mechanism is most important in low mass galaxies, for which the gravitational potential well is not deep enough to retain the gas.

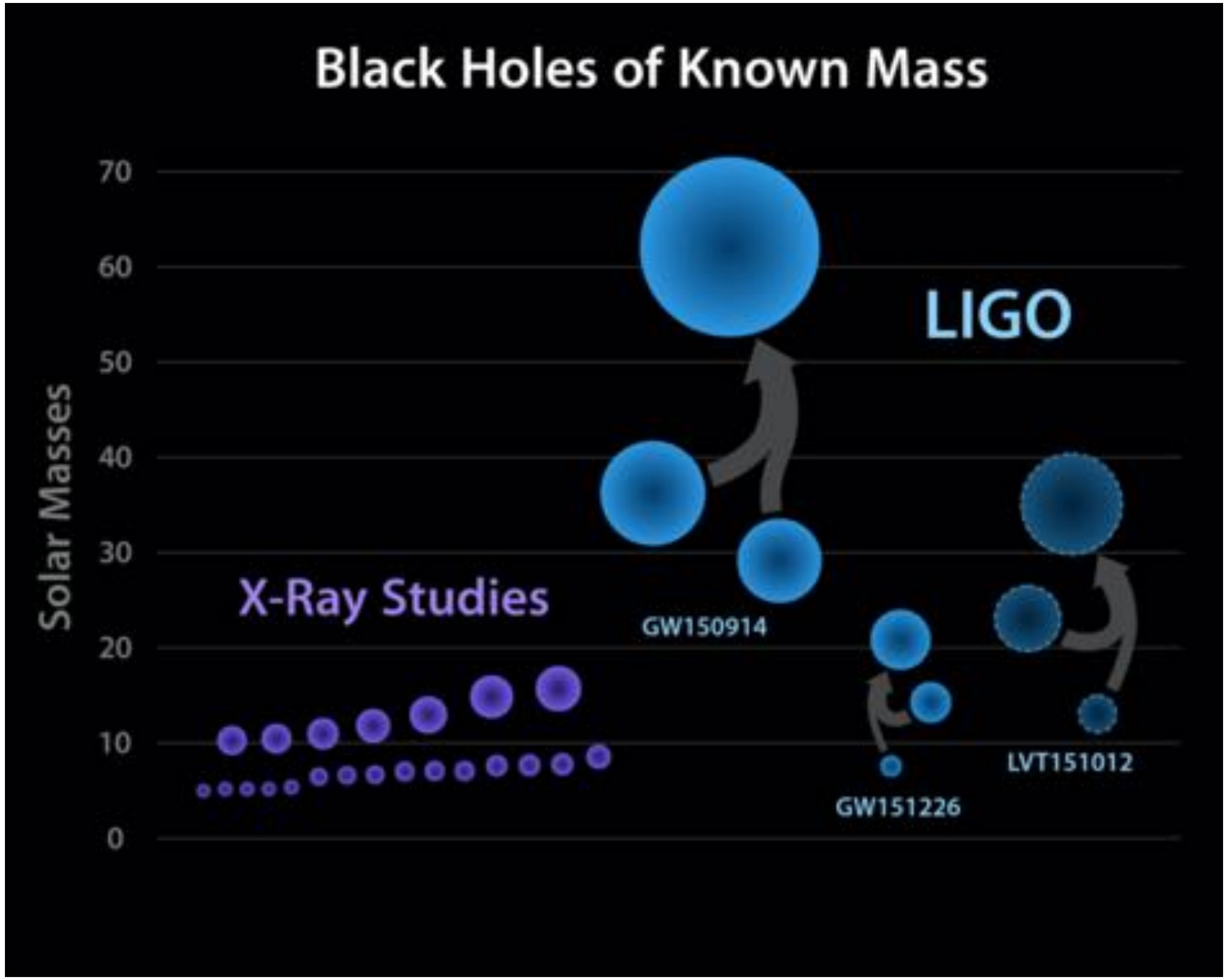
# Feedback from Supermassive Accreting Black Holes

Stellar black holes result from the collapse of massive stars : result from the collapse of a star whose mass , after the mass loss during its evolution is larger than $3-4\ M_\odot$ (which can result from a progenitor MS star of $M > 8M_\odot$

The most massive stellar black holes have masses a few /several ten $M_\odot$(see LIGO results)

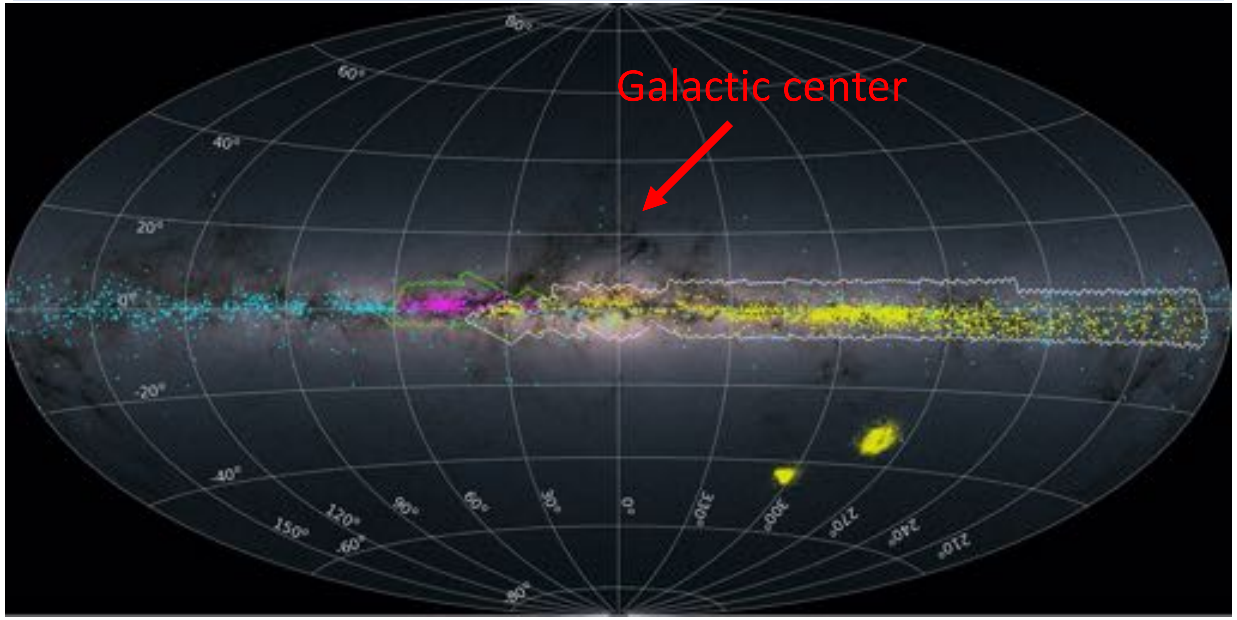
Collapse of very massive primordial (pop III) stars are expected to generate much more massive black holes up to a few $\sim 100M_\odot$ (both because the progenitors are much more massive and because the low metallicity greatly reduces the mass loss during their evolution).

Supermassive black holes are in galactic nuclei and have masses $M > 10^5 M_\odot$. They must have originated from accretion or merging of smaller black holes.



# Feedback from Supermassive Accreting Black Holes

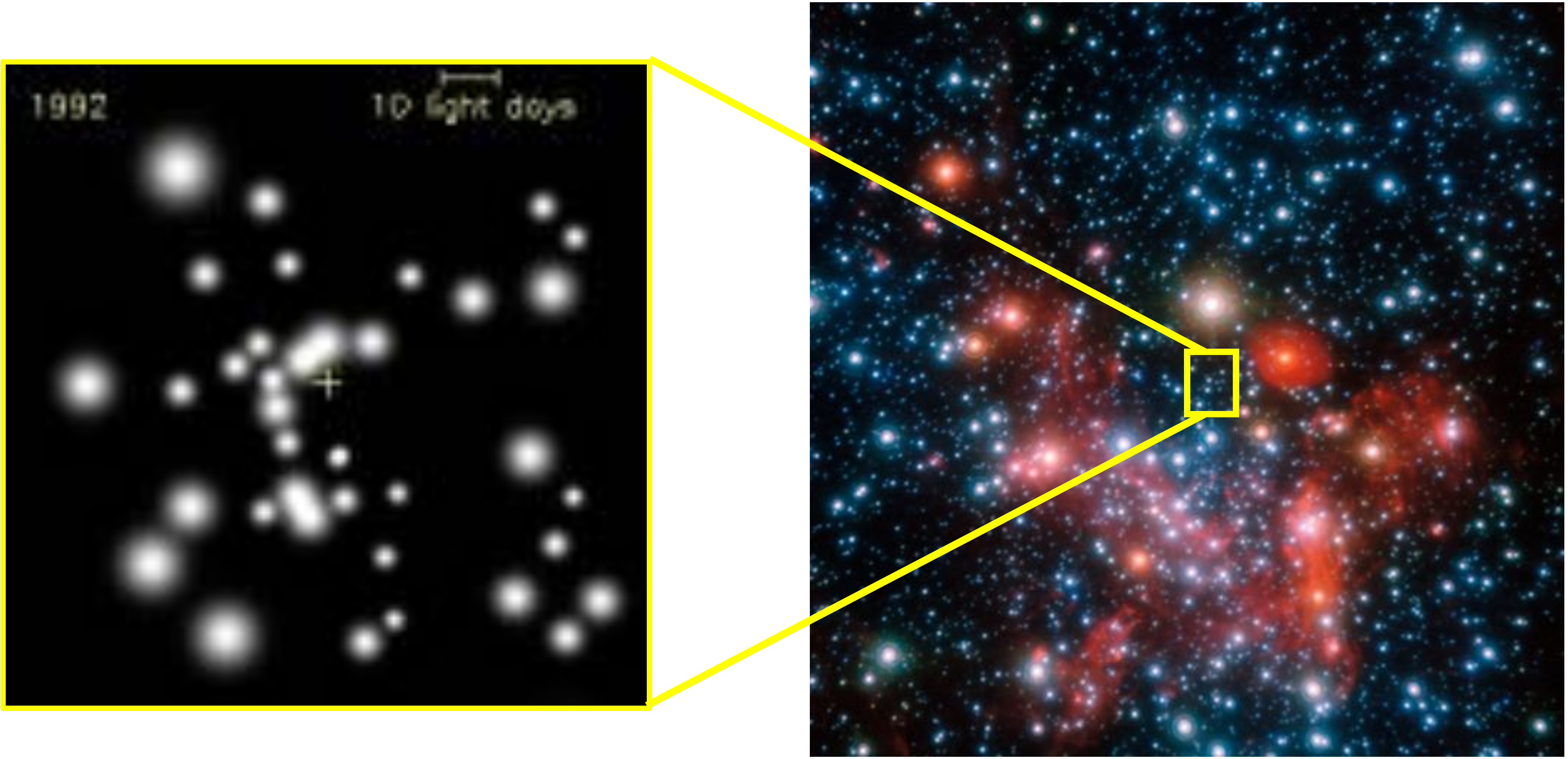
**Do we have observational probes of the presence of a supermassive black hole in the Milky Way ?**





The luminosity in the Milky Way seems brighter at the galactic center.

# Feedback from Supermassive Accreting Black Holes

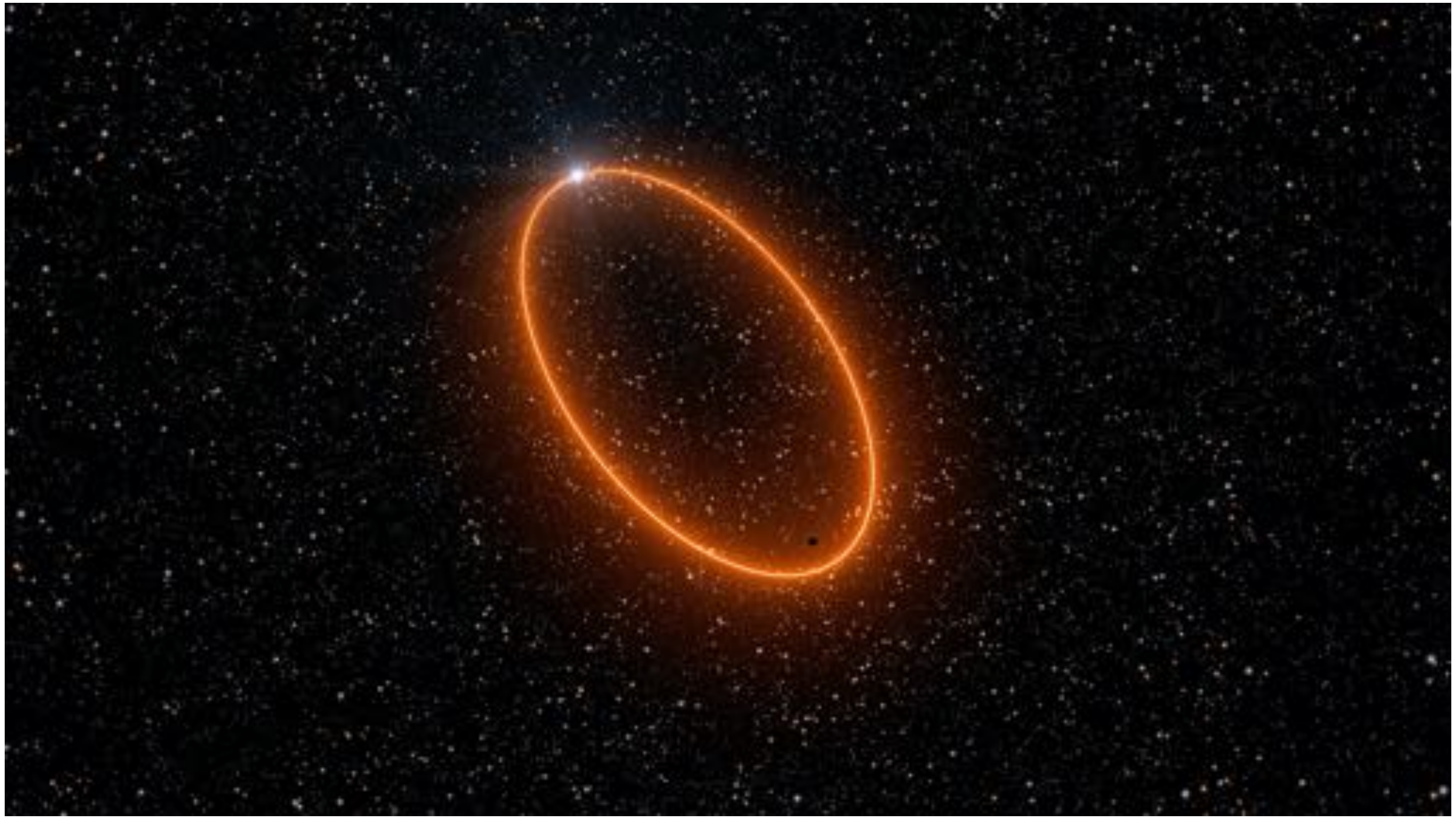


# Feedback from Supermassive Accreting Black Holes

The stars are orbiting around an object with a mass of $2{\times}10^6 M_\odot$ with a size $R < 125\ AU$

leading to a density of $5{\times}10^{15} M_\odot pc^{-3}$

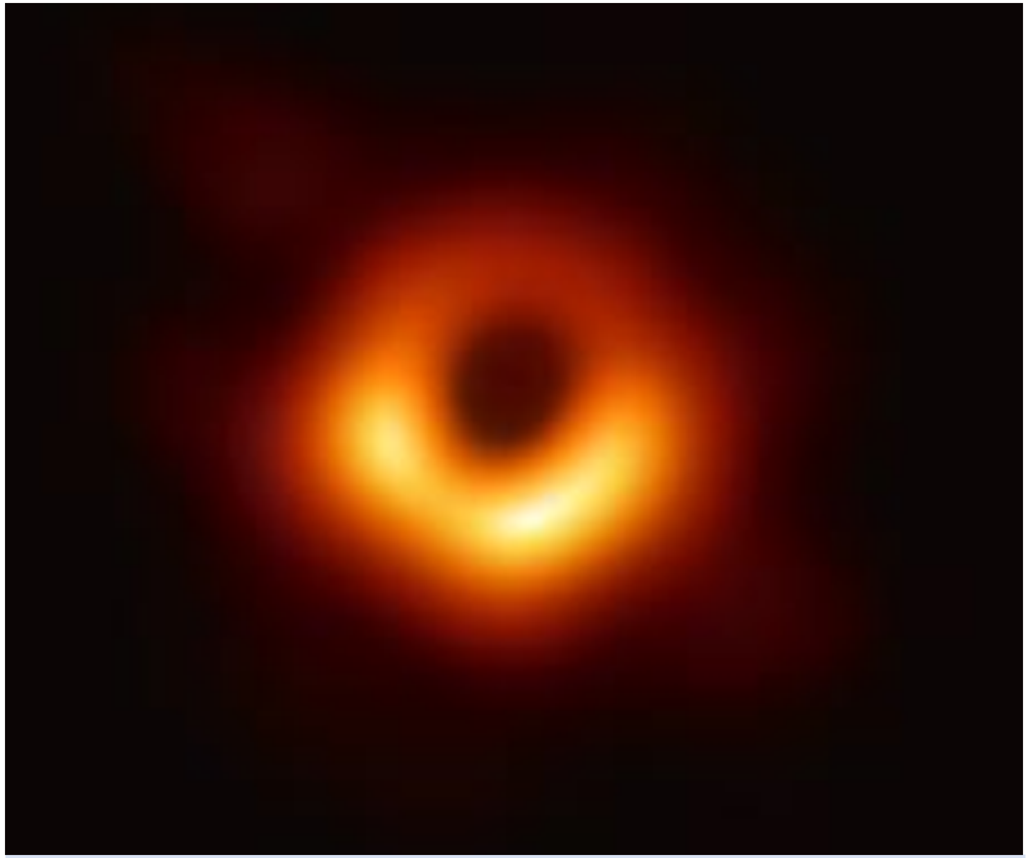
**Supermassive black hole**



# Feedback from Supermassive Accreting Black Holes

**Do we have observational probes of the presence of supermassive black holes in other galaxies ?**

Supermassive Black Holes are present in the nuclei of most galaxies, and their masses are found to be proportional to the mass of the stellar spheroid (bulge or whole galaxy in the case of an elliptical galaxy) in a ratio of $M_{BH} \sim 10^{-3} M_{sph}$



First picture of a supermassive black hole in M87
$M_{BH} = 6.5{\times}10^{9} M_{\odot}$

# Feedback from Supermassive Accreting Black Holes

**Accretion onto supermassive black hole**

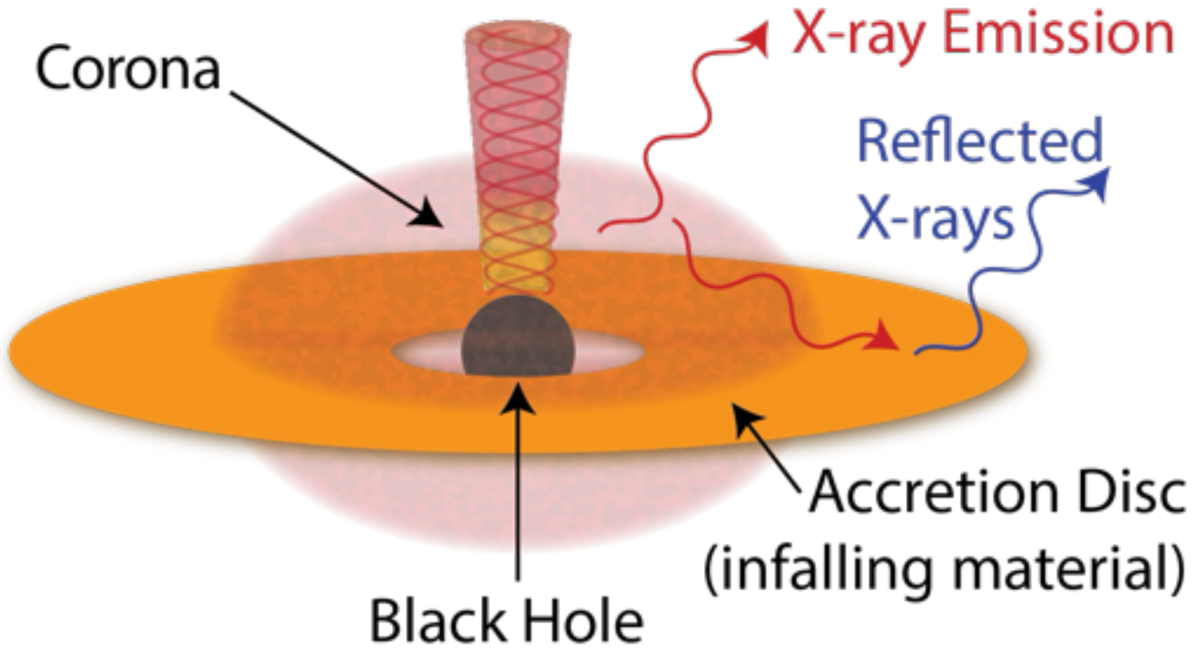
Because of angular momentum conservation gas accreting onto a black hole forms **accretion disc**.

Viscosity allows elements of gas to convert gravitational energy into thermal energy
→ gas particles moves towards inner orbits within the disc

Thermal energy heats the disc to temperatures in excess of $10^5$K

Strong UV thermal (black body like) radiation

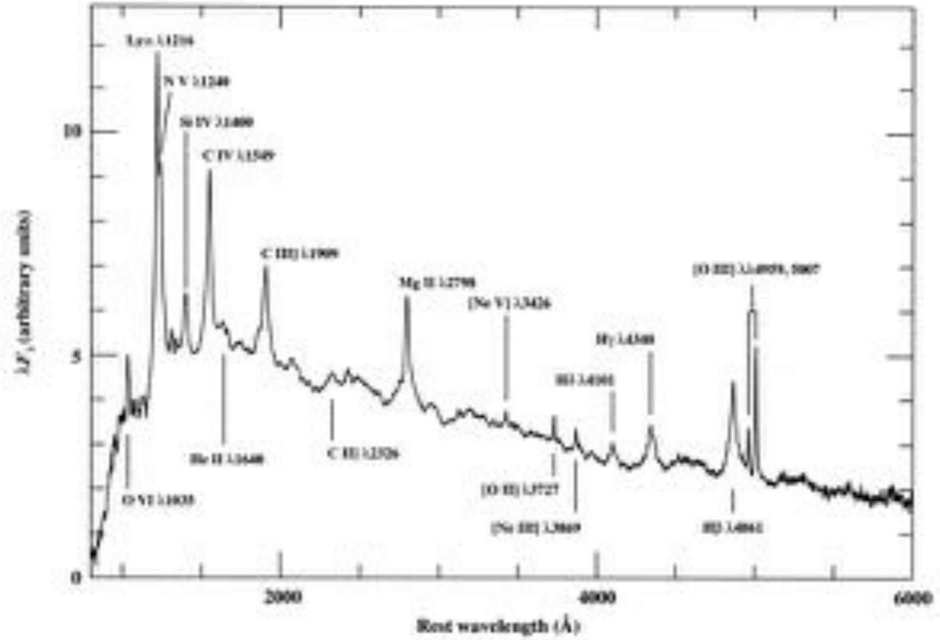
Ionisation of the circumnuclear medium
→ strong nebular emission line

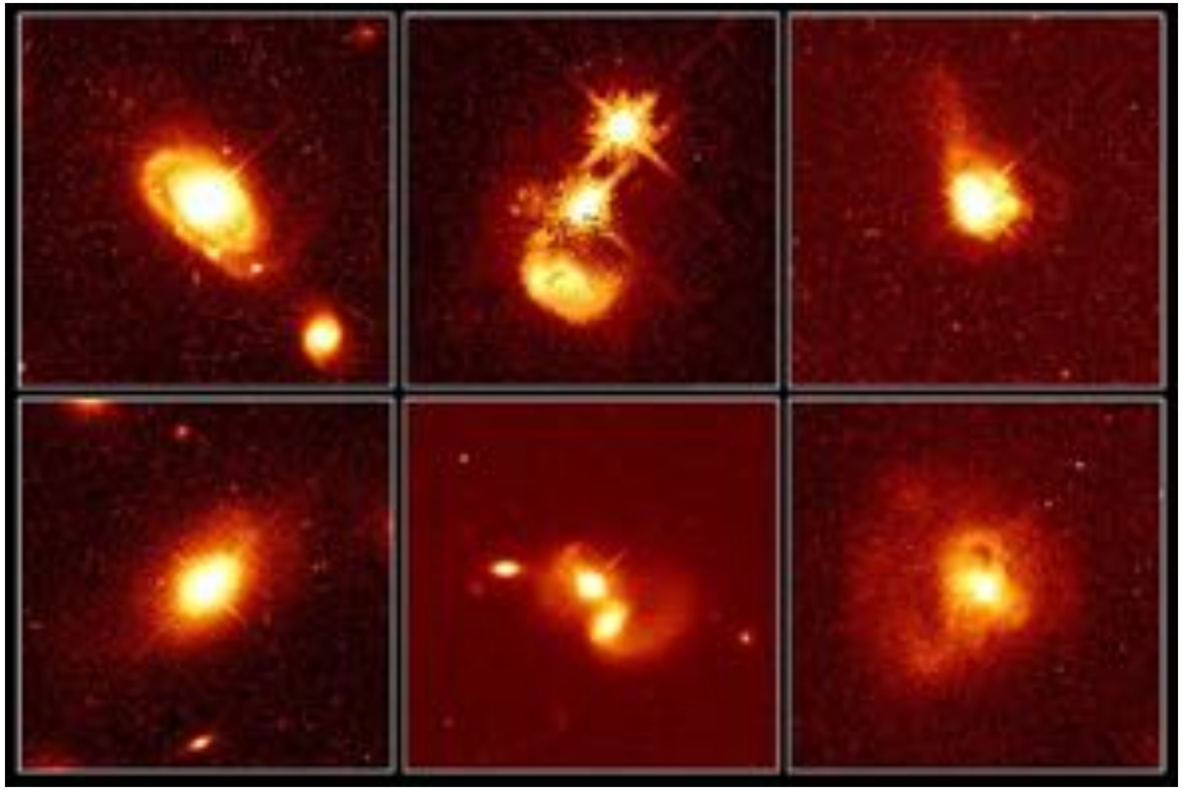


# Feedback from Supermassive Accreting Black Holes

The resulting energy production can be so powerful to outshine the light from all stars in the host galaxy
→ Quasars !

More generally , nuclear phenomena associated with Supermassive Black Hole accretion are called Active Galactic Nuclei (AGN)



Typical quasars spectrum, showing prominent nebular lines emitted by the circumnuclear gas photoionised by the UV radiation emitted by the accretion disc.

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... Part III Minor Topic Formation of Structure in the Universe - please complete this questionnaire and be as constructive as possible in order that the course may be improved in future years.

1. What fraction of the lectures did you attend?

| 0% | 10% | 20% | 30% | 40% | 50% | 60% | 70% | 80% | 90% | 100% |
|---|---|---|---|---|---|---|---|---|---|---|
| ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |

2. How difficult did you find the course?

| too easy | a little too easy | just right | challenging | very challenging |
|---|---|---|---|---|
| ○ | ○ | ○ | ○ | ○ |

3. How interesting did you find the material covered in the course?

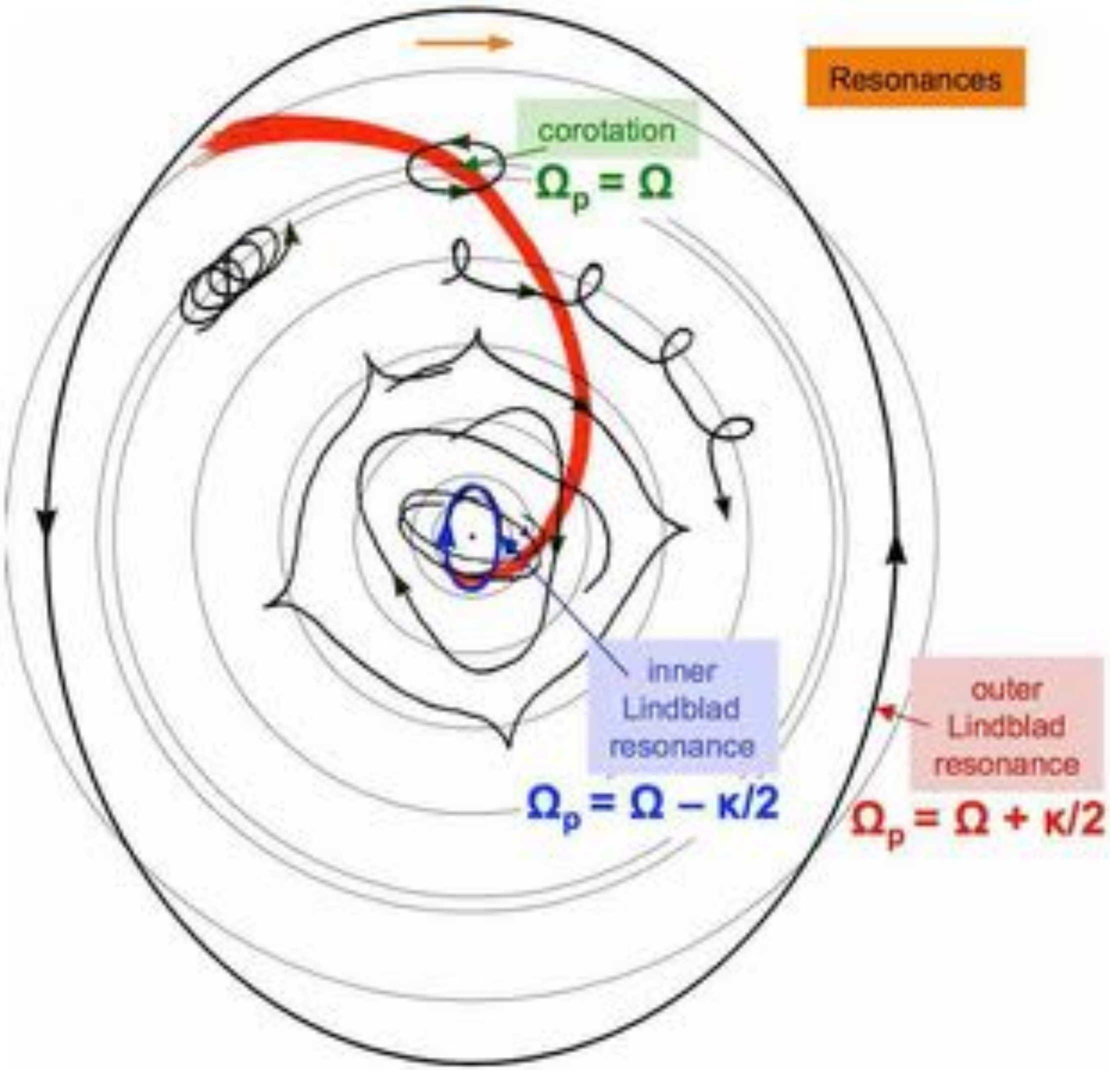
| very dull | quite dull | Okay | quite interesting | very interesting |
|---|---|---|---|---|
| ○ | ○ | ○ | ○ | ○ |

**Deadline to answer the survey (13 questions)**

**12th March**

# Summary of Friday's lecture

- $\Omega_p = \Omega - \frac{\kappa}{n}$: inner Lindblad resonance
- $\Omega_p = \Omega + \frac{\kappa}{n}$: outer Lindblad resonance
- $\Omega_p = \Omega$ is the corotation

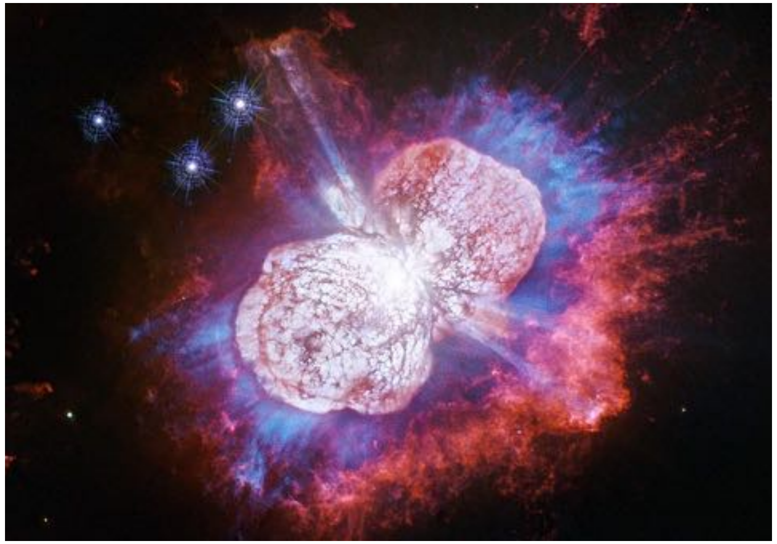


# Summary of Friday's lecture

**AN ISSUE WITH OUR CURRENT MODEL**

- 4% of the baryons are in stars.
- Whereas 80% should have been in stars

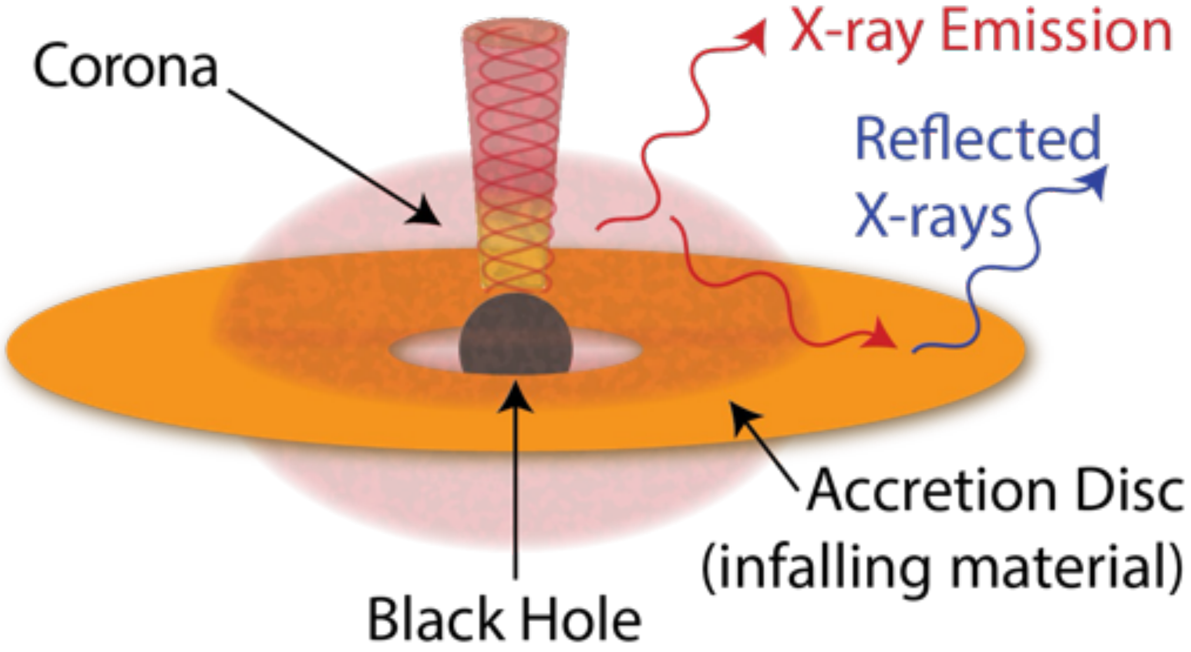
**Negative Feedback**



Kinetic energy is converted into heat, and therefore suppress the star-formation…

# Summary of Friday's lecture

**ACCRETING BY A BLACK HOLE**



# Feedback from Supermassive Accreting Black Holes

**The Eddington Limit**

To have accretion, the gravitational force must be larger than the radiation pressure.

The radiation pressure is dominated by Thomson scattering of photons on electrons

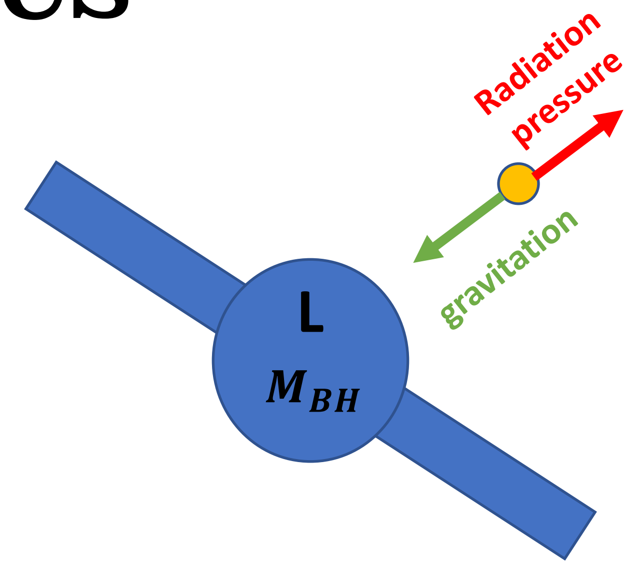
Gravitational pull on the gas dominated by protons

Therefore, the condition for accretion is :

$$\frac{L}{4\pi r^2}\frac{\sigma_e}{c} < \frac{Gm_pM_{BH}}{r^2}$$

Then the condition for accretion is given by :

$$\boldsymbol{L < L_{Edd} = \frac{4\pi cGm_p}{\sigma_e}M_{BH}}$$




Adding some numerical values gives :

$$L_{Edd} \approx 3\times10^4\left(\frac{M_{BH}}{M_\odot}\right)L_\odot$$

which could be inverted to give the minimum black hole mass associated with an accretion luminosity :

$$M_{BH} > 3\times10^{-5}\left(\frac{L}{L_\odot}\right)M_\odot$$

# Feedback from Supermassive Accreting Black Holes

**Mass to energy efficiency**

First, let's compute the efficiency for nuclear fusion reaction in stellar interiors :

p-p chain inside the Sun $\mathbf{4\ {}^{1}H \rightarrow\ {}^{4}He + 2\gamma + 2\nu_e}$

Initial mass : $4{\times}m_p$=4x1.0078 amu=4.0312 amu

Final mass : $m_{He}$=4.0026 amu

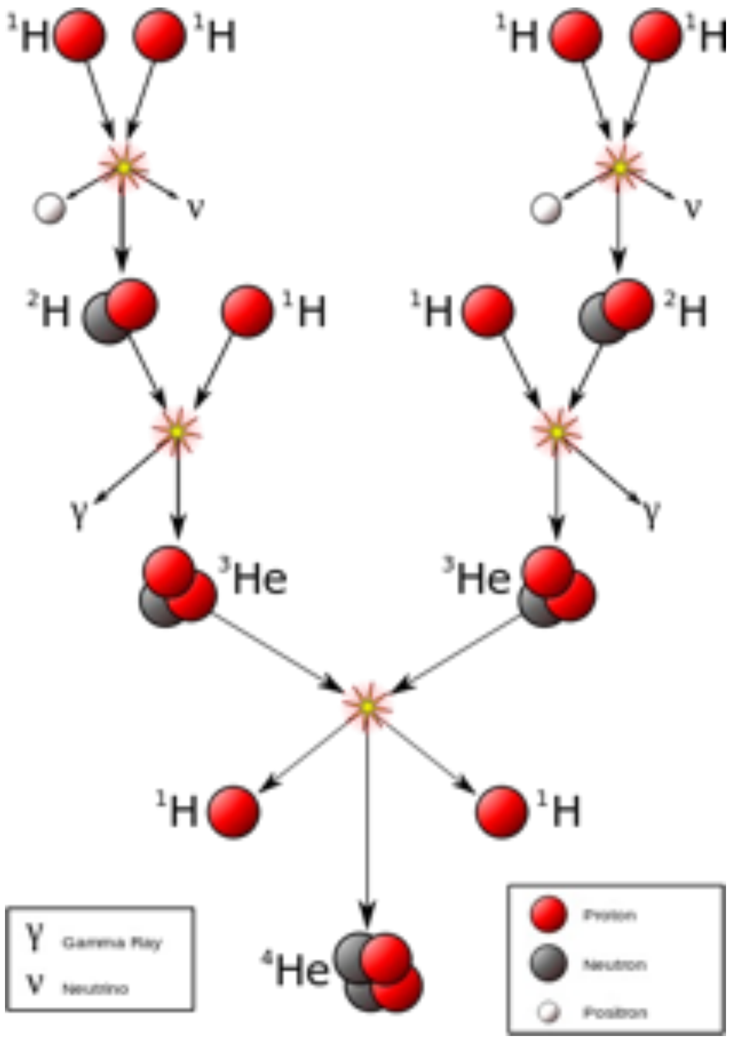
Mass converted into energy : $\Delta m$=0.0286 amu

Atomic mass unit $\approx 1.66{\times}10^{-27}$kg

Efficiency of Mass to Energy conversion :

$$\epsilon_{nuc} = \frac{\Delta m}{4m_p} = 0.007 = 0.7\%$$

**Nuclear reaction inside stars are not very efficient in converting mass into energy**



# Feedback from Supermassive Accreting Black Holes

**Mass to energy efficiency**

Now consider an element of mass $dm$ in the accretion disc moving from orbit with radius $r + dr$ to orbit with radius $r$.

According to the Virial theorem, half of the variation of gravitational potential energy must be radiated away :

$$dE_{rad} = -\frac{1}{2}\frac{GMdm}{r+dr} - \left(-\frac{1}{2}\frac{GMdm}{r}\right)$$

where $M$ is the mass of the central black hole.

The resulting luminosity is therefore :

$$dL = \frac{dE_{rad}}{dt} = \frac{1}{2}GM\frac{dM}{dt}\left(\frac{1}{r} - \frac{1}{r+dr}\right) = \frac{1}{2}G\dot{M}M\frac{dr}{r^2}$$

Accretion rate

Integrating :

$$L = \int_{R_{out}}^{R_{in}} dL = \frac{1}{2}GM\dot{M}\left(\frac{1}{R_{in}} - \frac{1}{R_{out}}\right) \approx \frac{1}{2}\frac{GM\dot{M}}{R_{in}}$$

The efficiency of conversion of matter into energy is given by :

$$L = \epsilon\dot{M}c^2$$

with

$$\epsilon = \frac{GM}{2c^2R_{in}}$$

From general relativity, for a non-rotating black hole, the innermost stable orbit is

$$R_{in} = 3R_{Sch} = \frac{6GM}{c^2}$$

Therefore $\epsilon \sim \frac{1}{12} \approx 0.1 \Rightarrow 10\%$

**Much larger than efficiency of nuclear reaction**

# Feedback from Supermassive Accreting Black Holes

$$\epsilon \sim \frac{1}{12} \approx 0.1 \Rightarrow 10\%$$

This is why BH accretion from a "tiny" region can outshine e stellar light of the whole galaxy.

Then, a black hole of mass $M_{BH}$ in the process of accreting matter must have radiated an amount of energy :

$$E_{BH} = 0.1\, M_{BH} c^2$$

Gravitational binding energy of a bulge or spheroidal galaxy of mass $M_{gal}$ is given by :

$$E_{gal} \sim M_{gal} \sigma^2$$

Stellar velocity dispersion (typically 400km/s)

Since $M_{BH} \sim 10^{-3} M_{sph}$, it follows that :

$$\frac{E_{BH}}{E_{gal}} = 10^{-4} \left(\frac{c}{\sigma}\right)^2$$

Then :

$$\frac{E_{BH}}{E_{gal}} > 80$$

**Much larger than efficiency of nuclear reaction**

The energy produced by the growth of the black hole exceeds the binding energy of the galaxy by a large factor. In principle, black hole accretion seriously harm its host galaxy.

# Feedback from Supermassive Accreting Black Holes

AGN can provide powerful feedback onto the host galaxy and surrounding medium in two forms :

- **when it's radiating at Quasar-like luminosities**, radiation pressure can produce a powerful nuclear wind, which shock the ISM in the host galaxy → heating and outlows expels the gas out of the galaxy. *Detailed calculations show that powerful quasars can completely clean a galaxy out of their ISM*

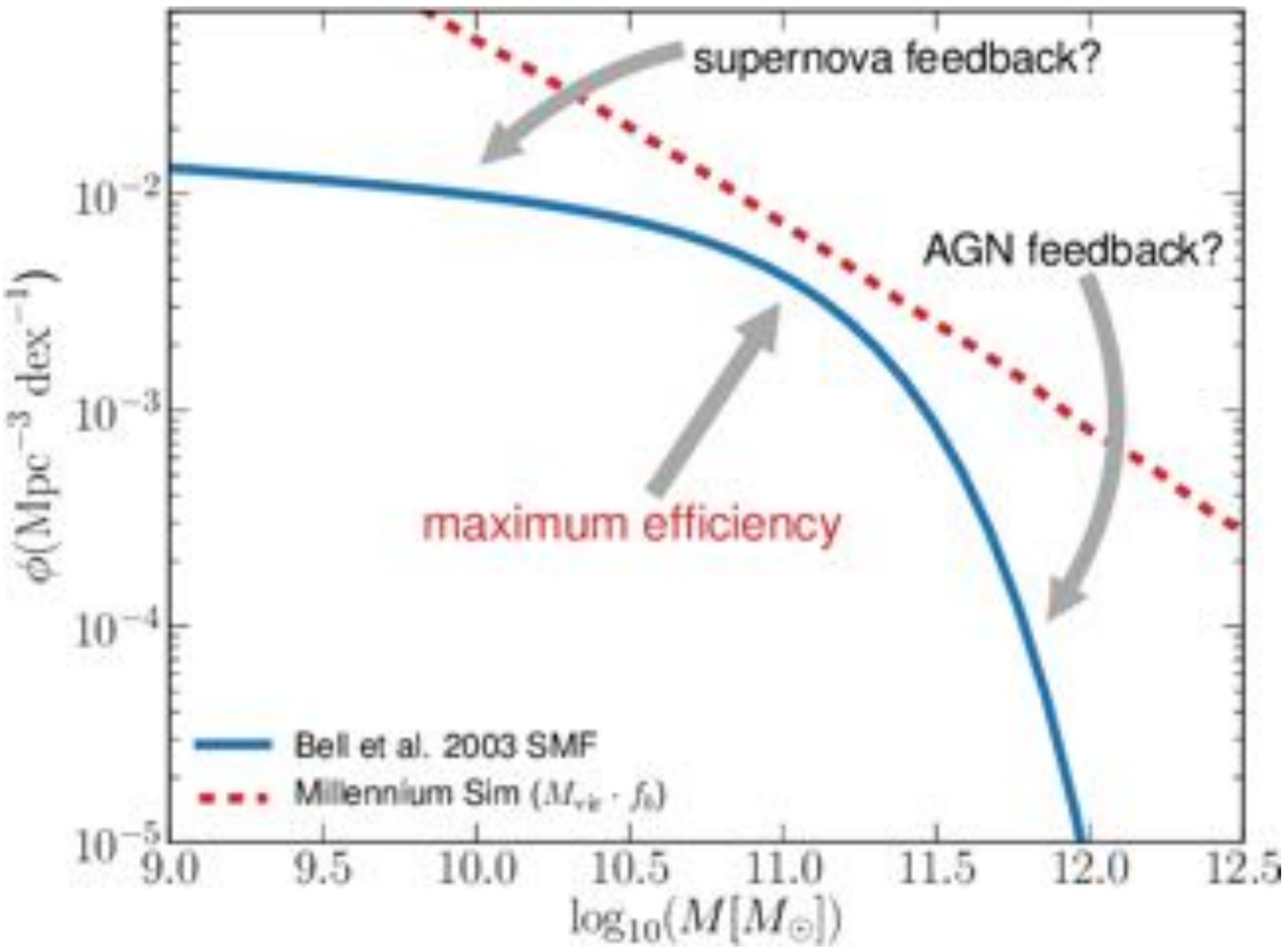
- **when the accreting black hole produces powerful radio-jet** then the energy injected into the intergalactic and intracluster medium keeps such medium hot, preventing it to cool and feed the central galaxy with new gas

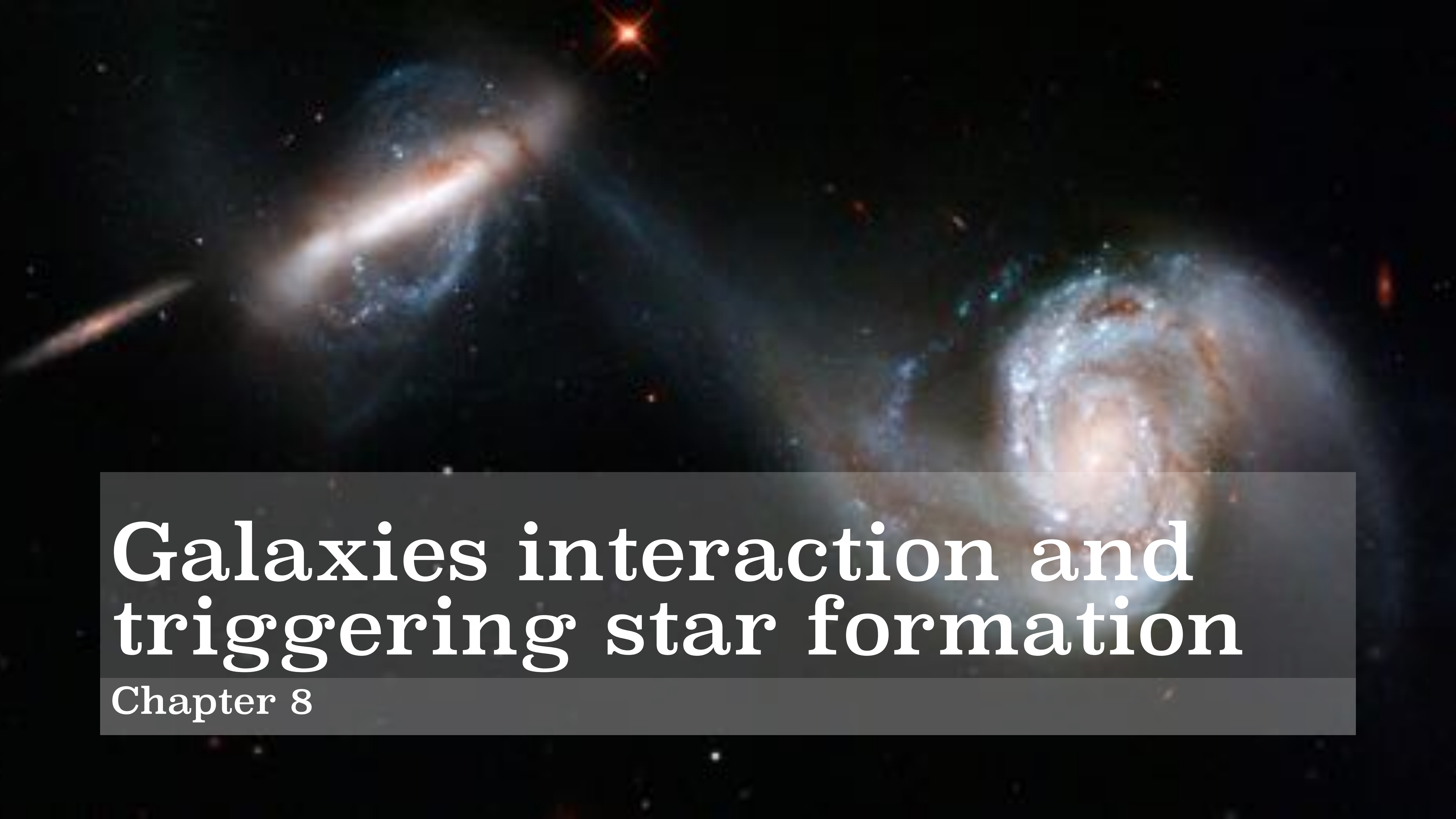
Such heating of the gas in the halo, preventing it to cool, results into "starvation" of the galaxy.

# Feedback from Supermassive Accreting Black Holes

**Which is the dominant effect ?**

Answering this question depends on the mass of the galaxy

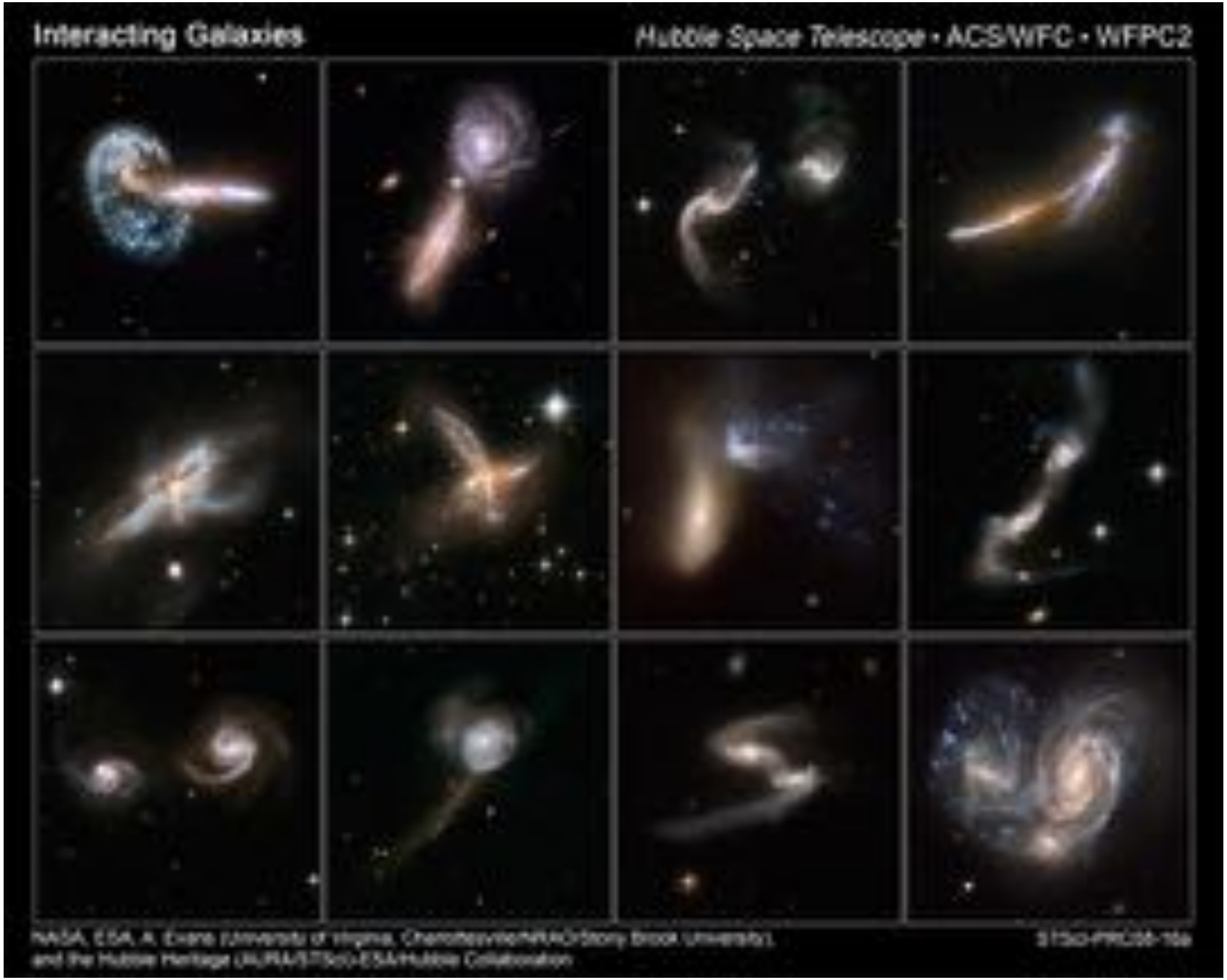




# Galaxies interaction and triggering star formation

## Chapter 8

The most luminous galaxies in the Universe (ULIRGs) and most powerful AGN always show highly distorted morphologies indicative of strong galaxies interactions or advanced galaxies merging.



# Collisions between stellar system

**Stellar collisional cross section and relaxation time**

We start by estimating the collision cross section for strong gravitation interactions as follows :

- We identify a pair-wise collision as one in which the star is **<u>significantly deflected</u>**
- This will occur if :

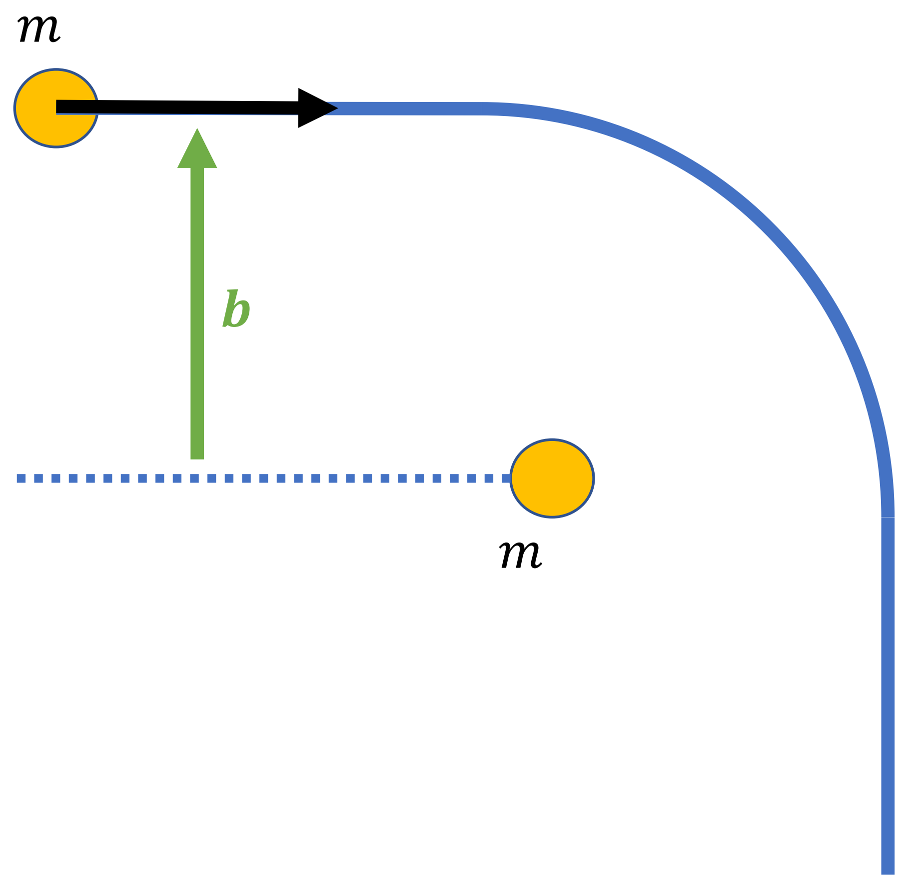
$$\frac{1}{2}mv^2 \sim \frac{Gm^2}{r}$$

which gives the scattering radius :

$$r_s \sim \frac{2Gm}{v^2}$$

- The effective cross section is therefore defined as :

$$\pi r_s^2 \sim \pi \left(\frac{2Gm}{v^2}\right)^2$$



# Collisions between stellar system

**Stellar collisional cross section and relaxation time**

<u>Note</u> :

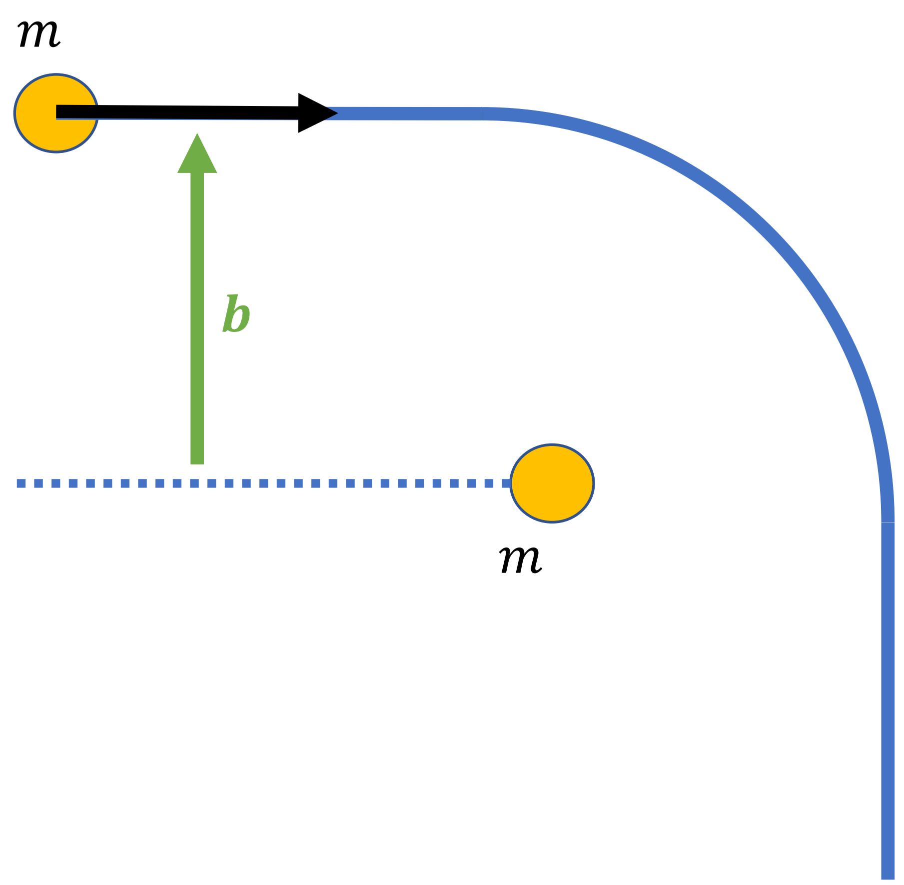
- The mean free path for strong collision is $\lambda = \frac{1}{n\sigma}$
- The collision time is $t_s = \frac{\lambda}{v}$

$$\sigma \sim \pi \left(\frac{2Gm}{v^2}\right)^2$$

or

$$\boxed{t_s = \frac{v^3}{4\pi G^2 m^2 n}} = 4\times 10^{12} yr \left(\frac{v}{10\ km/s}\right)^3 \left(\frac{m}{M_\odot}\right)^{-2} \left(\frac{n}{1pc^{-3}}\right)^{-1}$$

**Much longer than Hubble time → strong gravitational collisions do not occur**



# Collisions between stellar system

**Stellar collisional cross section and relaxation time**

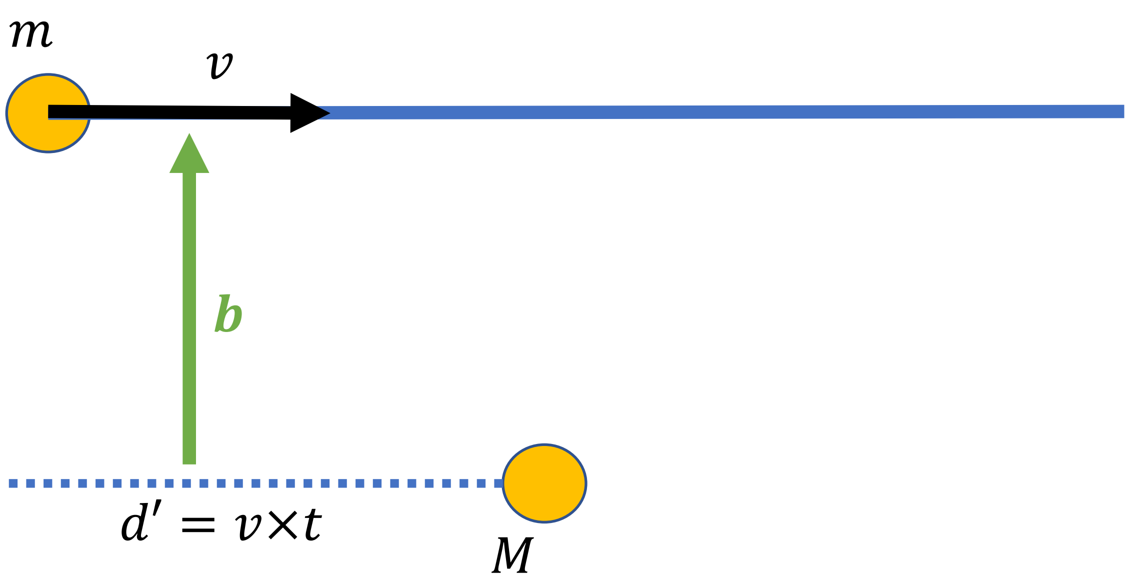
Now we consider interactions which give rise to small interaction, e.g., a star which is <u>**almost undeflected**</u>.

In this configuration the gravitational force is given by :

$$F = \frac{GmM}{d^2} = \frac{GmM}{b^2 + v^2t^2}$$

The net force over the interaction is perpendicular to the direction of travel ($F_\perp = F\sin\theta = F\frac{b}{d}$) and for an impact parameter $b$, we have :

$$F_\perp = \frac{GmMb}{(b^2 + v^2t^2)^{3/2}}$$




Then, according to the Newton's second law of motion :

$$F = m\frac{dv}{dt}$$

Then

$$F_\perp = \frac{GmMb}{(b^2 + v^2t^2)^{3/2}} = m\frac{dv_\perp}{dt}$$

# Collisions between stellar system

**Stellar collisional cross section and relaxation time**

Integrating over the interaction gives :

$$F_\perp = \frac{GmMb}{(b^2+v^2t^2)^{3/2}} = m\frac{dv_\perp}{dt}$$

$$\Delta v_\perp = \frac{1}{M}\int_{-\infty}^{+\infty} F_\perp dt = \frac{2Gm}{bv}$$

The momentum is conserved by the fact that the second star (here assumed stationary) must have a $\Delta v_\perp$ in the opposite sense.

As the star undergoes many small deflections, we may assume they are in random directions giving :

$$< \Delta v_\perp > = \ 0$$

Integrating over all encounters :

- The number of collisions with $b \rightarrow b + db$ is :

$$dN = n \times vt \times 2\pi bdb$$

Density of stars

Distance travels by a star in time t

Area of the annulus between impact parameter b and b+db

Hence :

$$< \Delta v_\perp^2 > = \int_{b_{min}}^{b_{max}} \left(\frac{2Gm}{bv}\right)^2 nvt2\pi bdb$$

$$= \frac{8\pi G^2 m^2 nt}{v} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

# Collisions between stellar system

**Stellar collisional cross section and relaxation time**

Defining :

$$\ln \Lambda = \ln\left(\frac{b_{max}}{b_{min}}\right)$$

After a long enough time, the star's perpendicular speed will (on average) grow to equal its original speed. Define this as **the <u>relaxation time</u>** - time required for the star to lose all memory of its initial orbit.

Set :

$$v^2 = <\Delta v_\perp^2> = \frac{8\pi G^2 m^2 n t}{v}\ln(\Lambda)$$

The relaxation time is therefore given by :

$$t_r = \frac{v^3}{8\pi G^2 m^2 n \ln\Lambda}$$

Remember that the **<u>collision time</u>** is defined by :

$$t_s = \frac{v^3}{4\pi G^2 m^2 n}$$

Hence the relaxation time is :

$$t_r = \frac{t_s}{2\ln\Lambda}$$

**Frequent distant interactions are much effective at changing the orbit that rare close encounters**

# Collisions in stellar systems

We can quantify the degree to which stars interact using the collisional relaxation time

$$t_r = \frac{t_s}{2 \ln \Lambda}$$

We first need to estimate :

$$\ln \Lambda = \ln\left(\frac{b_{max}}{b_{min}}\right)$$

For an isolated system of $N$ stars, and size $R$, we can estimate :

- $b_{max} \sim R$
- $b_{min} \sim r_s$ for consistency ($r_s \sim \frac{2Gm}{v^2}$)

Scattering radius

From the virial theorem :

$$2 \times \frac{1}{2} N m v^2 = \frac{G(Nm)^2}{R}$$

which gives :

$$v^2 R = GNm$$

Then :

$$\Lambda = \frac{b_{max}}{b_{min}} = \frac{R}{r_s} = R\frac{v^2}{Gm} = \frac{1}{2}N$$

The typical time for a star to cross the system is defined as **the crossing time** :

$$t_c \approx \frac{R}{v}$$

# Collisions in stellar systems

The density of stars per unit volume is given by :

$$n = \frac{N}{\frac{4}{3}\pi R^3}$$

Then :

$$\frac{t_r}{t_c} = \frac{v}{R}\frac{v^3 \frac{4}{3}\pi R^3}{8\pi G^2 m^2 N \ln \Lambda} = \frac{N}{6 \ln \frac{N}{2}}$$

$$t_c \approx \frac{R}{v}$$

$$t_r = \frac{t_s}{2 \ln \Lambda}$$

$$t_s = \frac{v^3}{4\pi G^2 m^2 n}$$

$$\Lambda = \frac{1}{2}N$$

$$v^2 R = GNm$$

For a typical galaxies with $N \sim 10^{11}$stars, the crossing time is $t_c = 10^7 - 10^8$ yr and the relaxation time $t_r \sim 10^8 t_c$ which is much longer than the Hubble time.

Similarly for a globular cluster, we find that $t_r \sim 10^{10}$yr

**Most of the stars systems are collisionless.**
**The Sun is a good example !**

# Collisions between stellar system

**Dynamical Friction**

The dynamical friction is loss of momentum and kinetic energy of moving bodies through gravitational interactions with surrounding matter in space.

Recall the result we obtained for the change in velocity perpendicular to nearly undeflected path of a particle of mass $M$ as it passes a mass of $m$ :

$$\Delta v_{\perp} = \frac{2Gm}{bv}$$

By conservation of momentum, the particles of mass $m$ should also suffer a change in perpendicular velocity of :

$$\Delta v_{\perp} = \frac{2GM}{bv}$$

The total change in kinetic energy of the system due to this change in perpendicular velocity is :

$$\Delta E_{k,\perp} = \frac{M}{2}\left(\frac{2Gm}{bv}\right)^2 + \frac{m}{2}\left(\frac{2GM}{bv}\right)^2 = \frac{2G^2mM(M+m)}{b^2v^2}$$

This energy can only come from the forward motion of $M$, i.e. $\Delta E_{k,\perp} + \frac{M}{2}\Delta v_{\parallel}^2 = 0$, but :

$$\frac{1}{2}\Delta\left(v_{\parallel}^2\right) = v_{\parallel}\Delta v_{\parallel} \approx v\,\Delta v_{\parallel}$$

Hence :

$$-\Delta v_{\parallel} \approx \frac{\Delta E_{k,\perp}}{Mv_{\parallel}} = \frac{2G^2m(M+m)}{b^2v^3}$$

# Collisions between stellar system

**Dynamical Friction**

Finally, we again integrate over all impact parameters and obtain :

$$-\frac{dv}{dt} = \int_{b_{min}}^{b_{max}} nv\, \frac{2G^2 m(M+m)}{b^2 v^3} 2\pi b db$$
$$= \frac{4\pi G^2 (M+m)\rho}{v^2} \ln \Lambda$$

We can now apply this model to the interaction of two galaxies :

- Take $M$ the mass of an entire galaxy
- $\rho = nm$ as the density within the interacting galaxy

The relaxation time :

$$\boxed{t_r = \frac{v}{|\dot{v}|} = \frac{v^3}{4\pi G^2 M \rho \ln \Lambda}}$$

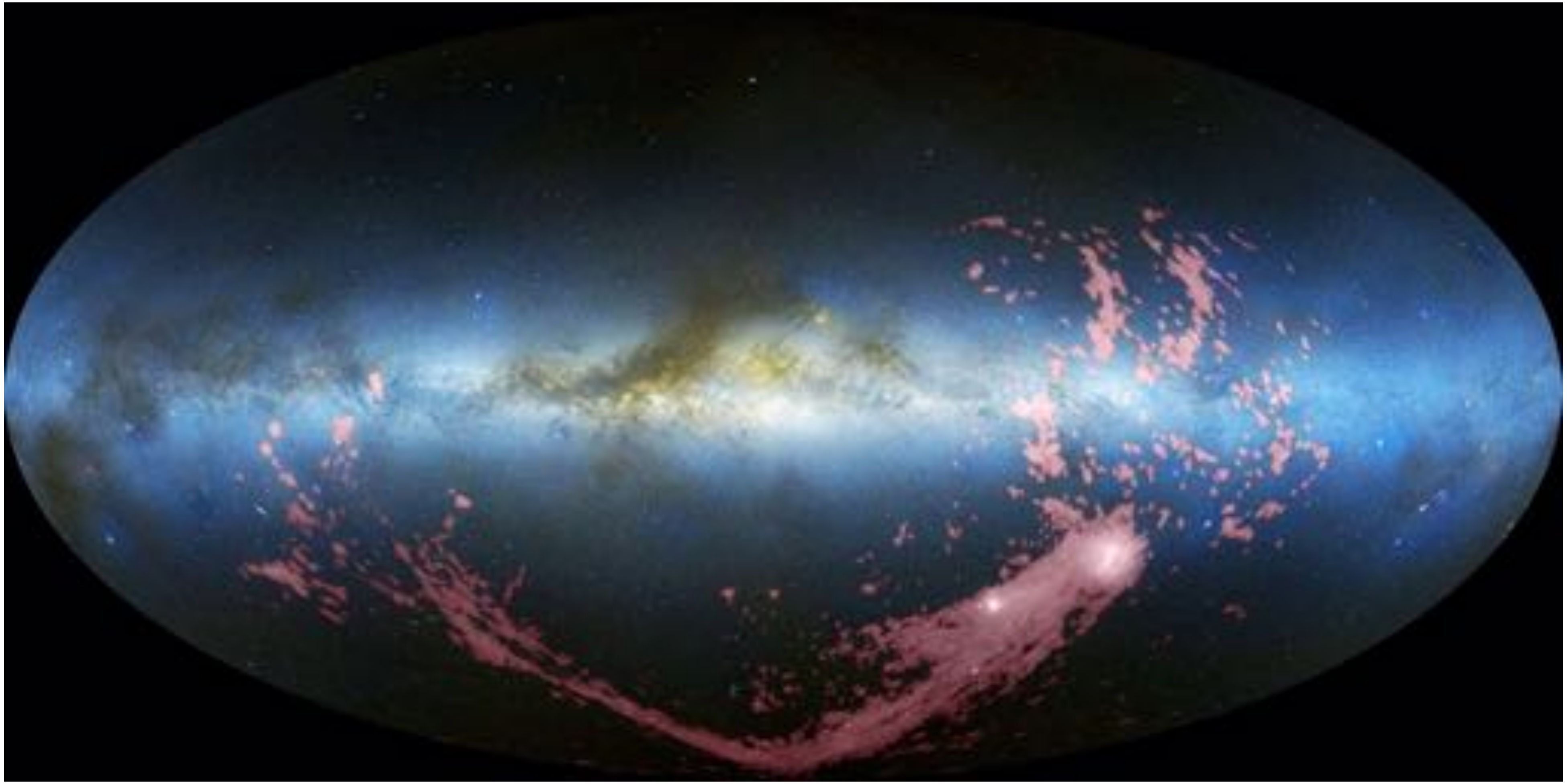
This will be the timescale on which dynamical friction acts to dissipate the bulk kinetic energy of the interacting systems and allow the galaxies to merge.

# Interacting galaxies

Many galaxies including our own are interacting gravitationally.

The Milky Way is interacting with several low mass systems including the LMC and the SMC.

In many cases, the effects of these interactions are relatively small.



# Interacting galaxies

M51 is probably the best example showing how interaction with smaller mass companion could influence the shape of the main galaxy.

The well defined arms in M51 are the consequence of the interaction with its companion on the right.

# Interacting galaxies

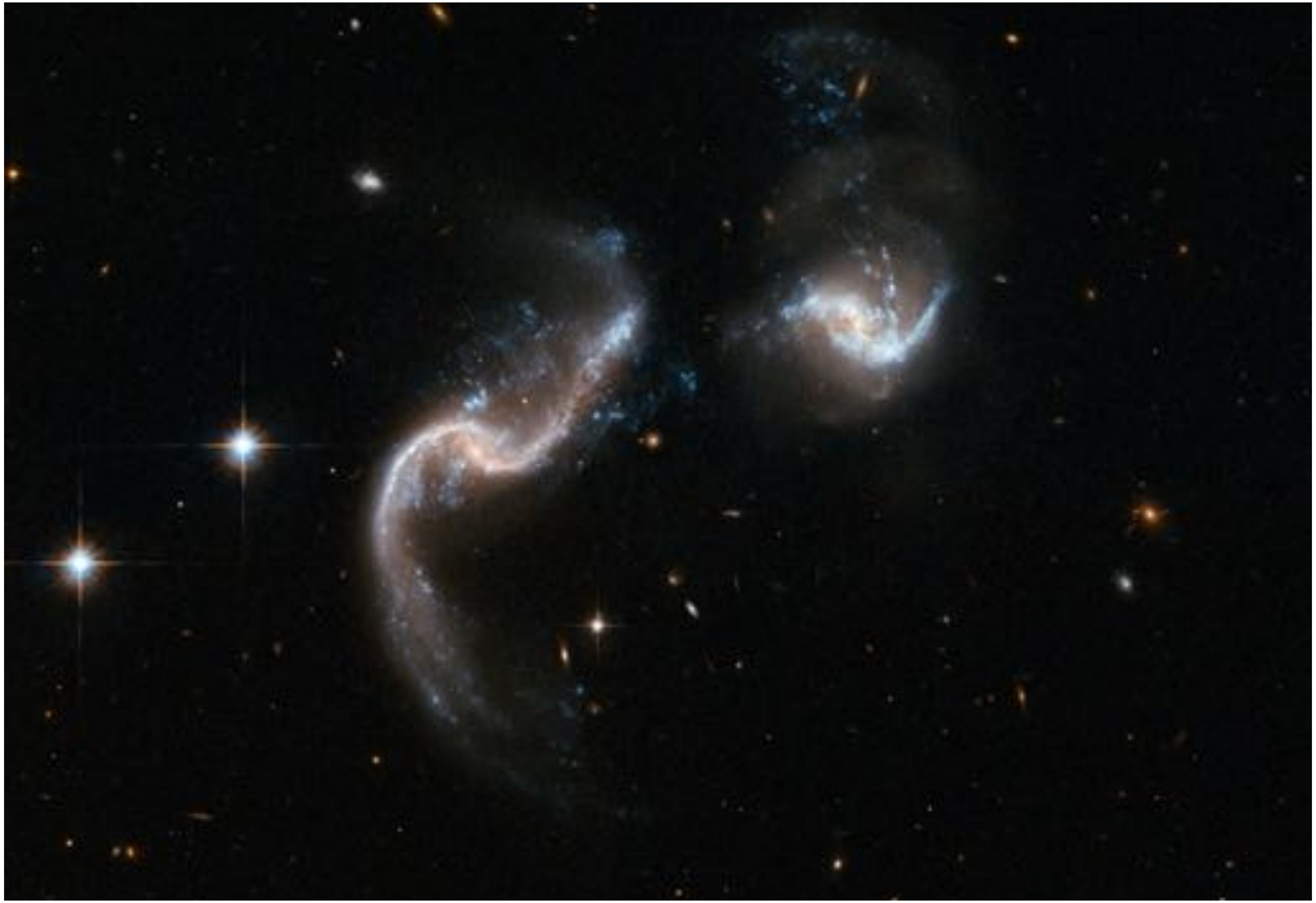
LEDA 62867 and NGC 6786 are two interacting spiral galaxies, with well defined arms.

Simulations of this system predict that it will lead to a merger in few billion years.

# Interacting galaxies

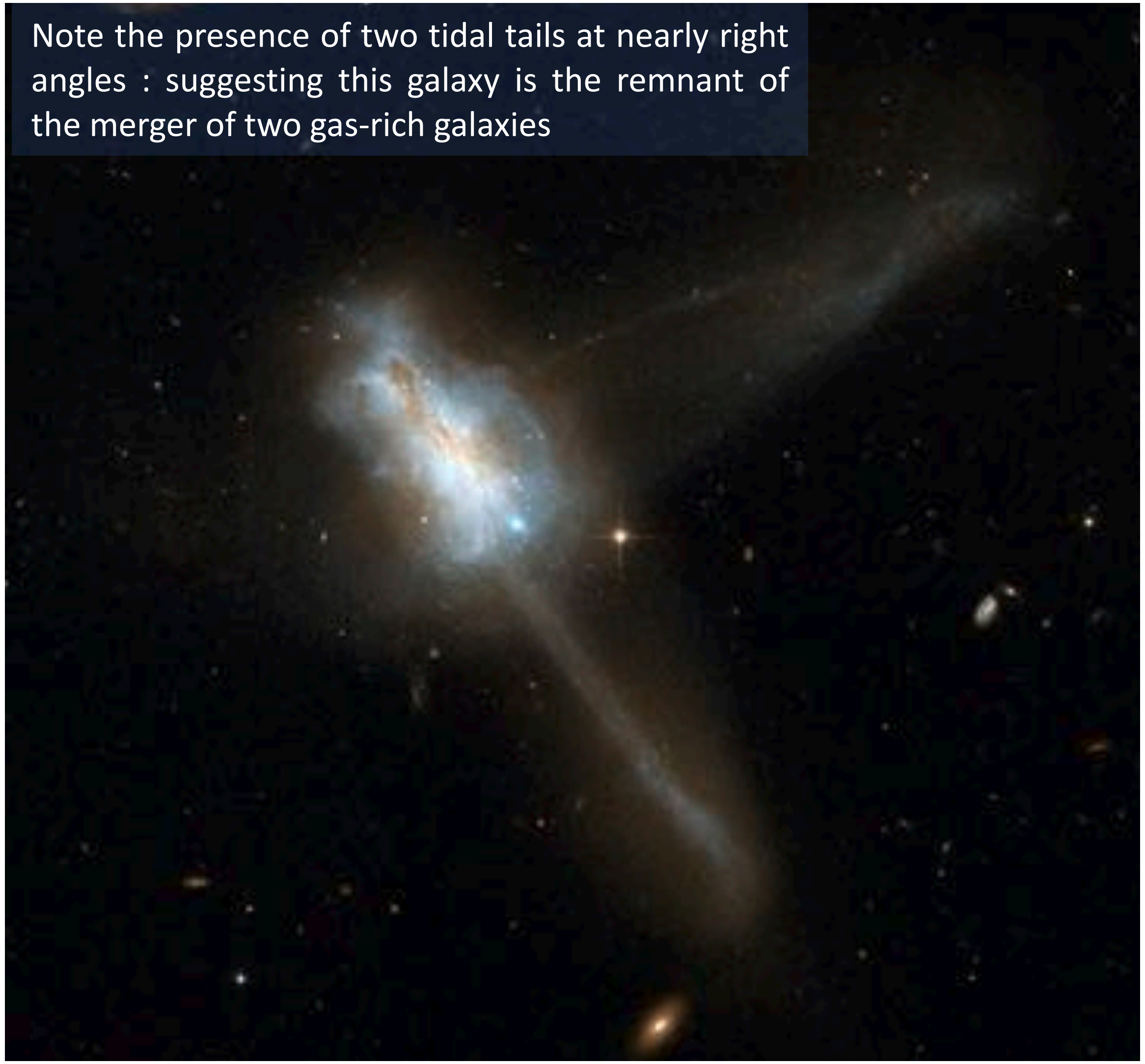
Arp 256 is another interacting system, for which Hubble images show blue knots of star formation which have been triggered by the interaction.

Note also the characteristic tidal tails on the left, which are a signature of a strong gravitational interaction.



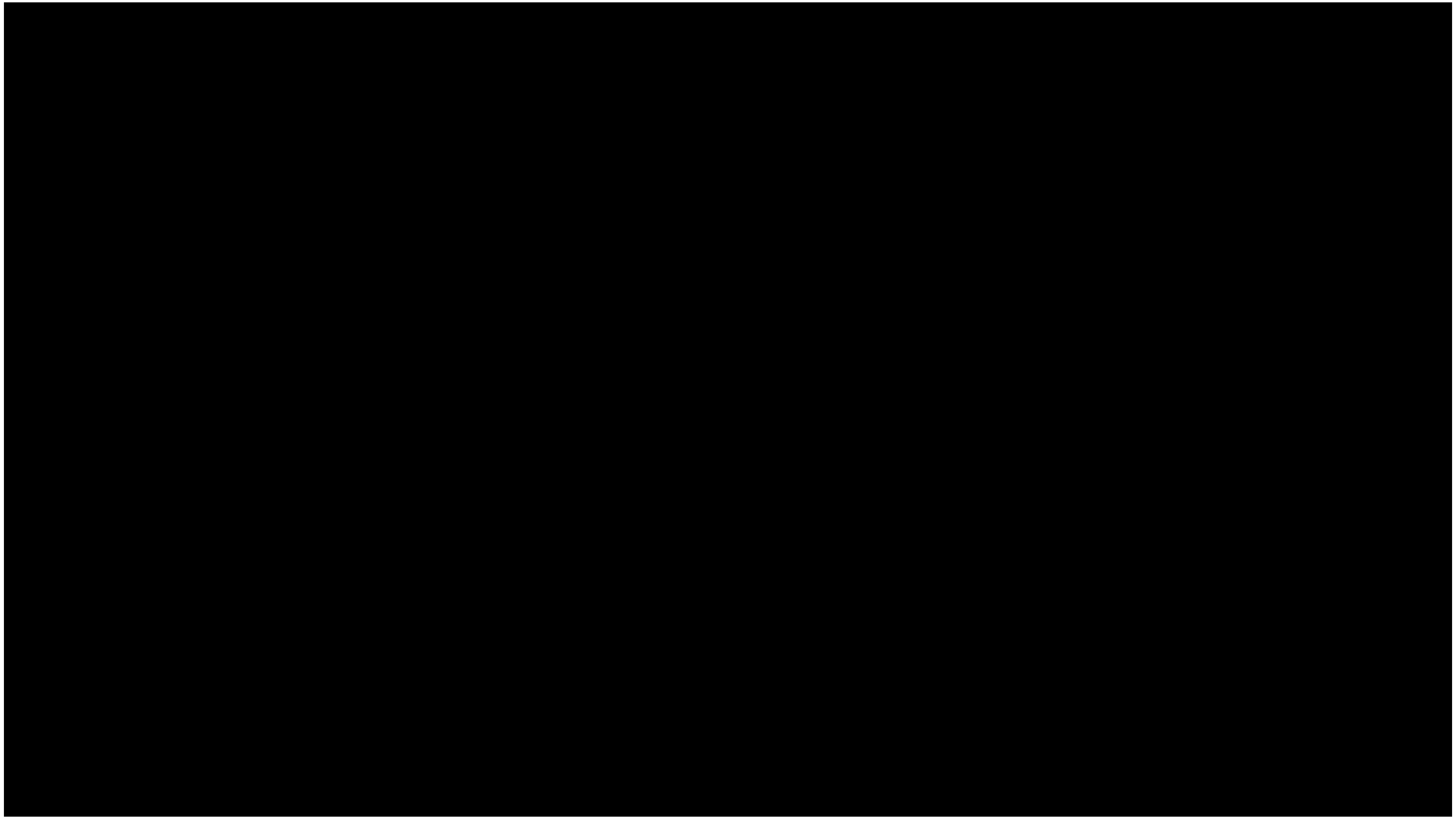
# Interacting galaxies

At an advanced stage of merging, intense star formation regions appears as long threadlike structure located between the main galaxy cores.



Note the presence of two tidal tails at nearly right angles : suggesting this galaxy is the remnant of the merger of two gas-rich galaxies

# Interacting galaxies



# Interacting galaxies

Despite the various morphologies observed in the Universe, the physics of these processes seems to be relatively simple, dominated by pure gravitation interactions.

We can successfully model galaxies in this context as being composed just of colisionless massive particles (stars or dark matter).

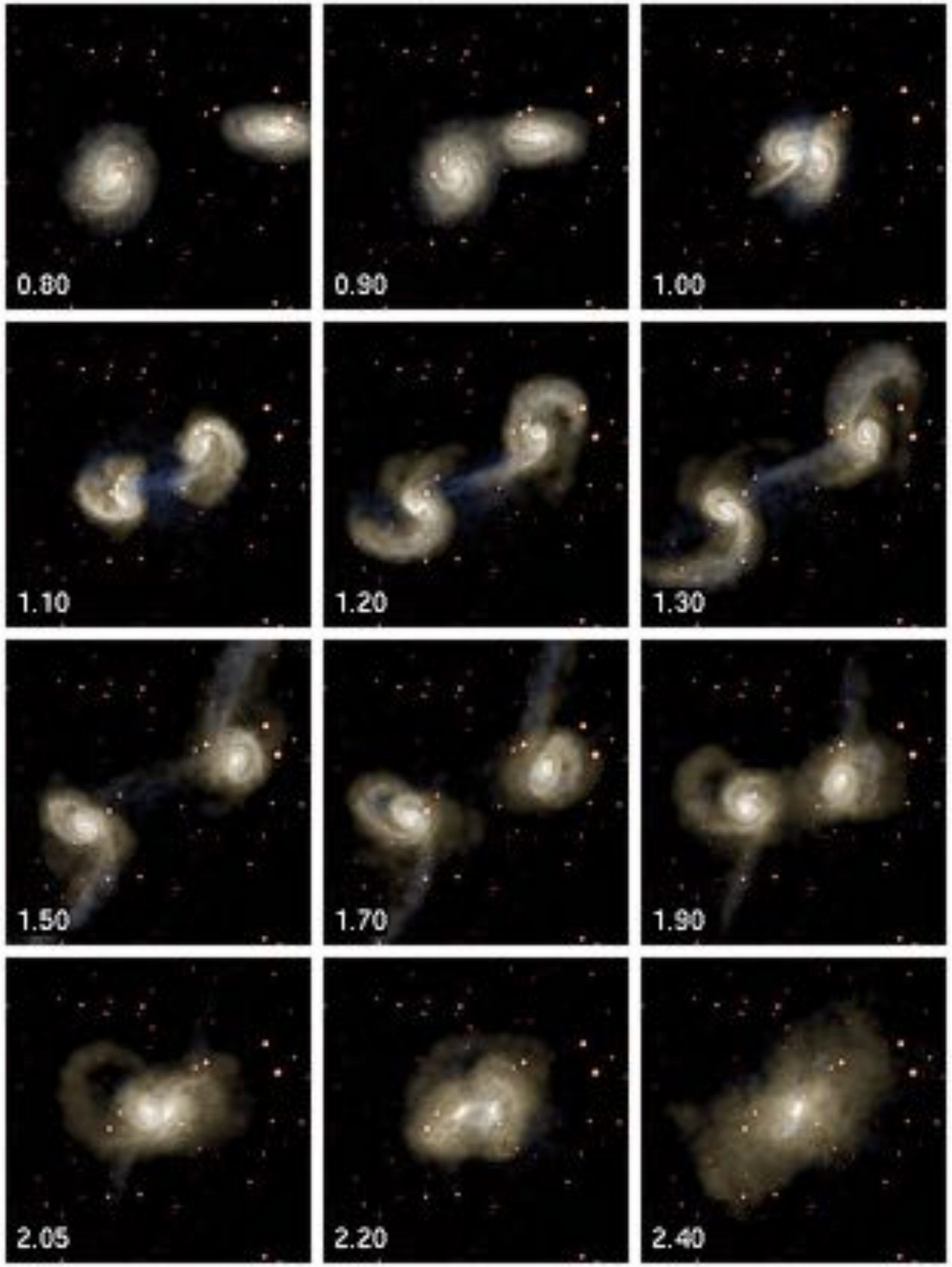
We can apply our analysis of dynamical friction to interacting galaxies. The relaxation time is given by :

$$t_r = \frac{v}{|\dot{v}|} = \frac{v^3}{4\pi G^2 M\rho \ln\Lambda}$$

To simplify the analysis, we assume $\ln\Lambda \sim 1$, and take similar values to those used in simulations :

- $v \sim 200 km/s$
- $M \sim 10^{10} M_\odot$
- $\rho \sim 10^8 M_\odot\ kpc^{-3}$

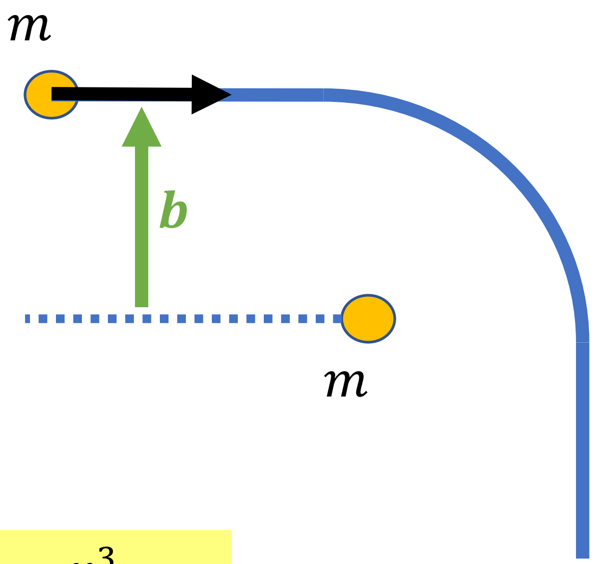
**$t_r \sim 8{\times}10^8 yr$ , consistent with the simulation, where the time between the first overlap of the discs and the completion of merger is $\sim 1{\times}10^9$yr.**



# Summary of Monday's lecture

COLLISION BETWEEN STELLAR SYSTEMS

STRONG INTERACTIONS

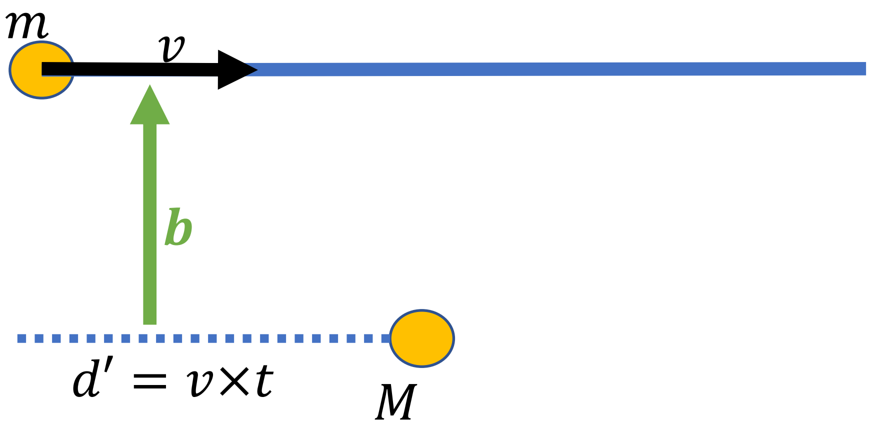



$$t_s = \frac{v^3}{4\pi G^2 m^2 n}$$

$\approx 4{\times}10^{12}$yr : much longer than the Hubble time

➔ Most stellar systems are collisionless.

SMALL INTERACTIONS




$$t_r = \frac{t_s}{2\ln\Lambda}$$

➔ Distant interactions are more effective to change stellar orbits

# Summary of Monday's lecture

**COLLISION BETWEEN STELLAR SYSTEMS**

$$\frac{t_r}{t_c} = \frac{N}{6 \ln \frac{N}{2}}$$

For a typical galaxies $t_r \sim 10^8 t_c$ : much longer than the Hubble time.

**Most of the stars systems are collisionless.**
**The Sun is a good example !**

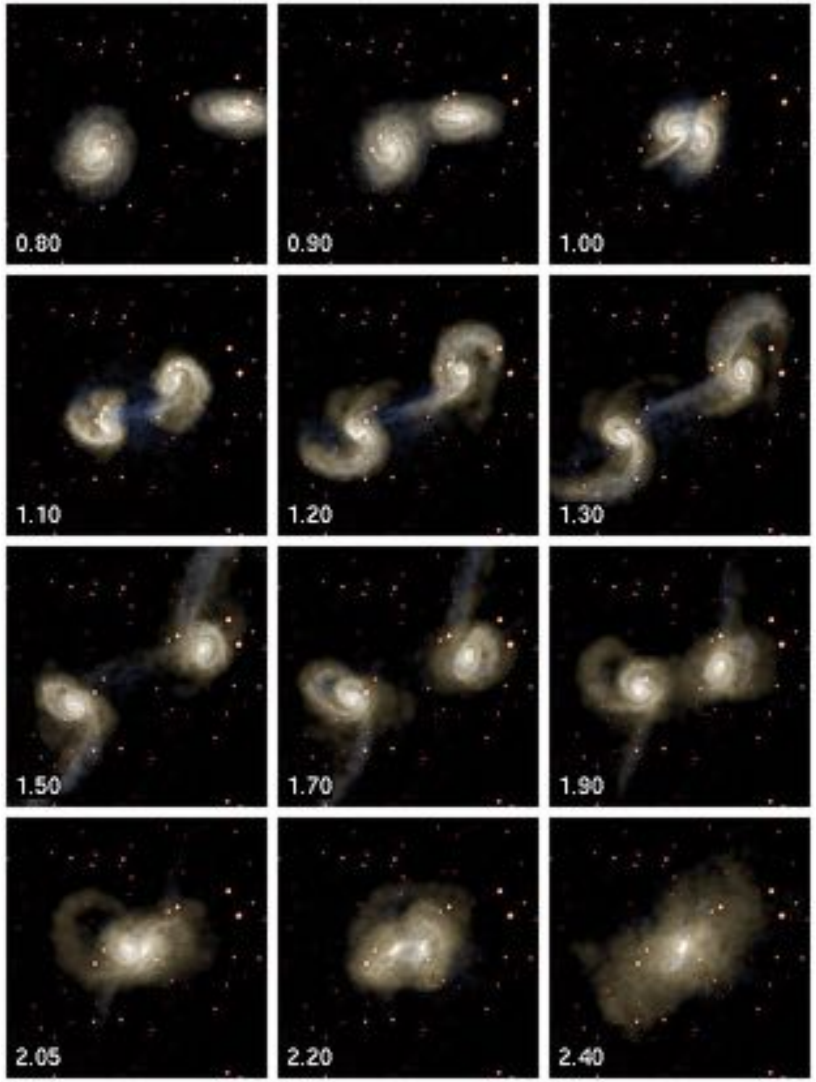
# Summary of Monday's lecture

**DYNAMICAL FRICTION**

The dynamical friction (also called the Chandrasekhar friction), is loss of momentum and kinetic energy of moving bodies through gravitational interactions with surrounding matter in space.

$$t_r = \frac{v}{|\dot{v}|} = \frac{v^3}{4\pi G^2 M\rho \ln \Lambda}$$

Timescale on which dynamical friction acts to dissipate the bulk kinetic energy of the interacting systems and allow the galaxies to merge.



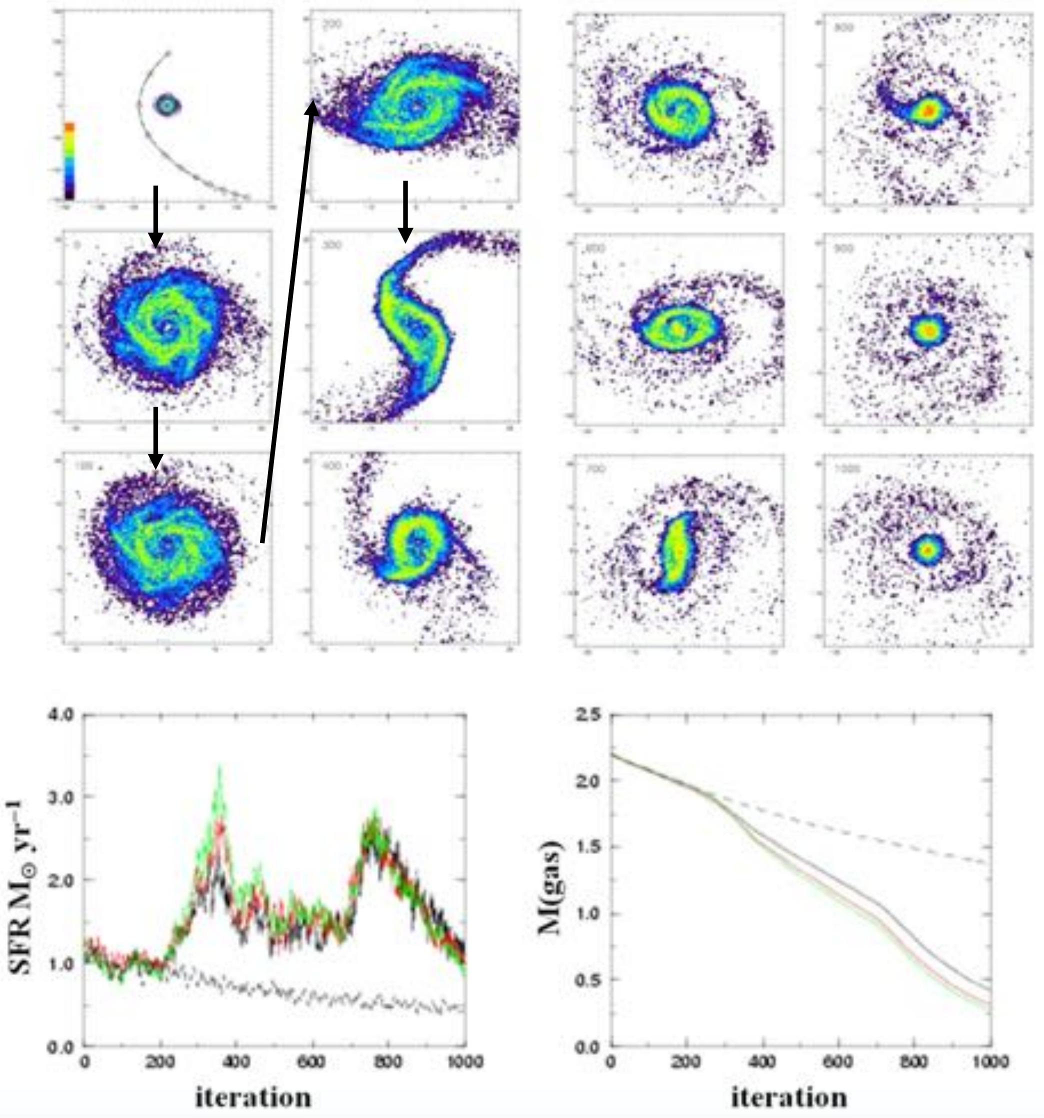
# Interacting galaxies

The simulation on the right includes a simple model for star formation which is determined by cloud build-up as well as following the gas in the galaxy.

The system is perturbed by the passage of a satellite galaxy, but this interaction will not start a merging process

The conclusions are the following :

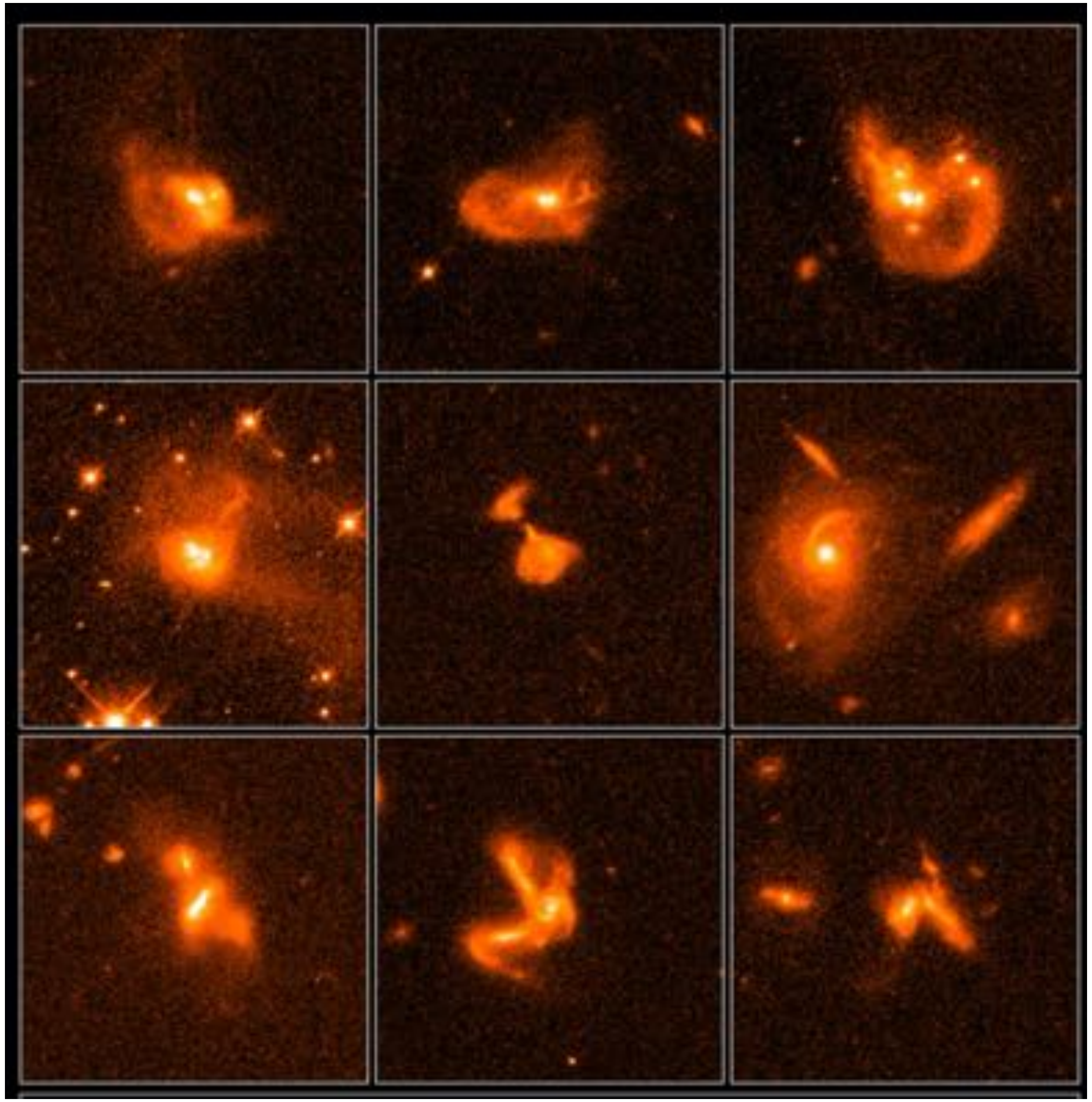
- Spiral density waves are induced with a very clear two armed spiral
- Shocks and cloud-collisions dissipate the kinetic energy of the cloud and gas accumulates on the inner Lindblad resonance
- The high-gas density leads to a burst of star formation
- After a further period of time, the gas dissipates more kinetic energy and accumulates in the nucleus of the galaxy giving rise to a second burst of star formation
- Fraction of the gas accumulated in the nuclear region can accrete onto the nuclear supermassive black hole.



# ULIRGs as merging systems

ULIRGs are the best examples of merging system : tidal interactions and merging after the circular motion of clouds causing cloud-cloud collisions resulting into loss of angular momentum, which allows clouds to flow towards the center of the galaxy to feed star formation in the central region, and often black hole accretion.

Indeed, we observe that, in ULIRGs, star formation is generally concentrated within few hundred parsecs and that they host AGN.



# Nuclear fueling through stellar bars

The formation of a stellar bar (often triggered by mild galaxy interaction) could also lead to increase the star formation in the galaxy :

- The bar is a very strong non-linear perturbation which can exist within the inner Lindblad resonance
- Gas on circular orbits encountering the bar at supersonic velocities is shocked
- Kinetic energy is dissipated and the gas accumulates in the bar
- Orbits in the bar are highly elongated bringing gas and stars to the nucleus.

# Gravitational instabilities in the cosmological context

Chapter 9

# Starting point

For the following analysis, we need to assume :

- **an expanding Universe** with the following cosmological field equations : $\epsilon \propto P/\rho c^2$

$$\frac{\ddot{R}}{R} + \frac{4\pi G\rho}{3}(1+\epsilon) - \frac{\Lambda}{3} = 0$$

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G\rho}{3} - \frac{\Lambda}{3} = -\frac{kc^2}{R^2}$$

- **the so-called ΛCDM cosmology** :
  - a flat geometry : $\Omega_\Lambda + \Omega_m = 1$
  - $\Omega_\Lambda \sim 0.7, \Omega_m \sim 0.3$ and $\Omega_b \sim 0.044$
  - the seeds of structure are quantum fluctuations which are amplified by inflation.

**DEFINITIONS TO REMEMBER**

Hubble constant; Scale factor

$$H = \frac{\dot{R}}{R}$$

Dark Energy density; Cosmological constant

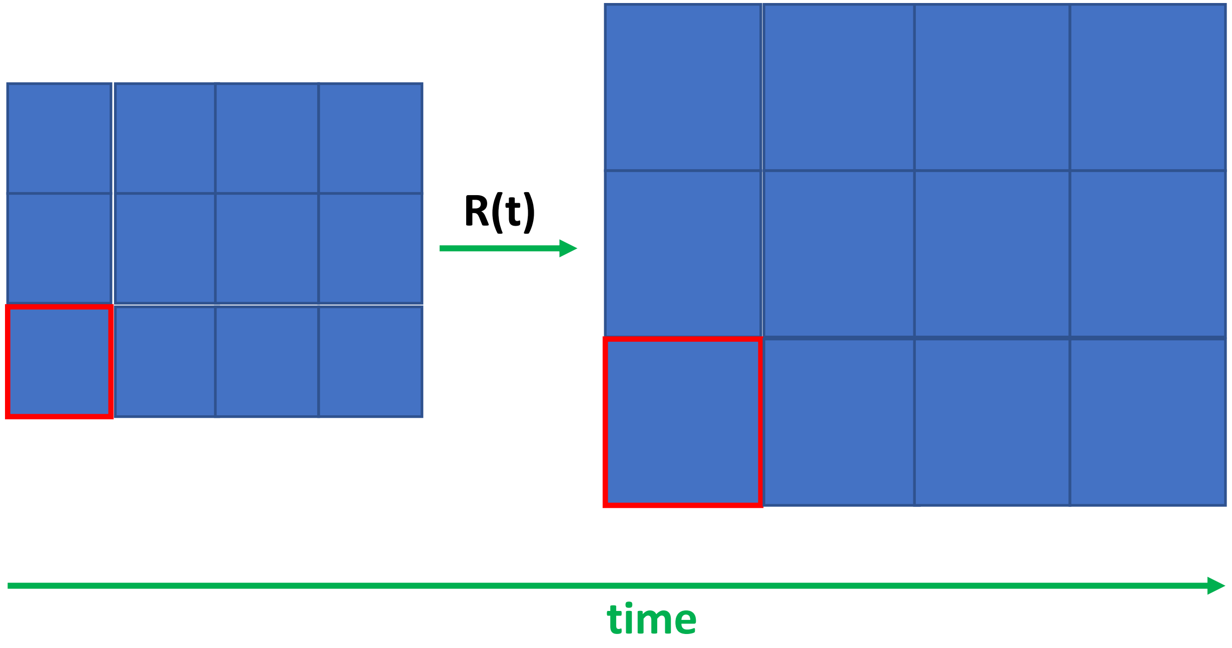
$$\Omega_\Lambda = \frac{\Lambda}{3H^2}$$

Matter density

$$\Omega_m = \frac{\rho}{3H^2/8\pi G}$$

Space curvature

$$1 - \Omega_m - \Omega_\Lambda = -\frac{kc^2}{R^2H^2}$$



www.spacetelescope.org

# Jeans instability in an expanding Universe

*A comoving coordinate system is a reference frame that expands in tandem with the expansion of the Universe, thus factoring out the effect of the Hubble expansion*

At the galactic scale, the Jeans instability focuses on the deviations from the smooth expansion of the Universe in a co-moving frame.

We expect the deviations to be small, and therefore :

- they can be approximated by a perturbation analysis
- we can use locally non-relativistic equations for the fluid
- we can treat gravitational perturbations as sufficiently small that a Newtonian approximation is justified.

# Jeans instability in an expanding Universe

**DEFINITIONS**

<u>Proper distance :</u> corresponds to where an object will be at a given cosmological time (*this distance can change over time because of the expansion of the Universe*)

<u>Comoving distance</u> takes into account the expansion of the Universe (*therefore it is not changing over time*)

The relation between *proper distance* and *comoving coordinates* is given by :

$$r = R(t)\chi$$

Proper distance

Scale factor

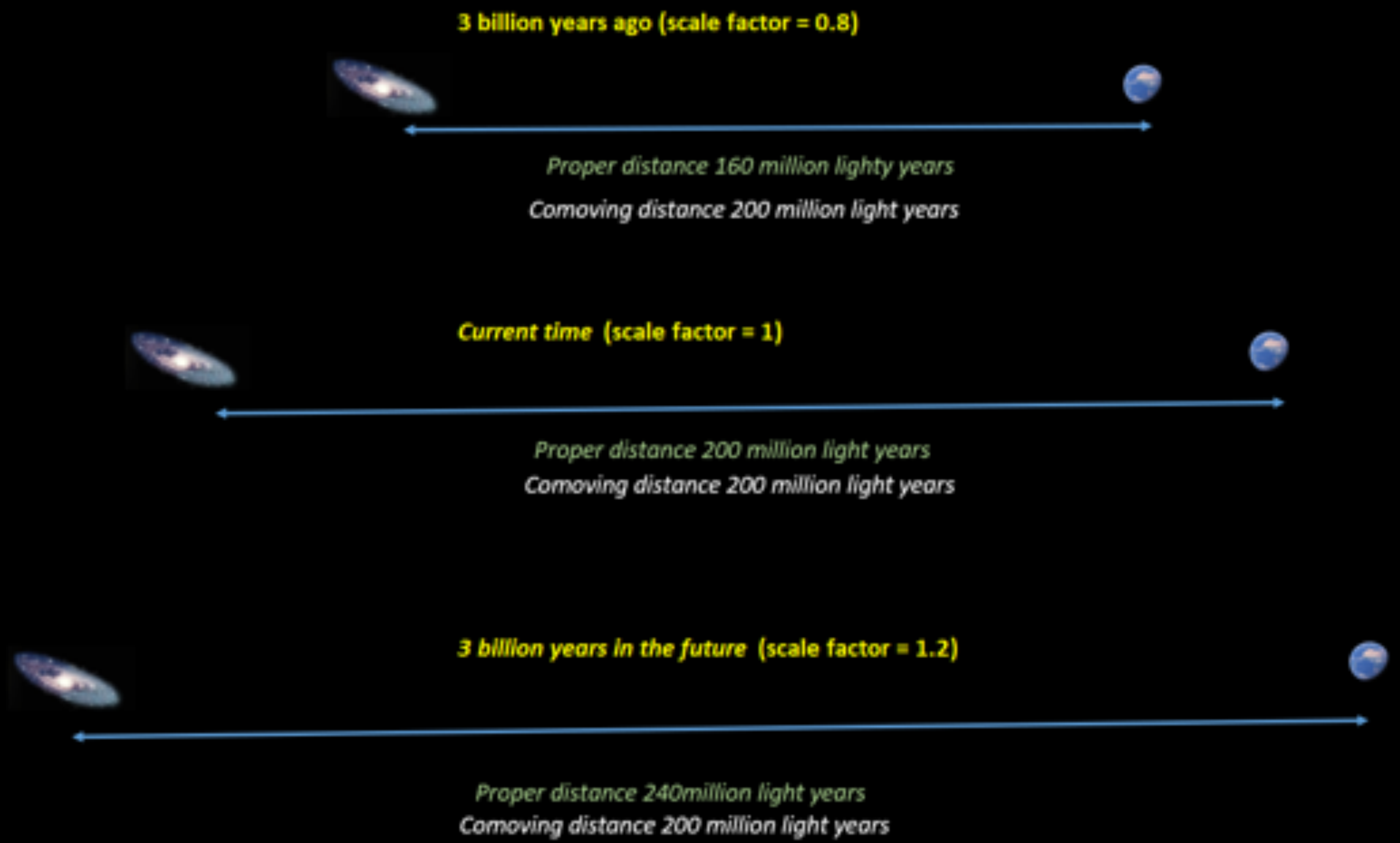
Comoving distance

Differentiating over time :

$$\frac{dr}{dt} = u = \underbrace{\dot{R}(t)\chi}_{\text{Velocity from the expansion of the Universe}} + R(t)\dot{\chi} = \dot{R}(t)\chi + v$$

Velocity in comoving coordinates

Peculiar velocity


3 billion years ago (scale factor = 0.8)
Proper distance 160 million lighty years
Comoving distance 200 million light years
Current time (scale factor = 1)
Proper distance 200 million light years
Comoving distance 200 million light years
3 billion years in the future (scale factor = 1.2)
Proper distance 240million light years
Comoving distance 200 million light years

# Jeans instability in an expanding Universe

In our approximation, we say that locally we can use non-relativistic equations for the fluid.

Therefore :

$$\left(\frac{\partial \rho}{\partial t}\right)_r + \nabla_r . (\rho u) = 0$$

**Equation of continuity**

**Euler's equation**

$$\left(\frac{\partial u}{\partial t}\right)_r + u \cdot \nabla_r u = -\frac{1}{\rho} \nabla_r P - \nabla \Phi$$

$$\nabla^2 \Phi = 4\pi G \rho$$

**Poisson's equation**

All these equations are with respect to the proper distance. We need to transform them to a co-moving frame.

$$r = R(t)\chi$$

Transforming the gradient to co-moving coordinates gives :

$$\nabla_r \rightarrow \frac{1}{R} \nabla_\chi$$

The time derivative becomes :

$$\left(\frac{\partial}{\partial t}\right)_r \rightarrow \left(\frac{\partial}{\partial t}\right)_\chi + \left(\frac{\partial \chi}{\partial t}\right)_r \cdot \nabla_\chi$$

***Tabulated !***

Hence :

$$\left(\frac{\partial}{\partial t}\right)_r = \left(\frac{\partial}{\partial t}\right)_\chi - \frac{\dot{R}}{R} \chi \cdot \nabla_\chi$$

# Jeans instability in an expanding Universe

$$u = \dot{R}(t)\chi + v$$

Let's start by rewriting the equation of continuity :

$$\left(\frac{\partial \rho}{\partial t}\right)_r + \nabla_r.(\rho u) = 0$$

Taking into account :

$$\nabla_r \rightarrow \frac{1}{R}\nabla_\chi$$

And

$$\left(\frac{\partial}{\partial t}\right)_r = \left(\frac{\partial}{\partial t}\right)_\chi - \frac{\dot{R}}{R}\chi \cdot \nabla_\chi$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{R}\nabla_\chi \cdot (\rho v) = -3\rho\frac{\dot{R}}{R}$$

Change of the density due to the cosmic expansion

Then the Euler's equation

$$\left(\frac{\partial u}{\partial t}\right)_r + u \cdot \nabla_r u = -\frac{1}{\rho}\nabla_r P - \nabla\Phi$$

which becomes :

$$\frac{\partial v}{\partial t} + \frac{1}{R}v \cdot \nabla_\chi v + \frac{\dot{R}}{R}v = -\frac{1}{R\rho}\nabla_\chi P - \frac{1}{R}\nabla_\chi\Phi - \ddot{R}\chi$$

The last term can be written as :

$$\ddot{R}\chi = \frac{1}{2}\ddot{R}\nabla_\chi\chi^2 = \frac{1}{R}\nabla_\chi\left(\frac{1}{2}R\,\ddot{R}\chi^2\right)$$

This helps to define a new potential :

$$\phi_g = \phi + \frac{1}{2}R\,\ddot{R}\chi^2$$

The Euler's equation can then be re-written as :

$$\frac{\partial v}{\partial t} + \frac{1}{R}v \cdot \nabla_\chi v + \frac{\dot{R}}{R}v = -\frac{1}{R\rho}\nabla_\chi P - \frac{1}{R}\nabla_\chi\Phi_g$$

# Jeans instability in an expanding Universe

$$\nabla_r \rightarrow \frac{1}{R} \nabla_\chi$$

We can now re-write the cosmological field equations and include the mean density of the Universe $\bar{\rho}$

$$\frac{\ddot{R}}{R} + \frac{4\pi G\bar{\rho}}{3}(1+\epsilon) - \frac{\Lambda}{3} = 0$$

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G\bar{\rho}}{3} - \frac{\Lambda}{3} = -\frac{k_b c^2}{R^2}$$

$\epsilon = 3P/\rho c^2$ and $\frac{P}{c^2} \ll \rho$

In our approximation of a non-relativistic fluid, and at enough early epoch ($\Lambda$ is negligible) we have :

$$\frac{\ddot{R}}{R} + \frac{4\pi G\bar{\rho}}{3} = 0$$

The Poisson equation

$$\nabla^2\Phi = 4\pi G\rho$$

Is now given by :

$$\frac{1}{R^2}\nabla_\chi^2\Phi_g = \frac{1}{R^2}\nabla_\chi^2\left(\Phi + \frac{1}{2}R\,\ddot{R}\chi^2\right)$$

But in spherical coordinates :

$$\nabla_\chi^2 \frac{1}{2}\chi^2 = \frac{1}{2\chi^2}\frac{\partial}{\partial\chi}\left(\frac{\chi^2\partial\chi^2}{\partial\chi}\right) = 3$$

Using the Poisson equation and the field equations , we get :

$$\frac{1}{R^2}\nabla_\chi^2\Phi_g = 4\pi G\rho - 4\pi G\bar{\rho} = 4\pi G\bar{\rho}\Delta$$

where we have for the density :

$$\rho = \bar{\rho}(1+\Delta)$$

Density contrast

$$\Delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

# Jeans instability in an expanding Universe

**SUMMARY OF THE FLUID EQUATIONS IN CO-MOVING COORDINATES**

The equation of continuity :

$$\frac{\partial \rho}{\partial t} + \frac{1}{R}\nabla_\chi \cdot (\rho v) = -3\rho\frac{\dot{R}}{R}$$

The Euler's equation :

$$\frac{\partial v}{\partial t} + \frac{1}{R} v \cdot \nabla_\chi v + \frac{\dot{R}}{R} v = -\frac{1}{R\rho}\nabla_\chi P - \frac{1}{R}\nabla_\chi \Phi_g$$

The Poisson equation :

$$\frac{1}{R^2}\nabla_\chi^2 \Phi_g = 4\pi G\bar{\rho}\Delta$$

# Jeans instability in an expanding Universe

We also need to determine the governing equations for the overdensity.

To simplify the analysis, we will consider that the pressure is only function of the density, therefore we can expand it as :

$$P \approx P(\bar{\rho}) + \frac{dP}{d\rho}\rho\Delta = P(\bar{\rho}) + c_s^2\bar{\rho}\Delta$$

From the cosmological principle we have :

$$\nabla P(\bar{\rho}) = 0$$

We can also assume that the sound speed is constant.

Then, to first order assuming small perturbations, we have :

$$\frac{\partial\Delta}{\partial t} + \frac{1}{R}\nabla_\chi \cdot v = 0$$

$$\frac{\partial v}{\partial t} + \frac{\dot{R}}{R}v = -\frac{c_s^2}{R}\nabla_\chi\Delta - \frac{1}{R}\nabla_\chi\Phi_g$$

$$\frac{1}{R^2}\nabla_\chi^2\Phi_g = 4\pi G\,\bar{\rho}\Delta$$

We now have a set of equations showing the evolution of the overdensity !

# Jeans instability in an expanding Universe

We now need to determine the time dependence of these equations (to see how the overdensities evolve with time).

As for the local case, we can determine how the perturbation evolve with time, making two assumptions :

- Small quantities
- Only first order

We have the following equations :

$$\frac{\partial \Delta}{\partial t} + \frac{1}{R}\nabla_\chi \cdot v = 0$$

$$\frac{\partial v}{\partial t} + \frac{\dot{R}}{R} v = -\frac{c_s^2}{R}\nabla_\chi \Delta - \frac{1}{R}\nabla_\chi \Phi_g$$

$$\frac{1}{R^2}\nabla_\chi^2 \Phi_g = 4\pi G \bar{\rho} \Delta$$

As in the local case, we can search for solutions of the form :

$$\Delta = \Delta(t)\exp(ik_c \cdot \chi)$$

and we can demonstrate that the following equation gives the time dependant overdensity for wave number $k = k_c/R$ :

$$\boxed{\frac{d^2\Delta}{dt} + 2\left(\frac{\dot{R}}{R}\right)\frac{d\Delta}{dt} = (4\pi G\rho - k^2 c_s^2)\Delta}$$

# Jeans instability in an expanding Universe

**THE GROWTH OF INSTABILITIES**

We can now contrast the growth of instabilities in a non-expanding and expanding universe.

We obtain the <u>static case</u> by making the following assumptions :

- $\dot{R} = 0$ (no expansion of the Universe)
- Solutions are of the form : $\Delta = \Delta_0 \exp i(k_c\chi - \omega t)$

Then :

$$\frac{d^2\Delta}{dt^2} = (4\pi G\rho - k^2 c_s^2)\Delta$$

Static universe

We get the same dispersion relation as before :

$$\omega^2 = c_s^2 k^2 - 4\pi G\rho$$

then , as previously, this gives exponentially growing modes for $c_s^2 k^2 < 4\pi G\rho$, or :

$$\lambda > \lambda_J = c_s \left(\frac{\pi}{G\rho}\right)^{1/2}$$

When $\lambda \gg \lambda_J$, the modes grow like $\exp(t/\tau)$ where $\tau \sim (4\pi G\ \rho)^{-1/2}$

# Jeans instability in an expanding Universe

**THE GROWTH OF INSTABILITIES**

We consider now the simplest model of an expanding Universe : the flat Universe described by the Einstein-de-Sitter universe.

**PROPERTIES OF THE EINSTEIN-DE-SITTER UNIVERSE**

$$\Omega_M = 1$$

$$\frac{R}{R_0} = \left(\frac{t}{t_0}\right)^{2/3} = \left(\frac{3}{2} H_0 t\right)^{2/3}$$

$$H^2 = \frac{8\pi G\rho}{3}$$

$$4\pi G\bar{\rho} = \frac{2}{3t^2}$$

To simplify, we also consider the case where the gravitational attraction is much stronger than the pressure force, i.e :

$$4\pi G\bar{\rho} \gg c_s^2 k^2$$

Then we can rewrite the equation for the time dependent overdensity :

$$\frac{d^2\Delta}{dt} + 2\left(\frac{\dot{R}}{R}\right)\frac{d\Delta}{dt} = (4\pi G\rho - k^2 c_s^2)\Delta$$

such as :

$$\frac{d^2\Delta}{dt} + \frac{4}{3t}\frac{d\Delta}{dt} - \frac{2}{3t^2}\Delta = 0$$

# Jeans instability in an expanding Universe

$$\frac{d^2\Delta}{dt} + \frac{4}{3t}\frac{d\Delta}{dt} - \frac{2}{3t^2}\Delta = 0$$

$$\frac{R}{R_0} = \left(\frac{t}{t_0}\right)^{2/3}$$

It is easy to verify that for solutions of the form $\Delta \propto t^n$, the growing modes have :

$$\Delta \propto t^{\frac{2}{3}} \propto R \propto (1+z)^{-1}$$

**CONCLUSIONS**

- Perturbations only grow algebraically (and not exponentially) with time
- This basic result is similar for other cosmologies as well.

We can modify this analysis to account for the early radiation dominated phase of the Universe as follow :

- $4\pi G$ must be replaced by $32\pi G/3$
- In the radiation dominated phase $R \propto t^{1/2}$

Then :

$$\Delta \propto t \propto R^2 \propto (1+z)^{-2}$$

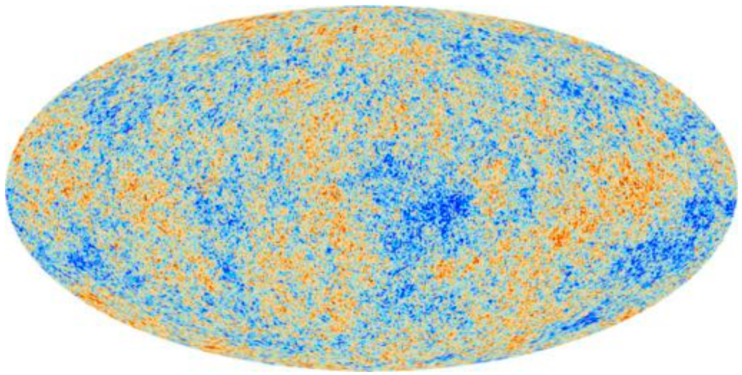
# The need for dark matter

In the early, radiation dominated, Universe when matter and radiation are strongly coupled, we have :

$$\rho \propto \frac{1}{R^3} \propto T^3$$

Then :

$$\Delta = \frac{\delta\rho}{\rho} \approx 3\frac{\delta T}{T}$$



According to Planck

$$\frac{\delta T}{T} \sim 10^{-5}$$

$$\Delta(z = 1500) = 3\times10^{-5}$$

After recombination, the matter perturbations grow via gravity approximately according to the results of the previous section. Since :

$$\Delta \propto t^{\frac{2}{3}} \propto R \propto (1+z)^{-1}$$

Then

$$\Delta(z = 0) \sim \Delta(z = 1500)\times(1 + 1500) \sim 0.05$$

$$\Delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

This is clearly not the case !
($\Delta > 1$)

# Summary of the last lecture

Main goal : study how overdensities evolve in a cosmological context (i.e. accounting for the expansion of the Universe)


$$r = R(t)\chi$$

Equation of continuity

$$\left(\frac{\partial \rho}{\partial t}\right)_r + \nabla_r . (\rho u) = 0$$

Euler's equation

$$\left(\frac{\partial u}{\partial t}\right)_r + u \cdot \nabla_r u = -\frac{1}{\rho}\nabla_r P - \nabla\Phi$$

Poisson's equation

$$\nabla^2\Phi = 4\pi G\rho$$

$$\nabla_r \rightarrow \frac{1}{R}\nabla_\chi$$

$$\left(\frac{\partial}{\partial t}\right)_r = \left(\frac{\partial}{\partial t}\right)_\chi - \frac{\dot{R}}{R}\chi \cdot \nabla_\chi$$

# Summary of the last lecture

$$\frac{\partial \rho}{\partial t} + \frac{1}{R}\nabla_\chi \cdot (\rho v) = -3\rho\frac{\dot{R}}{R}$$

**Equation of continuity**

**Euler's equation**

$$\frac{\partial v}{\partial t} + \frac{1}{R} v \cdot \nabla_\chi v + \frac{\dot{R}}{R} v = -\frac{1}{R\rho}\nabla_\chi P - \frac{1}{R}\nabla_\chi \Phi_g$$

$$\frac{1}{R^2}\nabla_\chi^2 \Phi_g = 4\pi G\bar{\rho}\Delta$$

**Poisson's equation**

$$\frac{\ddot{R}}{R} + \frac{4\pi G\bar{\rho}}{3} = 0$$

$$\Omega_\Lambda = \frac{\Lambda}{3H^2}$$

**!!! Typos!!!**

**APPROXIMATIONS**

- Instabilities can be treated by a perturbation analysis
- we can use locally non-relativistic equations for the fluid
- we can treat gravitational perturbations as sufficiently small that a Newtonian approximation is justified.

$$\rho = \bar{\rho}(1 + \Delta)$$

Main goal

# Summary of the last lecture

$$\frac{\partial \Delta}{\partial t} + \frac{1}{R}\nabla_\chi \cdot v = 0$$

**Equation of continuity**

**Euler's equation**

$$\frac{\partial v}{\partial t} + \frac{\dot{R}}{R} v = -\frac{c_s^2}{R}\nabla_\chi \Delta - \frac{1}{R}\nabla_\chi \Phi_g$$

$$\frac{1}{R^2}\nabla_\chi^2 \Phi_g = 4\pi G\, \bar{\rho}\Delta$$

**Poisson's equation**

Solution of the form :

$$\Delta = \Delta(t)\exp(ik_c \cdot \chi)$$

$$k = k_c/R$$

$$\frac{d^2\Delta}{dt} + 2\left(\frac{\dot{R}}{R}\right)\frac{d\Delta}{dt} = (4\pi G\rho - k^2 c_s^2)\Delta$$

**Static Universe :**

$$\frac{d^2\Delta}{dt^2} = (4\pi G\rho - k^2 c_s^2)\Delta$$

$$\exp\left(\frac{t}{(4\pi G\,\rho)^{-\frac{1}{2}}}\right)$$

Modes grow exponentially

**Expanding Universe**

***flat Einstein – de Sitter Universe ; no DM***

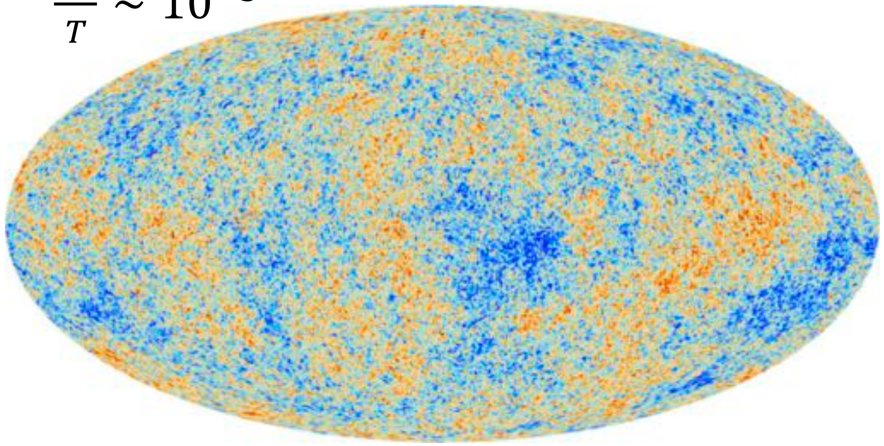
$$\frac{d^2\Delta}{dt} + \frac{4}{3t}\frac{d\Delta}{dt} - \frac{2}{3t^2}\Delta = 0$$

$$\Delta \propto t^{\frac{2}{3}} \propto R \propto (1+z)^{-1}$$

Modes grow algebraically

# Summary of the last lecture

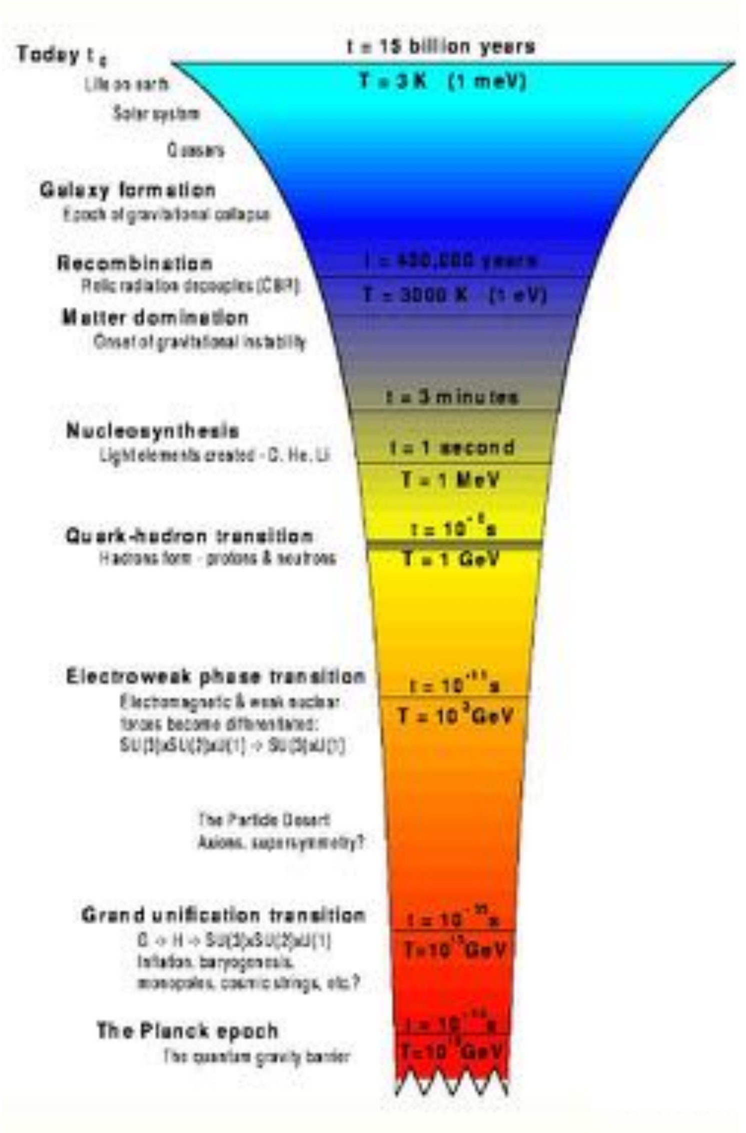
$$\frac{\delta T}{T} \sim 10^{-5}$$



$$\Delta \propto t^{\frac{2}{3}} \propto R \propto (1+z)^{-1}$$

Matter dominated

Radiation dominated era




$$\Delta(z_{rec} = 1500) = 3\times 10^{-5}$$

$$\Delta \propto t \propto R^2 \propto (1+z)^{-2}$$

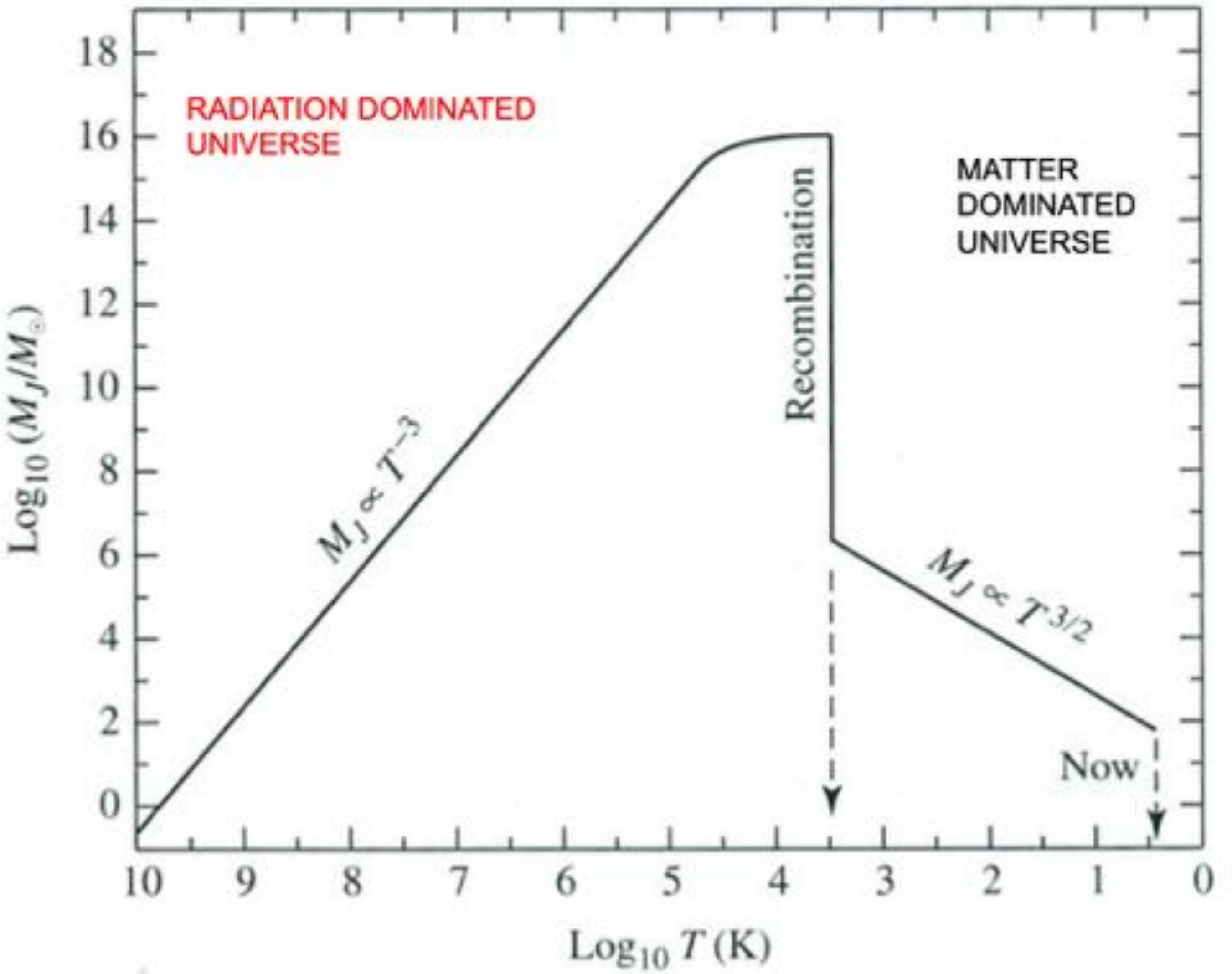
$$\rho \propto \frac{1}{R^3} \propto T^3$$

$$\Delta(z = 0) \sim \Delta(z = 1500)\times(1 + 1500) \sim 0.05$$

Different from what is observed today !

$DARK\ MATTER$

# Summary of the last lecture



# Perturbations with dark matter

$$\frac{d^2\Delta}{dt} + \frac{4}{3t}\frac{d\Delta}{dt} - \frac{2}{3t^2}\Delta = 0$$

$$\frac{d^2\Delta}{dt} + 2\left(\frac{\dot{R}}{R}\right)\frac{d\Delta}{dt} = (4\pi G\rho - k^2c_s^2)\Delta$$

If we now consider the presence of dark matter and baryonic matter, then we need to solve the following coupled equations :

$$\frac{d^2\Delta_B}{dt^2} + 2\left(\frac{\dot{R}}{R}\right)\frac{d\Delta_B}{dt} = A\overline{\rho_B}\Delta_\mathrm{B} + A\overline{\rho_D}\Delta_D$$

$$\frac{d^2\Delta_D}{dt^2} + 2\left(\frac{\dot{R}}{R}\right)\frac{d\Delta_D}{dt} = A\overline{\rho_B}\Delta_\mathrm{B} + A\overline{\rho_D}\Delta_D$$

where $A$ is a constant

Valid for both matter dominated universe and for the radiation dominated case depending on the choice of the constant A.

First case : epoch immediately after recombination

- $A = 4\pi G$
- $\Omega_D \approx 1$ and $\Omega_B \ll 1$ ; then $\overline{\rho_B}\Delta_B \ll \overline{\rho_D}\Delta_D$ the second equation is therefore :
$$\frac{d^2\Delta_D}{dt} + 2\left(\frac{\dot{R}}{R}\right)\frac{d\Delta_\mathrm{D}}{dt} = (4\pi G\rho_D)\Delta_\mathrm{D}$$
we have just discussed
- We write the solution as $\Delta_D = B\times R$ where $B$ is a constant.

We can demonstrate that the equation for $\Delta_B$ is then :

$$R^{\frac{3}{2}}\frac{d}{dR}\left(R^{-\frac{1}{2}}\frac{d\Delta_B}{dR}\right) + 2\frac{d\Delta_B}{dR} = \frac{3}{2}B$$

# Perturbations with dark matter

$$R^{\frac{3}{2}}\frac{d}{dR}\left(R^{-\frac{1}{2}}\frac{d\Delta_B}{dR}\right)+2\frac{d\Delta_B}{dR}=\frac{3}{2}B$$

which has the solution $\Delta_B = B(R - R_0)$

$R \propto (1+z)^{-1}$

$\Delta_D = B \times R$

then $B = \Delta_D / R$

$$\Delta_B \sim \Delta_D\left(1-\frac{z}{z_0}\right)$$

This implies that the amplitude of the baryonic perturbation quickly grows to that of the dark matter, no matter how small the baryonic perturbation is at $z = z_0$, e.g. at recombination.

# Evolution of perturbations

Now we can <u>summarize the process of structure formation</u> at the **cosmological scale**

In the very early Universe, any initial perturbations on scales larger than the Jeans length grow as $R^2$ since radiation dominated.

$$\Delta \propto t \propto R^2 \propto (1+z)^{-2}$$

At this stage, the amplitudes of perturbations in baryonic, dark matter, and radiation are equal since they are coupled

In the $\Lambda CDM$ cosmologies, we assume that dark matter is made of heavy particles

The cold dark matter decouples from the radiation at an early epoch

At a redshift $z_{eq} \approx 4{\times}10^4 \Omega h^2$ the Universe switches from radiation to matter dominated

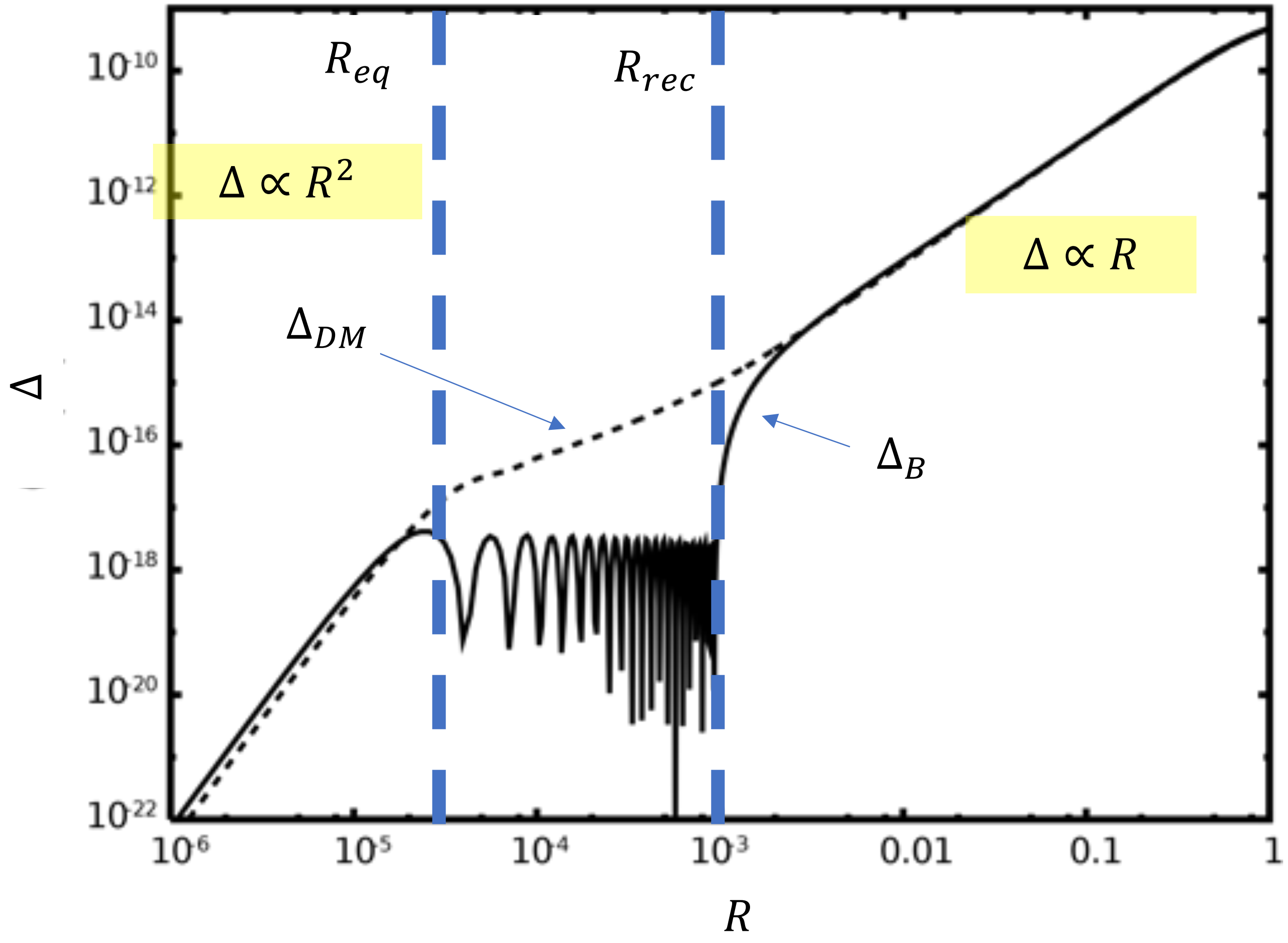
The perturbations now grow as R, however perturbations in the radiation and baryonic gas are damped

$$\Delta \propto t^{2/3} \propto R \propto (1+z)^{-1}$$

After the epoch of recombination, the baryonic matter decouples from the radiation

Quickly perturbations in the baryonic gas start to follow those in the CDM as we just have shown

# Evolution of perturbations



# Evolution of perturbations

To get a first estimate of the typical masses of the first structure, we can determine the Jeans mass just after recombination.

$$c_s^2 k_j^2 = 4\pi G\rho$$

$$\lambda_J = \frac{2\pi}{k_j}$$

$$\lambda_J = c_s \left(\frac{\pi}{G\rho}\right)^{1/2}$$

The sound speed has been defined at the beginning of this course as :

$$c_s^2 = \frac{5}{3}\frac{k_B T}{\mu} = \frac{5}{3}\frac{k_B T}{m_H}$$

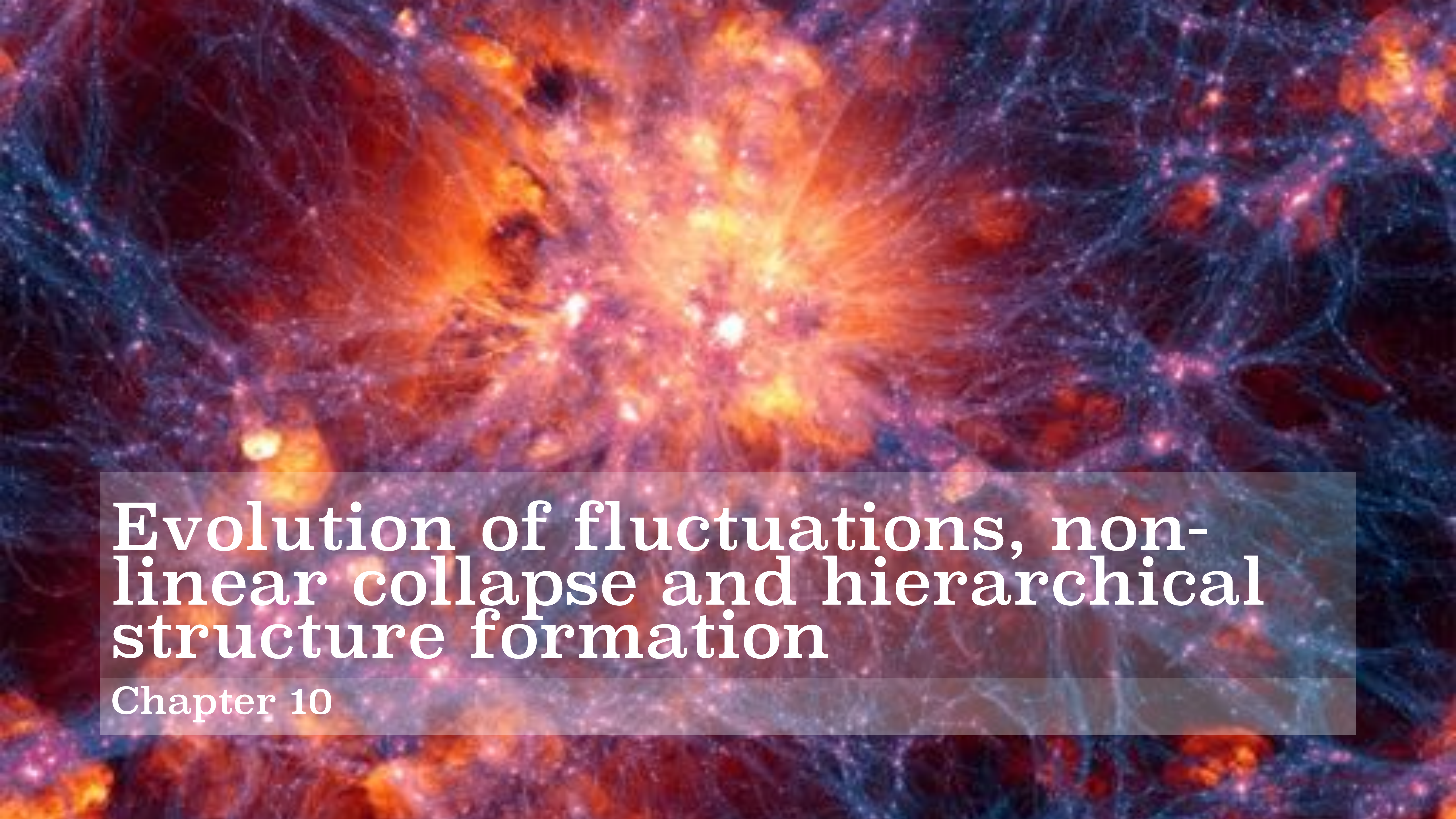
The Jeans mass is given by :

$$M_J = \frac{4\pi}{3}\left(\frac{\lambda_J}{2}\right)^3 \rho_m$$

with $\rho_m = \left(1 + z_{eq}\right)^3 \rho_c \Omega_m$

Redshift of matter-radiation equality

Numerical application gives that the minimum mass of the first structures in the Universe is $M_J \sim 3\times 10^5 M_{\odot}$



# Evolution of fluctuations, non-linear collapse and hierarchical structure formation

Chapter 10

# The Power Spectrum of fluctuations

**THE TWO-POINT CORRELATION FUCNTION**

The power spectrum determines the mass-spectrum of the initial perturbations and the initial spatial distribution of structure in the Universe.

To determine the power spectrum, we first need to define the two-point correlation function : excess probability of finding a galaxy (or density enhancement) at a distance $r$ from a galaxy (or density enhancement) randomly selected in a uniform, random distribution.

The number of galaxies in volume $dV$ at $r$ from any galaxy :

$$dN(r) = N_0[1 + \xi(r)]dV$$

Number of galaxies in dV

Average number density of galaxies

Two-point correlation function

Which could be rewritten as the probability of finding pairs :

$$dN_{pair} = N_0^2[1 + \xi(r)]dV_1dV_2$$

# The Power Spectrum of fluctuations

**THE TWO-POINT CORRELATION FUCNTION**

The two-point correlation function can be directly related to the density contrast, and we can write the density as :

$$\rho = \rho_0[1 + \Delta(x)]$$

Density contrast

$$\Delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

And then the pairwise numbers of galaxies separated by $r$ is :

$$dN_{pair}(r) = \rho(x)dV_1\rho(x + r)dV_2$$

Hence

$$dN_{pair}(r) = \rho_0^2[1 + \Delta(x)][1 + \Delta(x + r)]dV_1dV_2$$

By averaging over a large volume ($< \Delta > = 0$ by definition => only the cross term remains)

$$dN_{pair}(r) = \rho_0^2[1 + < \Delta(x)\Delta(x + r) >]dV_1dV_2$$

Inserting :

$$dN_{pair} = N_0^2[1 + \xi(r)]dV_1dV_2$$

Within the previous equation, gives :

$$\boxed{\xi(r) = < \Delta(x)\Delta(x + r) >}$$

$$dN_{pair} = N_0^2[1 + \xi(r)]dV_1dV_2$$

# The Power Spectrum of fluctuations

**THE POWER-SPECTRUM OF FLUCTUATIONS**

The Fourier transform for $\Delta(r)$ is defined as :

$$\Delta(r) = \frac{V}{2\pi^3}\int \Delta_k \exp(-i\vec{k}\cdot\vec{r})d^3k$$

with

$$\Delta_k = \frac{1}{V}\int \Delta(r)\exp(-i\vec{k}\cdot\vec{r})d^3x$$

Comoving wavevector

Remember that the Parseval's theorem gives :

$$\frac{1}{V}\int \Delta^2(r)d^3x = \frac{V}{(2\pi)^3}\int |\Delta_k|^2 d^3k$$

$< \Delta^2 >$

Density contrast
$\Delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$

Also noted $P(k)$ :
Power spectrum of the fluctuations

Then :

$$< \Delta^2 > = \frac{V}{(2\pi)^3}\int |\Delta_k|^2 d^3k = \frac{V}{(2\pi)^3}\int P(k)d^3k$$

The two point correlation function is spherically symmetric $(d^3k = 4\pi k^2 dk)$:

$$< \Delta^2 > = \frac{V}{2\pi^2}\int |\Delta_k|^2 k^2 dk = \frac{V}{2\pi^2}\int P(k)k^2 dk$$

# The Power Spectrum of fluctuations

$$< \Delta^2 > = \frac{V}{2\pi^2} \int |\Delta_k|^2 k^2 dk$$

We can also write $\Delta(x)$ as a Fourier series such as :

$$\Delta(x) = \Sigma_k \Delta_k \exp(-i\vec{k} \cdot \vec{x})$$

Remember that, the two point correlation function is given by :

$$\xi(r) =< \Delta(x)\Delta(x+r) >$$

Hence :

$$\xi(r) =< \Sigma_k \Sigma_{k'} \Delta_k \Delta_{k'} \exp\left(-i(\vec{k} - \vec{k}') \cdot \vec{x}\right) \exp(i\vec{k}' \cdot \vec{r}) >$$

Given the orthogonality of the Fourier basis, the cross terms vanish except those with k=k', therefore :

$$\xi(r) = \Sigma |\Delta_k|^2 \exp(i\vec{k} \cdot \vec{r})$$

and (in terms of Fourier integral):

$$\xi(r) = \frac{V}{(2\pi)^3} \int |\Delta_k|^2 \exp\left(\mathrm{i}\vec{k} \cdot \vec{r}\right) d^3k$$

The two-point correlation function is real, therefore we are only interested in the integral of the real part of the exponential, i.e $\cos(k \cdot r) = \cos(kr \cos\theta)$

Moreover, because of the spherical symmetry of the two-point correlation function, we integrate over the angular part of the volume element $\frac{1}{2}\sin\theta d\theta$ :

Spherical coordinates

$$\xi(r) = \frac{V}{2\pi^2} \int |\Delta_k|^2 \frac{\sin kr}{kr} k^2 dk$$

$$= \frac{V}{2\pi^2} \int P(k) \frac{\sin kr}{kr} k^2 dk$$

Only allows wavenumbers $k \leq \frac{1}{r}$ to contribute to the amplitude of fluctuations

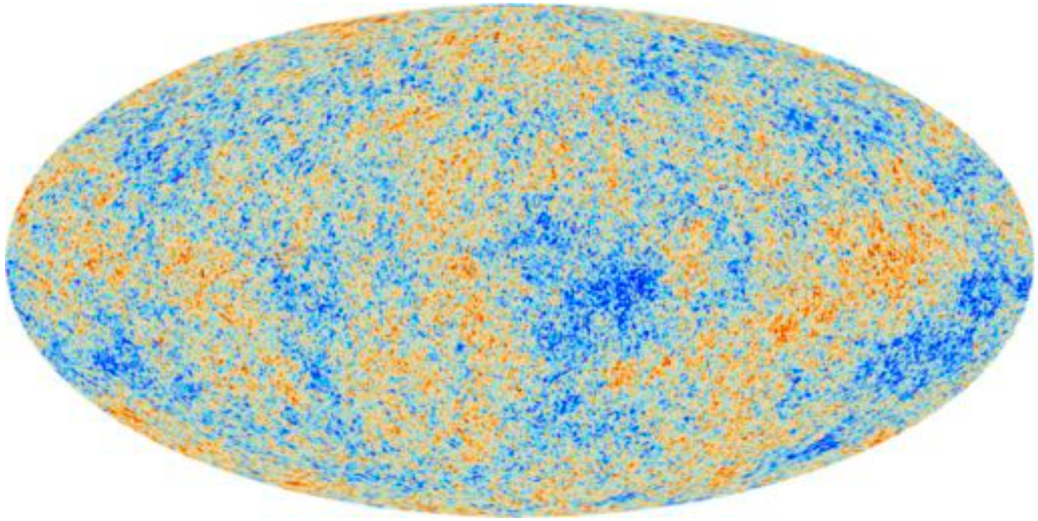
The inverse Fourier transform gives the Power Spectrum :

$$\boxed{P(k) = \frac{1}{V} \int_0^\infty \xi(r) \frac{\sin kr}{kr} 4\pi r^2 dr}$$

# The Power Spectrum of fluctuations

$$\xi(r) = \frac{V}{2\pi^2}\int |\Delta_k|^2 \frac{\sin kr}{kr} k^2 dk$$

**THE INITIAL POWER-SPECTRUM**



From CMB observations, we know that the form of the initial perturbation is a power-law with no preferred scale as :

$$P(k) = |\Delta_k|^2 \propto k^n$$

Then, the two-point correlation function has the following form :

$$\xi(r) \propto \int \frac{\sin kr}{kr} k^{n+2} dk$$

$$\frac{\sin kr}{kr} \approx 1$$

for $kr \ll 1$

We can therefore integrate k from 0 to $k_{max}$ =1/r to estimate the dependence of the amplitude of the correlation function :

$$\xi(r) \propto r^{-(n+3)}$$

But the mass in the perturbation is $M \approx \rho r^3$, then :

$$\xi(M) \propto M^{-(n+3)/3}$$

And the density contrast for a mass scale M :

$$\Delta(M) = < \Delta^2 >^{1/2} \propto M^{-(n+3)/6}$$

# The Power Spectrum of fluctuations

## THE HARISSON-ZEL'DOVICH POWER SPECTRUM

The interest of this particular spectrum is that the density contrast $\Delta(M)$ has the same amplitude on all scales when perturbations come through their particle horizon.

This is the special case where the index of the power law n=1 :

$$P(k) = |\Delta_k|^2 \propto k^{n=1}$$

Hence :

$$\Delta(M) \propto M^{-2/3}$$

And

$$\xi \propto r^{-4} \propto M^{-4/3}$$

## EVOLUTION OF THE POWER SPECTRUM

The Power Spectrum is modified from its initial form as the Universe evolves. It interacts with 3 main components :

- Baryonic matter
- Non-baryonic matter
- Photons

We define the *Transfer Function* as the function describing how the shape of the initial Power Spectrum in the dark matter is modified by different processes :

$$\Delta_k(z=0) = T(k) f(z) \Delta_k(z)$$

Power spectrum at the present epoch

Transfer function

The linear growth factor between the scale factor at z and z=0

Initial Power Spectrum

# The Power Spectrum of fluctuations

Consider an initial power-law power spectrum :

$$P(k) = |\Delta_k|^2 \propto k^n$$

A critical point in the evolution of the perturbations is <u>when their size is equal to the horizon size</u> (i.e. the Universe size).
For a perturbation of size r, this happens when $r \approx ct$- we say that the perturbation has entered the horizon

Before the perturbation entered the horizon (during the radiation dominated era), their density contrasts grew as $\Delta_k \propto R^2$ on all scales (*Monday's lecture*)

T<47 kyr after BB

If the perturbations came through the horizon during the *radiation dominated phase*, the dark matter perturbations were gravitationally coupled to the radiation dominated plasma, and their amplitudes were stabilised.

Therefore as soon as the perturbations came through the horizon the perturbations ceased to grow until the epoch of equality.

After that time all perturbations grew as $\Delta_k \propto R$

Between crossing their particle horizons at scale factor $R_H$ and the epoch of equality $R_{eq}$, the amplitudes of the perturbations were damped by a factor $\left(\frac{R_H}{R_{eq}}\right)^2$ relative to the unmodified spectrum :

$$\Delta_k \propto k^{\frac{n}{2}} \left(\frac{R_H}{R_{eq}}\right)^2$$

Transfer function

# The Power Spectrum of fluctuations

Since $k \propto R_H^{-1}$, it follows that the transfer function $T(z)$ has the asymptotic forms :

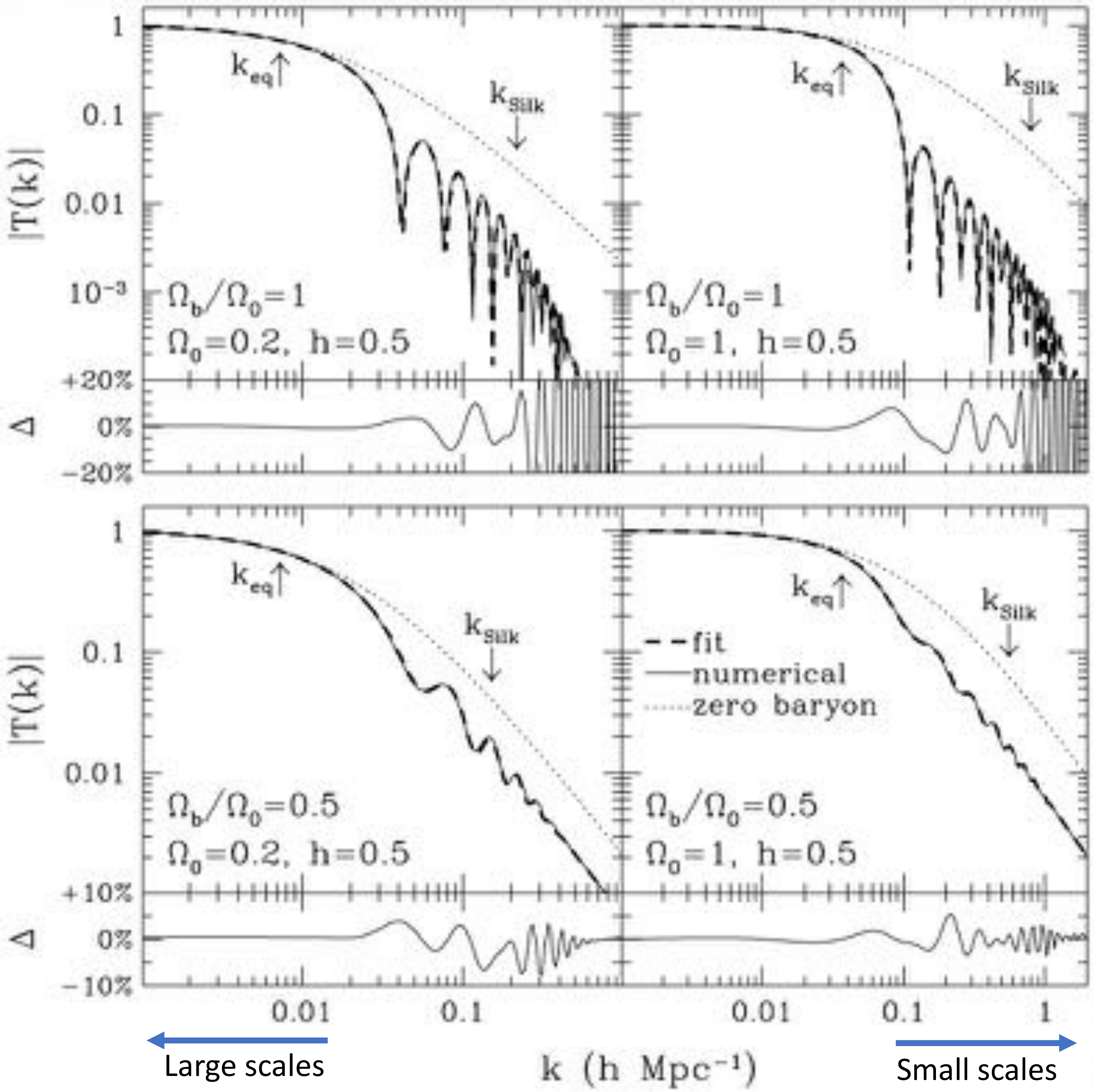
- $T_k = 1$ for $M \geq M_{eq}, k \leq k_{eq}$
- $T_k \propto k^{-2}$ for $M \leq M_{eq}, k \geq k_{eq}$

Thus, for small masses, the 'processed' power spectrum $P(k) \propto T_k^2$ is flatter than the input spectrum of perturbations by a power $k^{-4}$ :

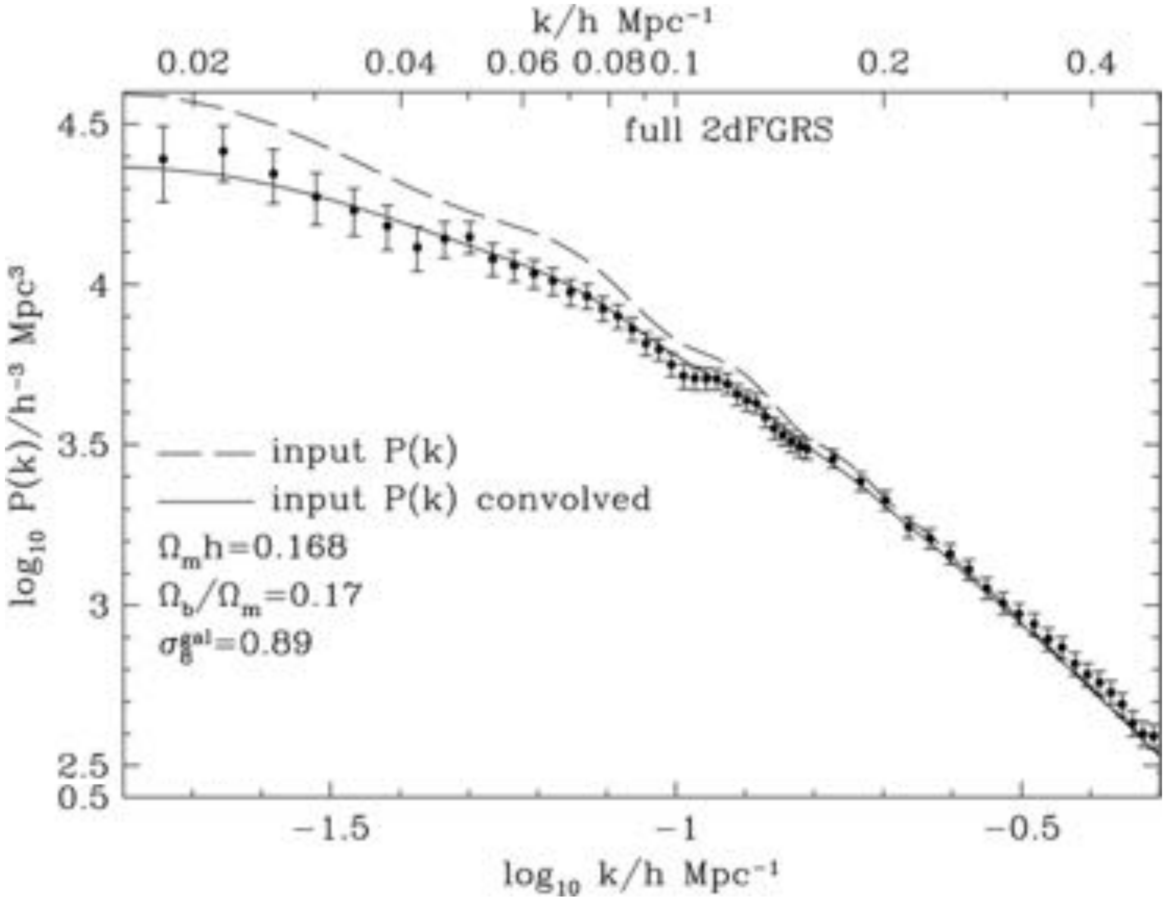
$$P(k) = |\Delta_k|^2 \propto k^{n-4}$$

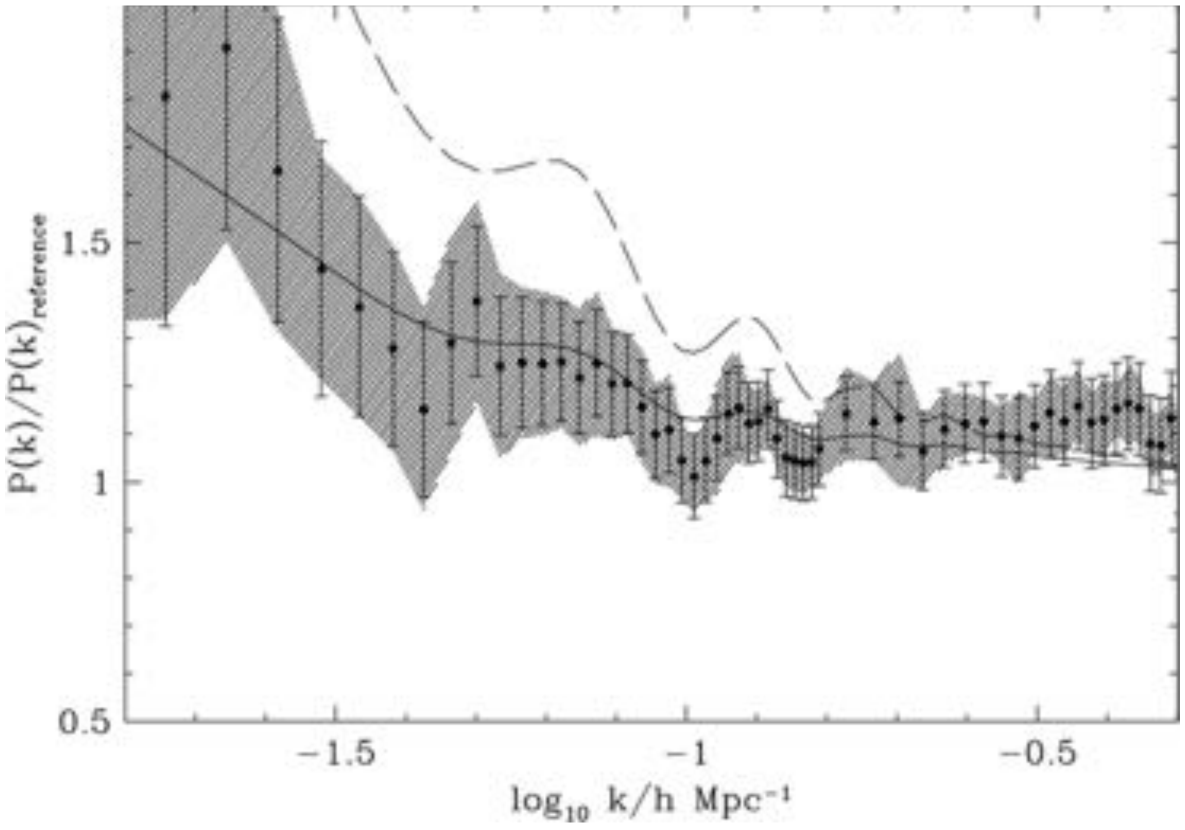
And

$$\xi(r) \propto r^{-(n-1)} \text{ or } \xi(M) \propto M^{-(n-1)/3}$$



# The Power Spectrum of fluctuations



# Summary of the last lecture

We defined the two-point correlation function as the excess probability of finding a galaxy at a distance $r$ from a galaxy randomly selected in a uniform, random distribution.

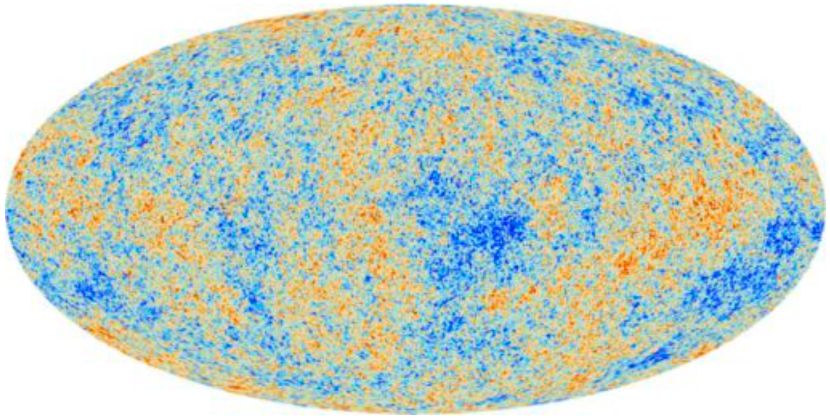
$$dN_{pair} = N_0^2[1+\xi(r)]dV_1 dV_2$$

$$\rho = \rho_0[1+\Delta(x)]$$

$$\xi(r) =< \Delta(x)\Delta(x+r) >$$

The power spectrum determines the mass-spectrum of the initial perturbations and the initial spatial distribution of structure in the Universe.

$$P(k) = \frac{1}{V}\int_0^\infty \xi(r)\,\frac{\sin kr}{kr}\,4\pi r^2 dr$$



From the CMB we know that : $P(k) = k^n$

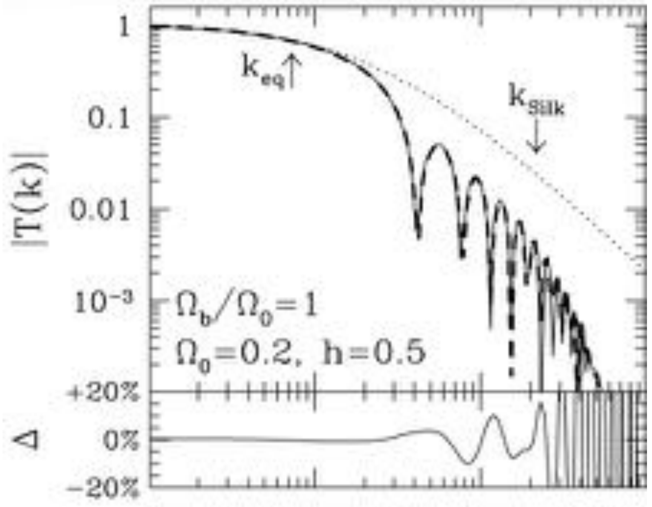
# Summary of the last lecture

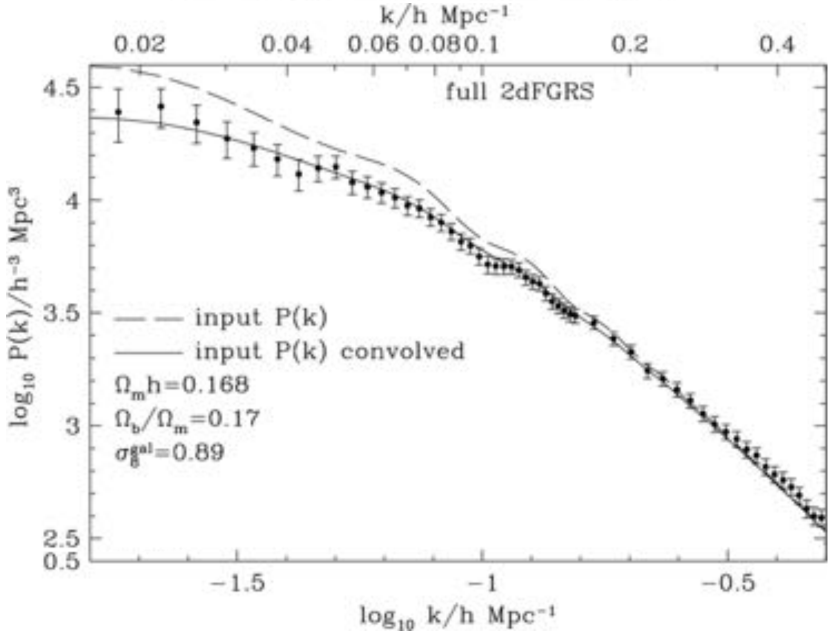
To study the evolution of the initial Power Spectrum, we defined the *Transfer Function*

$$\Delta_k(z=0) = T(k) f(z) \Delta_k(z)$$

$$\Delta_k \propto k^{\frac{n}{2}} \left(\frac{R_H}{R_{eq}}\right)^2$$

- $T_k = 1$ for $M \geq M_{eq}, k \leq k_{eq}$
- $T_k \propto k^{-2}$ for $M \leq M_{eq}, k \geq k_{eq}$



# Non-linear collapse of a spherical overdensity

Main idea : to model the evolution of perturbations into the non-linear regime, we consider a single spherical overdensity

*As in the classical Gauss's theorem, in general relativity for a perfectly spherical geometry, the mass inside a sphere is not influenced by the material outside the sphere*

Assumptions :

- One spherical overdensed region of radius $R_s$ and density $\rho_s$
- A spatially flat Universe

  => Einstein-de-Sitter Universe with mean density $\bar{\rho}$ with a scale factor $R$
- Take recombination as a reference epoch
  - With a mean density in the universe : $\rho_0 = \bar{\rho}(t_0)$
  - Scale factor $R_0$
  - Mass $M$, Radius $R_{S_0}$ and density :

$$\rho_{S_0} = \frac{3M}{4\pi \mathrm{R}_{S_0}^3}$$

# Non-linear collapse of a spherical overdensity

$r = R(t)\chi$

<u>Assumptions [..]</u>:

- To simplify the algebra, we choose a co-moving coordinates such that the ***outer radius of our spherical overdensity*** has $\chi = 1$

=> hence the scale factor is just the radius the overdensity would have if it expended with the Hubble flow in our flat Universe

=> At $t = t_0$, therefore $R_{S_0} = R_0$

The evolution of both Universe as a whole and the overdensity are described by the field equations :

$$\frac{\ddot{R}}{R} + \frac{4\pi G\rho}{3}(1+\epsilon) - \frac{\Lambda}{3} = 0$$

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G\rho}{3} - \frac{\Lambda}{3} = -\frac{kc^2}{R^2}$$

# Non-linear collapse of a spherical overdensity

**<u>Background Universe</u>** : *Einstein-de-Sitter*

- $k = 0, \Lambda = 0, \epsilon = 0$
- $\Omega_m = 1$
- $\frac{R}{R_0} = \left(\frac{t}{t_0}\right)^{2/3} = \left(\frac{3}{2}H_0 t\right)^{2/3}$
- $H^2 = \frac{8\pi G\bar{\rho}}{3}$
- $4\pi G\bar{\rho} = \frac{2}{3t^2}$

The evolution of both Universe as a whole and the overdensity are described by the field equations :

$$\frac{\ddot{R}}{R} + \frac{4\pi G\rho}{3}(1+\epsilon) - \frac{\Lambda}{3} = 0$$

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G\rho}{3} - \frac{\Lambda}{3} = -\frac{kc^2}{R^2}$$

<u>Evolution of the overdensity</u> :

*The overdensity means that for the region of the Universe within the radius $R_s$ the Universe must be closed (because we have the overdensity) with $k = 1$*

*The equation for the radius of the overdensity is :*

$$\left(\frac{\dot{R}_s}{R_s}\right)^2 - \frac{8\pi G\rho_{S_0} R_{S_0}^3}{3\, R_S^3} = -\frac{kc^2}{R_S^2}$$

# Non-linear collapse of a spherical overdensity

<u>Assumptions</u> :
- One spherical overdense region of radius $R_s$ and density $\rho_s$
- An Einstein-de-Sitter Universe with mean density $\bar{\rho}$ with a scale factor $R(t)$
- Take recombination as a reference epoch
  - mean density in the universe : $\rho_0 = \bar{\rho}(t_0)$
  - Scale factor $R_0$
  - Mass $M$, Radius $R_{S_0}$ and density : $\rho_{S_0} = \frac{3M}{4\pi R_{S_0}^3}$

At $t = t_0$ :
- The radius of the overdensity is $R_{S_0}$
- The mean cosmological density is $\rho_0$

Introducing variables changes :
- We define the conformal time as : $\eta = \int_0^t c \frac{dt'}{R(t')}$
- Dimensionless radius : $a = R_s / R_{S_0}$
- A constant : $a_m = \frac{8\pi G \rho_{S_0} R_{S_0}^2}{3c^2}$
- We also define the two cosmological densities :

$$\Omega_S = \frac{8\pi G \rho_S}{3H_S^2} \text{ and } \Omega_{S_0} = \frac{8\pi G \rho_{S_0}}{3H_{S_0}^2}$$

We assume that the perturbation is still approximately following the Hubble flow at $t = t_0$, then :

$$H_{S_0} \approx H_0 \text{ and } \rho_{S_0} \approx \Omega_{S_0} \rho_0$$

$$\left(\frac{\dot{R}_s}{R_s}\right)^2 - \frac{8\pi G \rho_{S_0} R_{S_0}^3}{3\, R_S^3} = -\frac{kc^2}{R_S^2}$$

$$\left(\frac{1}{a}\frac{da}{d\eta}\right)^2 = \frac{a_m}{a} - 1$$

*Solution with boundary conditions for Big-Bang cosmology*

$$a = \frac{R_S}{R_{S_0}} = \frac{a_m}{2}(1 - \cos\eta)$$

$$t = \frac{R_{S_0}}{c}\frac{a_m}{2}(\eta - \sin\eta)$$

# Non-linear collapse of a spherical overdensity

<u>Assumptions</u> :
- One spherical overdense region of radius $R_s$ and density $\rho_s$
- An Einstein-de-Sitter Universe with mean density $\bar{\rho}$ with a scale factor $R(t)$
- Take recombination as a reference epoch
  - mean density in the universe : $\rho_0 = \bar{\rho}(t_0)$
  - Scale factor $R_0$
  - Mass $M$, Radius $R_{S_0}$ and density : $\rho_{S_0} = \frac{3M}{4\pi R_{S_0}^3}$

$$a = \frac{R_S}{R_{S_0}} = \frac{a_m}{2}(1 - \cos\eta)$$
$$t = \frac{R_{S_0}}{c}\frac{a_m}{2}(\eta - \sin\eta)$$

$$\Omega_{S_0} = \frac{8\pi G \rho_{S_0}}{3H_{S_0}^2}$$

$$a_m = \frac{8\pi G \rho_{S_0} R_{S_0}^2}{3c^2}$$

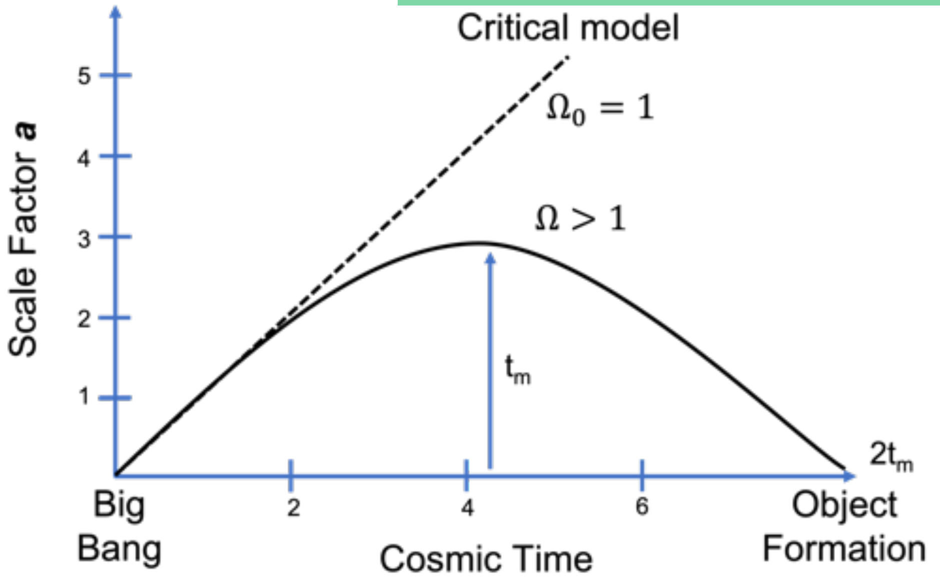
The overdensity will reach a maximum radius $R_m$ at a time $t_m$ where $(\eta = \pi)$ :

$$R_m = a_m R_{S_0} = \frac{\Omega_{S_0}}{\Omega_{S_0} - 1} R_{S_0}$$

And

$$t_m = \frac{\pi}{2}\frac{R_{S_0}}{c} a_m = \frac{\pi \Omega_{S_0}}{2H_{S_0}\left(\Omega_{S_0} - 1\right)^{3/2}}$$

where $R_{S_0} = \frac{c}{H_{S_0}}\left(\Omega_{S_0} - 1\right)^{-1/2}$




- Overdensity collapses to its final state at a time $2t_m(\eta = 2\pi)$
- Point where the overdensity reaches its maximum is turn around
- Density within the collapsing overdense region at turn around is

$$\rho_s(t_m) = \rho_{S_0}\left(\frac{R_{S_0}}{R_m}\right)^3 \approx \rho_0 \Omega_{S_0}\left(\frac{R_{S_0}}{R_m}\right)^3 = \rho_0 \Omega_{S_0}\left(\frac{\Omega_{S_0} - 1}{\Omega_{S_0}}\right)^3$$

# Non-linear collapse of a spherical overdensity

<u>Assumptions</u> :

- One spherical overdense region of radius $R_s$ and density $\rho_s$
- An Einstein-de-Sitter Universe with mean density $\bar{\rho}$ with a scale factor $R(t)$
- Take recombination as a reference epoch
  - mean density in the universe : $\rho_0 = \bar{\rho}(t_0)$
  - Scale factor $R_0$
  - Mass $M$, Radius $R_{S_0}$ and density : $\rho_{S_0} = \frac{3M}{4\pi R_{S_0}^3}$

$$\rho_s(t_m) = \rho_{S_0}\left(\frac{R_{S_0}}{R_m}\right)^3 \approx \rho_0\Omega_{S_0}\left(\frac{R_{S_0}}{R_m}\right)^3 = \rho_o\Omega_{S_0}\left(\frac{\Omega_{S_0}-1}{\Omega_{S_0}}\right)^3$$

$$\frac{\rho_s(t_m)}{\bar{\rho}(t_m)} \approx \frac{\left(\Omega_{S_0}-1\right)^3\Omega_{S_0}^{-2}}{\left(\frac{3H_0t_m}{2}\right)^{-2}}$$

- The mean density in the Universe at turn around is :

$$\bar{\rho}(t_m) = \rho_0\left(\frac{R_{S_0}}{R(t_m)}\right)^3$$

But $H_0t_m \approx H_{S_0}t_m = \frac{\pi\Omega_{S_0}}{2(\Omega_{S_0}-1)^{3/2}}$

Radius a sphere of radius $R_{S_0}$ at time $t_0$ would have if following the smooth cosmological expansion at $t_m$

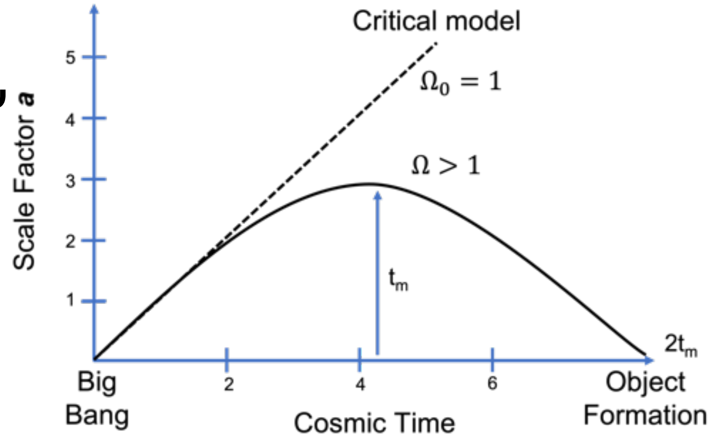
For Einstein de Sitter cosmology

$$\frac{R(t_m)}{R_0} = \left(\frac{t_m}{t_0}\right)^{2/3} = \left(\frac{3}{2}H_0t_m\right)^{2/3}$$

$$\frac{\rho_s(t_m)}{\bar{\rho}(t_m)} \approx \left(\frac{3\pi\Omega_{S_0}}{4\left(\Omega_{S_0}-1\right)^{\frac{3}{2}}}\right)^2 \times \frac{\left(\Omega_{S_0}-1\right)^3}{\Omega_{S_0}^2} = \left(\frac{3\pi}{4}\right)^2$$

**Independent of the initial overdensity**

# Non-linear collapse of a spherical overdensity




The collapsed object forms at a time $t_f = 2t_m$ :
- The redshift of turn around : $z_m$
- The redshift of formation : $z_f$

Related in the Einstein de Sitter Universe by :

$$\frac{1+z_m}{1+z_f} = \frac{R(2t_m)}{R(t_m)} = 2^{2/3} \approx 1.59$$

$R \propto t^{2/3}$

The collapse is halted by the internal pressure and the end state will be determined by the virial equilibrium, the object will be virialised :

$$2K_v + \Phi_v = 0$$

Internal energy (thermal + turbulent)

Gravitational potential energy

If the collapsing object has little kinetic energy at turn around (i.e. its peculiar velocity is small), the conservation of energy gives :

$$-\frac{GM^2}{R_m} = K_v - \frac{GM^2}{R_v} = -\frac{1}{2}\frac{GM^2}{R_v}$$

hence $R_v = \frac{1}{2} R_m$

- The density will be eight times the density at turn around ($\rho \propto R^{-3}$)
- Object has now fully decoupled from the Hubble flow

# Non-linear collapse of a spherical overdensity

The density of the Universe at the formation epoch is given by :

$$\bar{\rho}(z_f) = \left(\frac{1+z_f}{1+z_m}\right)^3 \bar{\rho}(z_m)$$

$$\frac{1+z_m}{1+z_f} = \frac{R(2t_m)}{R(t_m)} = 2^{2/3} \approx 1.59$$

Hence the virialised density is given by :

$$\rho_v \sim 5.6\times8\times\bar{\rho}(z_m) = (1.59)^3\times5.6\times8\times\bar{\rho}(z_f)$$

$$\frac{\rho_{s(t_m)}}{\bar{\rho}(t_m)} \sim 5.6$$

$$R_v = \frac{1}{2}R_m$$

Finally $\rho_v \sim 200\,\bar{\rho}(z_f) \sim 200\,\bar{\rho}(z=0)(1+z_f)^3$

The final de-coupled virialised object has a density 200 times larger than the density of the universe at the epoch of its formation

# Application to the Milky Way

We can apply the previous analysis to our Milky Way. It applies to all the matter and is therefore dominated by dark matter.

For the Milky Way we know that :

- $M_{DM} \approx 3{\times}10^{11} M_{\odot}$
- $R_{DM} \approx 50\ kpc$

Calculating $\rho_{DM}(MW)$ and comparing to $\bar{\rho}(z=0)$ gives $z_f \sim 2.5$

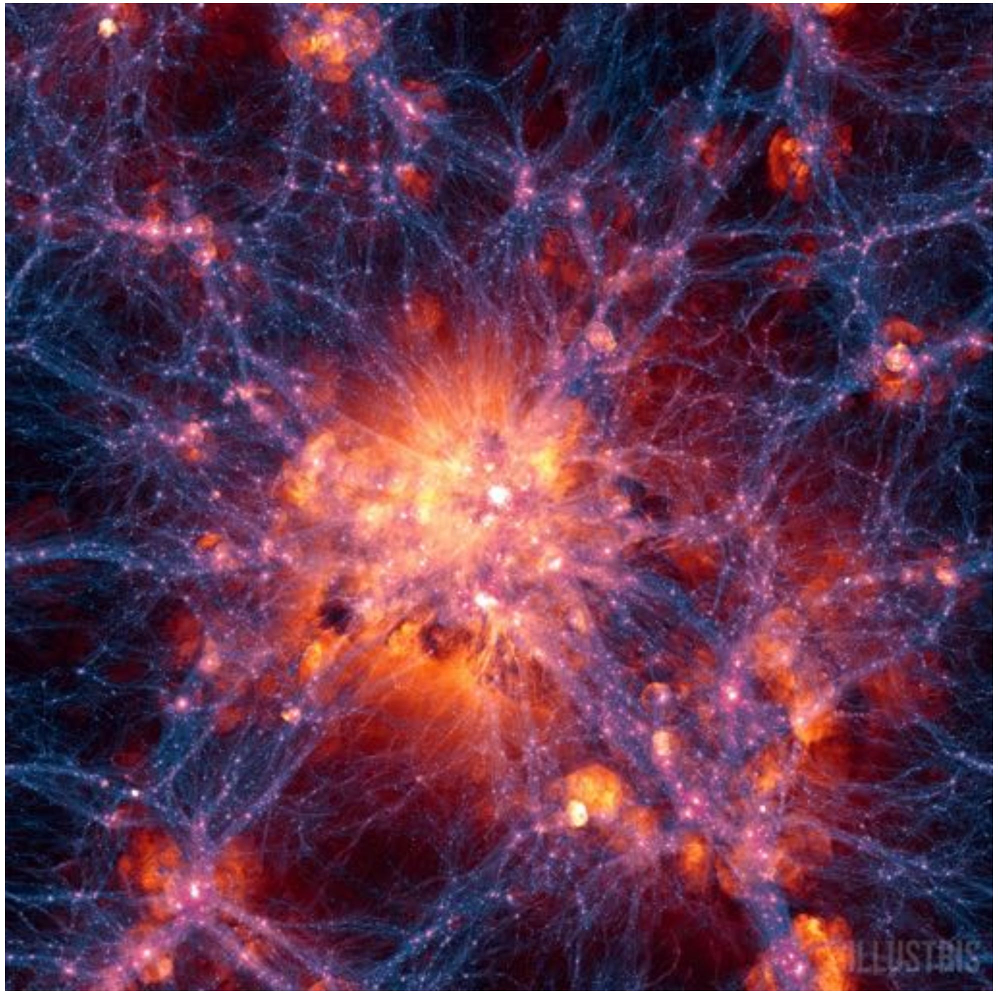
# Hierarchical structure formation

The typical Jeans mass we found is crucial in understanding the process of galaxy formation.

The structure which initially form in a CDM cosmology are much smaller than the scale of a typical galaxy we can observe in the local Universe.

Indeed the initial structures collapse under their own self-gravity to form dark matter halos

These dark matter halos are of course subject to gravitational interactions and can merge under their mutual gravitational interaction to form larger structures




*Illustris* is one of the best simulations following this process.

# The Press-Schechter Mass Function

Assumption : primordial density perturbations are ***Gaussian fluctuations***

Random phases of perturbation modes

Gaussian amplitude distribution of the probabilities :

$$p(\Delta) = \frac{1}{\sqrt{2\pi}\sigma(M)} \exp\left[-\frac{\Delta^2}{2\sigma^2(M)}\right]$$

Density contrast :

$$\Delta = \frac{\delta\rho}{\rho}$$

$$< \Delta^2 > = < \left(\frac{\delta\rho}{\rho}\right)^2 \geq \sigma^2(M)$$

Assumptions :

- Perturbations grow according to the linear theory until they reach a critical density contrast $\Delta_c$ (density contrast at $t_m$)
- The perturbations had a power-law power spectrum $P(k) = k^n$
- In a Einstein-de-Sitter Universe :

$$\Omega_0 = 1 \, , \Omega_\Lambda = 0$$

Perturbations grow as $\Delta \propto R \propto t^{2/3}$

# The Press-Schechter Mass Function

$\xi(M) \propto M^{-(n+3)/3}$

For fluctuations of a given mass $M$, the fraction $F(M)$ of those which became bound at a particular epoch have $\Delta > \Delta_c$ :

$$F(M) = \frac{1}{\sqrt{2\pi}\sigma(M)} \int_{\Delta_c}^{+\infty} \exp\left[-\frac{\Delta^2}{2\sigma^2(M)}\right] d\Delta = \frac{1}{2}[1 - \Phi(t_c)]$$

with $t_c$, the threshold density contrast in units of the rms density fluctuation :

$$t_c = \frac{\Delta_c}{\sqrt{2}\sigma}$$

and $\Phi(x)$is the probability integral :

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

The mean-squared density contrast in terms of the power-spectrum of fluctuations is given by :

$$\sigma^2(M) = < \left(\frac{\delta\rho}{\rho}\right)^2 > = < \Delta^2 > = AM^{-(3+n)/3}$$

We can also express $t_c$ in terms of the mass distribution :

$$t_c = \frac{\Delta_c}{\sqrt{2}\sigma(M)} = \frac{\Delta_c}{\sqrt{2}A^{1/2}} M^{(3+n)/6} = \left(\frac{M}{M^*}\right)^{(3+n)/6}$$

$$M^* = \left(\frac{2A}{\Delta_c^2}\right)^{3/(3+n)}$$

The amplitude of the perturbation grew as $\Delta(M) \propto R \propto t^{2/3}$

Hence, $\sigma^2(M) = \Delta^2(M) \propto t^{\frac{4}{3}}$,or $A \propto t^{4/3}$, then :

$$M^* \propto A^{\frac{3}{3+n}} \propto t^{\frac{4}{3+n}}$$

and $M^* = M_0^* \left(\frac{t}{t_0}\right)^{4/(3+n)}$

# The Press-Schechter Mass Function

The fraction of perturbations with masses in the range $M$ to $M + dM$ exceeding $\Delta_c$ is :

$$dF = -\left(\frac{\partial F}{\partial M}\right) dM$$

*because F is decreasing function of increasing M*

In the linear regime, the mass of the perturbation is $M = \bar{\rho}V$ where $\bar{\rho}$ is the mean density of the Universe.

Once the perturbation became non-linear collapse start and then a bound object of mass $M$ is formed.

The space density per unit mass of perturbations in the mass range $M$ and $M + dM$ which will become bound is :

$$N(M) = \frac{dn(M)}{dM} = \frac{1}{V}\frac{dF}{dM} = -\frac{\bar{\rho}}{M}\frac{\partial F}{\partial M}$$

$$F(M) = \frac{1}{2}[1 - \Phi(t_c)]$$

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2} dt$$

$$t_c = \left(\frac{M}{M^*}\right)^{(3+n)/6}$$

$$\frac{d\Phi}{dx} = \frac{2}{\sqrt{\pi}} e^{-x^2}$$

$$N(M) = \frac{1}{2\sqrt{\pi}}\left(1 + \frac{n}{3}\right)\frac{\bar{\rho}}{M^2}\left(\frac{M}{M^*}\right)^{(3+n)/6} \exp\left[-\left(\frac{M}{M^*}\right)^{\frac{3+n}{3}}\right]$$

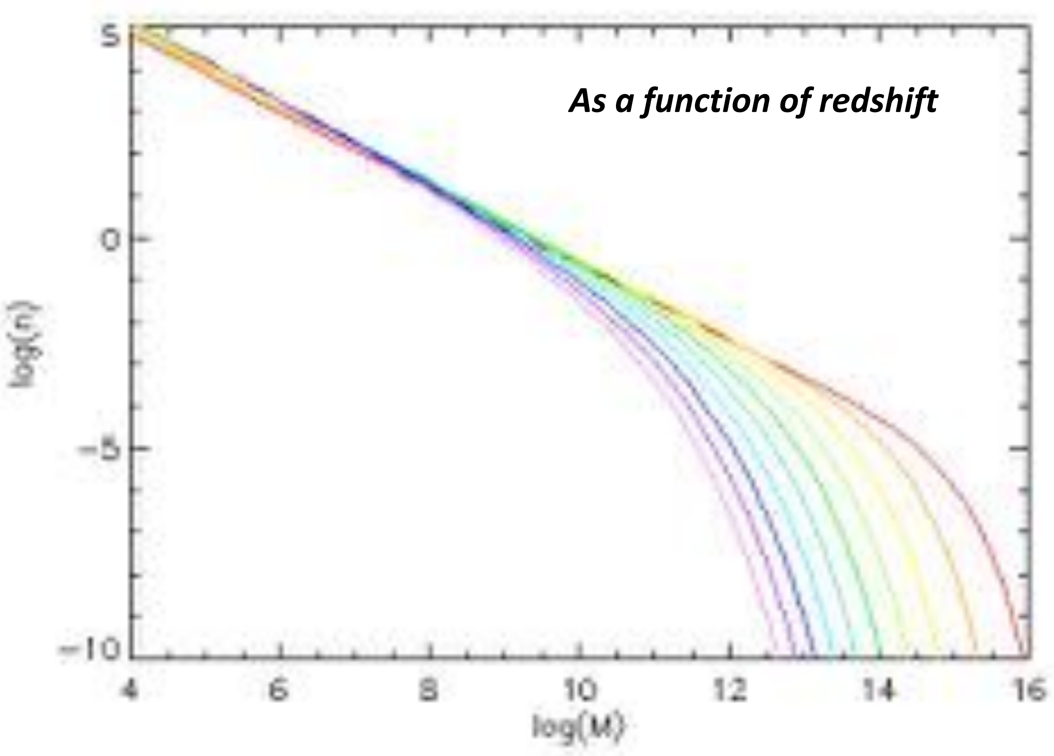
# The Press-Schechter Mass Function

$$N(M) = \frac{1}{2\sqrt{\pi}}\left(1+\frac{n}{3}\right)\frac{\overline{\rho}}{M^2}\left(\frac{M}{M^*}\right)^{(3+n)/6} \exp\left[-\left(\frac{M}{M^*}\right)^{\frac{3+n}{3}}\right]$$

- This formalism results in only half the total mass density being condensed into bound objects because of the fact that, according to this simple analysis, only the positive density fluctuations developed into bound systems

- Underlying cause of this factor of two discrepancy is the fact that the above analysis is based upon the linear theory of the growth of the perturbations

- Once perturbations developed to large amplitude, mass was accreted from the vicinity of the perturbation and N-body simulations show that most of the mass was indeed condensed into discrete structures.



# From the Feedback survey :

***what is the difference between Jeans Mass and the Bonnor Ebert Mass ?***

$$\frac{k_B T}{\mu} = a_T^2$$

$$M = c_g^{1/2}\left(\frac{k_B T}{\mu}\right)^2 \frac{1}{G^{3/2} P_0^{1/2}} = c_g^{1/2} \frac{a_T^3}{\rho_0^{1/2} G^{3/2}}$$

Bonnor-Ebert Mass

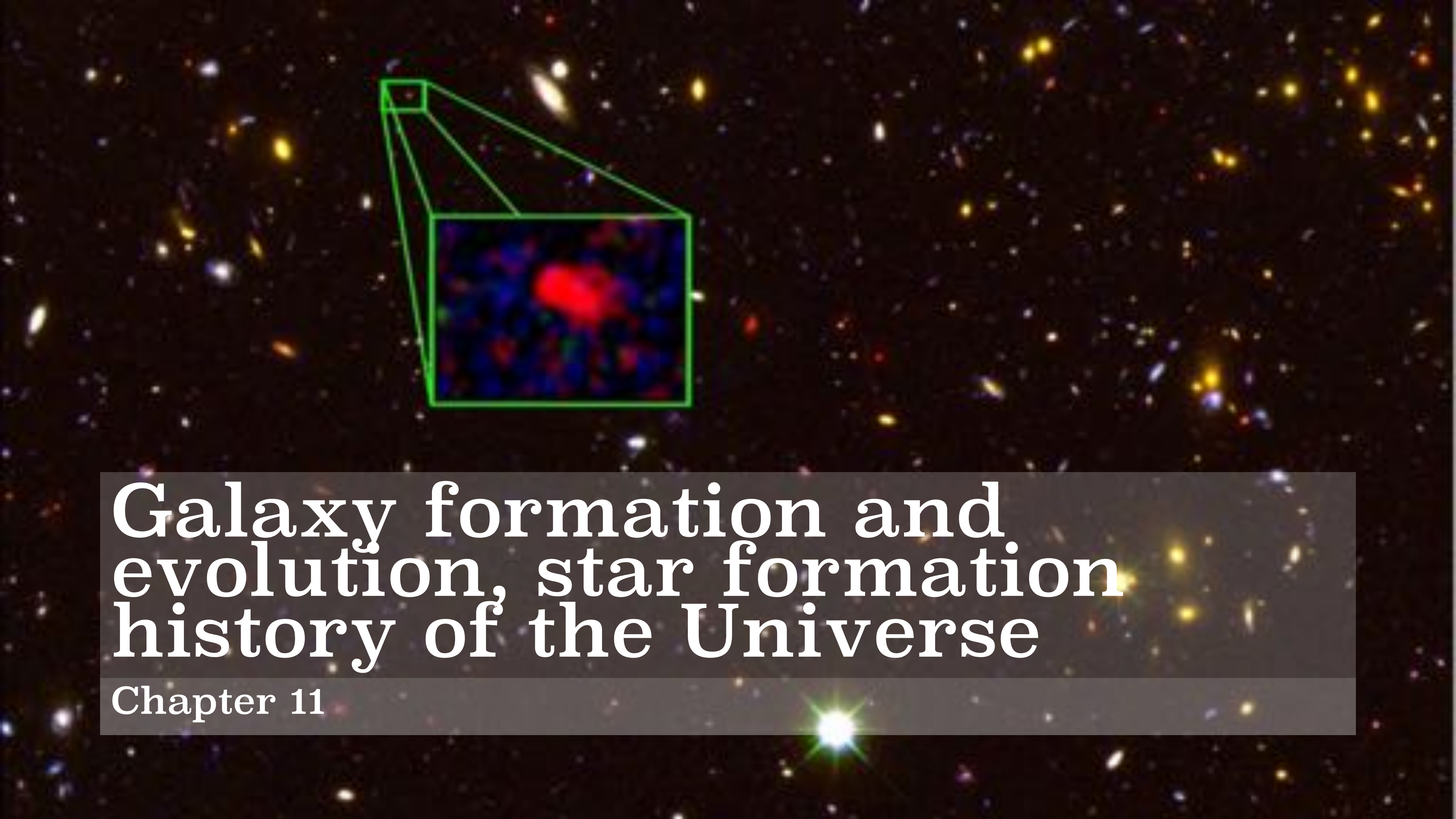
$$\frac{M_J}{M_\odot} = 1.0 \times \left(\frac{T}{10K}\right)^{3/2} \times \left(\frac{n_H}{2 \times 10^{10} m^{-3}}\right)^{-1/2}$$

Jeans Mass

**By definition : mass above which gravity dominates**

*Obtained from a singular isothermal sphere and the equation of the virial equilibrium*

*Obtained from a perturbation analysis of the fluid equations*



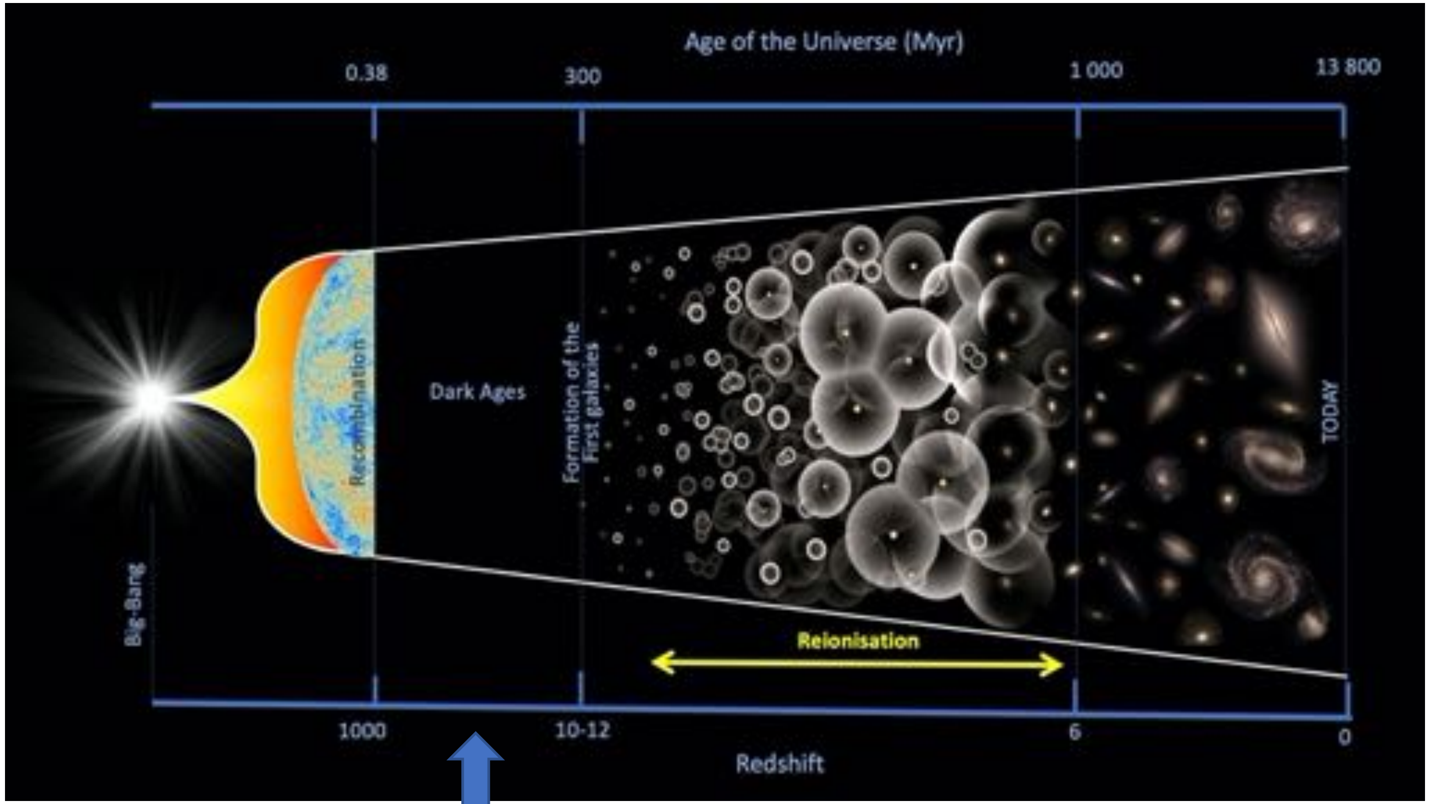
# Galaxy formation and evolution, star formation history of the Universe

Chapter 11

# The first objects

Following recombination baryonic gas is neutral. As the Universe expands a number of things occur :

- The CMB cools proportional to $1/R$ hence its temperature is given by : $T_{CMB}(z) = T_{CMB}(z=0)\times(1+z)$
- The baryonic gas not associated with self-gravitating objects cools adiabatically and faster than this proportional to $1/R^2$ since its adiabatic index is 5/3
- The process of hierarchical structure formation is occurring all the time with the merger of dark matter halos to form ever larger self-gravitating objects




At first however, there is no star formation or formation of black holes and AGN : the universe is dark apart from the CMB radiations : **the dark ages .**

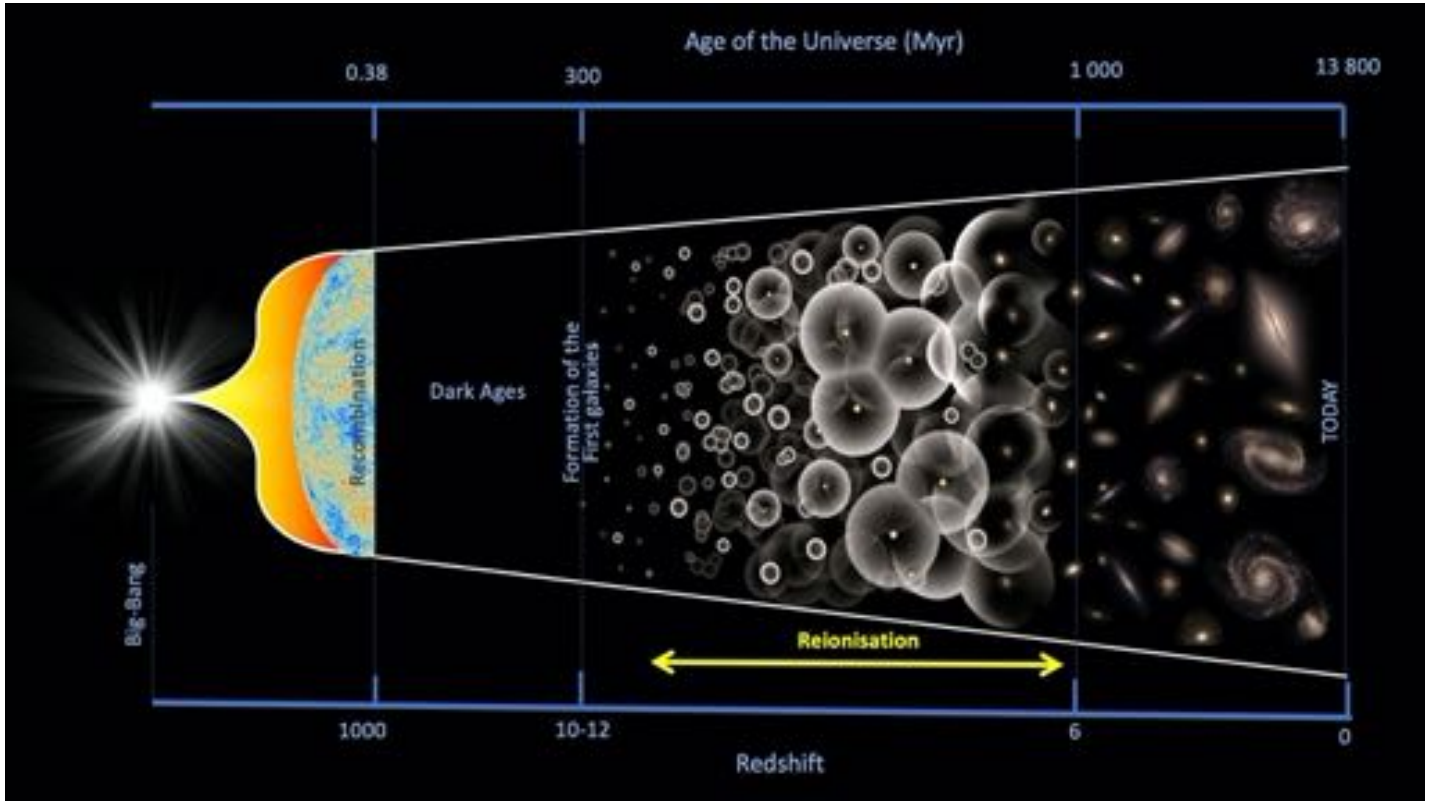
# The first objects

Following recombination baryonic gas is neutral. As the Universe expands a number of things occur :

- At some point the density of baryonic gas in halos becomes sufficiently large that the first stars and/or AGNs form

- The first objects produce UV and/or X-Ray emission which starts to ionise the neutral hydrogen outside of the densest regions. This epoch, when the first objects form is therefore called **the Epoch of reionisation.**

- Importantly, it is observable via the 21-cm transition of hydrogen redshifted to low frequencies corresponding to this epoch.



# The first objects

**WHAT IS THE EFFECT OF STAR FORMATION ON THE HYDROGEN SPIN TEMPERATURE ?**

At high densities, collisions keep the spin temperature equal to the kinetic temperature of the gas.

As the density falls, collisions become less important, and interaction with the photon field determines the spin temperature

Initially these are CMB photons, but once the first objects start to form, a small UV flux is present and this acts to excite neutral hydrogen to an excited electronic energy state. Then, the hydrogen returns to the ground state. The probability of returning to either of the split levels is determined by the kinetic temperature of the gas

The spin temperature is directly related to the populations of the ground state of hydrogen :

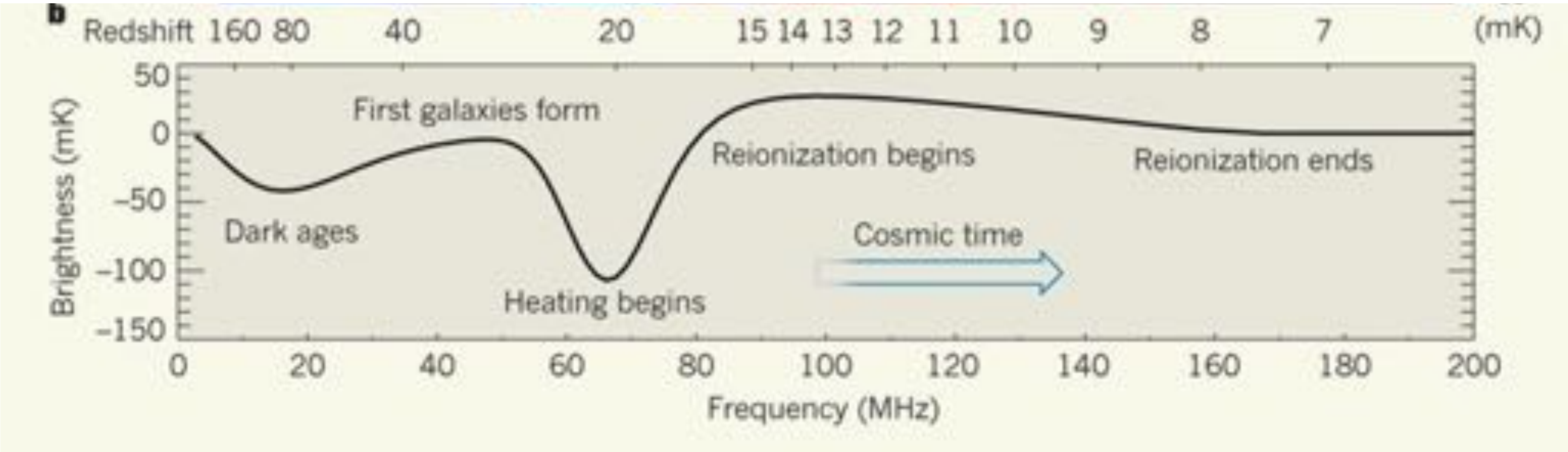
$$\frac{n_2}{n_1} = \frac{g_2}{g_1}\exp(-\frac{h\nu_{21}}{k_B T})$$

# The first objects

- After recombination gas and CMB cool adiabatically, the kinetic temperature is $T_K < T_{CMB}$
- Initially, the density is large enough that collisions ensure $T_S \sim T_K$
- As density falls, CMB determines $T_S \rightarrow T_{CMB}$
- Overdensity collapse forming the first objects

- As first objects form, UV resonant scattering couples $T_S \rightarrow T_{CMB}$
- Gas temperature increases, and so does $T_S$ due to heating from stars and AGN
- UV and X-Ray flux ionise the gas and $T_S \rightarrow 0$

$T_S - T_K$



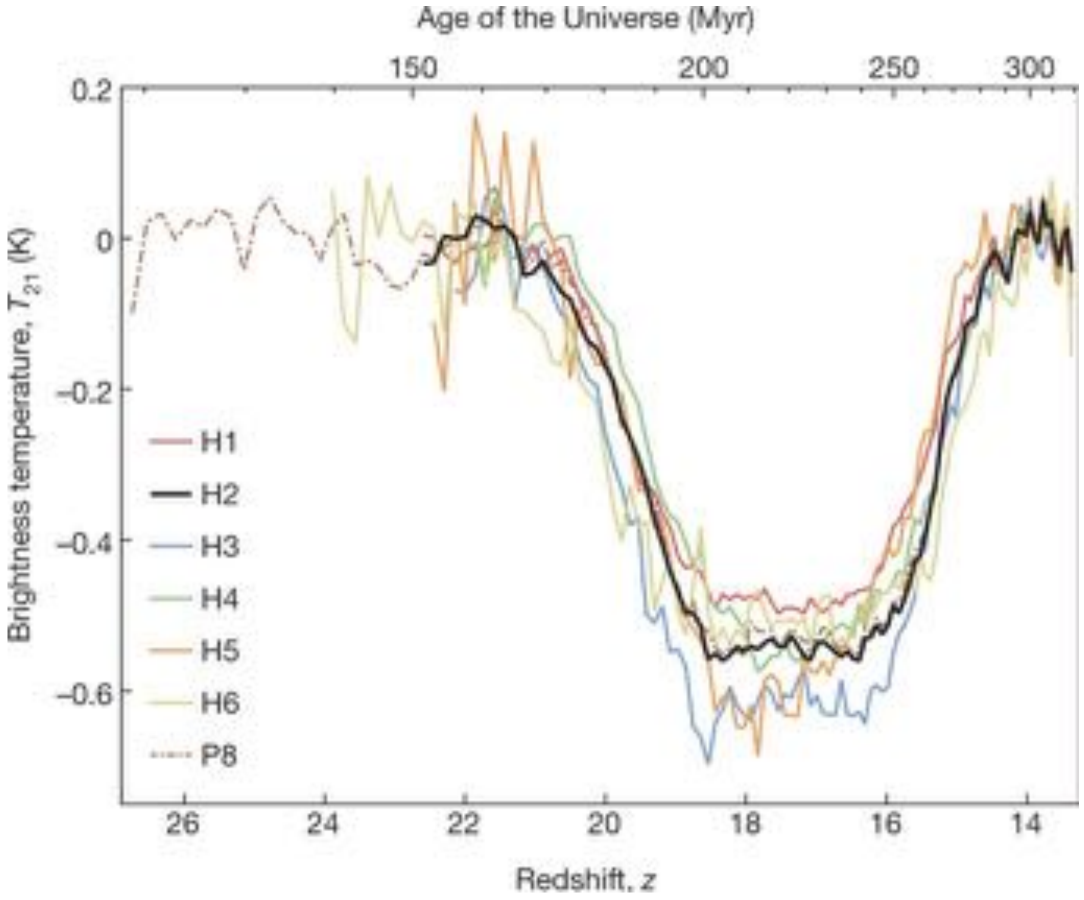
# The first objects

- What we observe is the hydrogen seen against the CMB : we are in the Rayleigh-jeans limit.

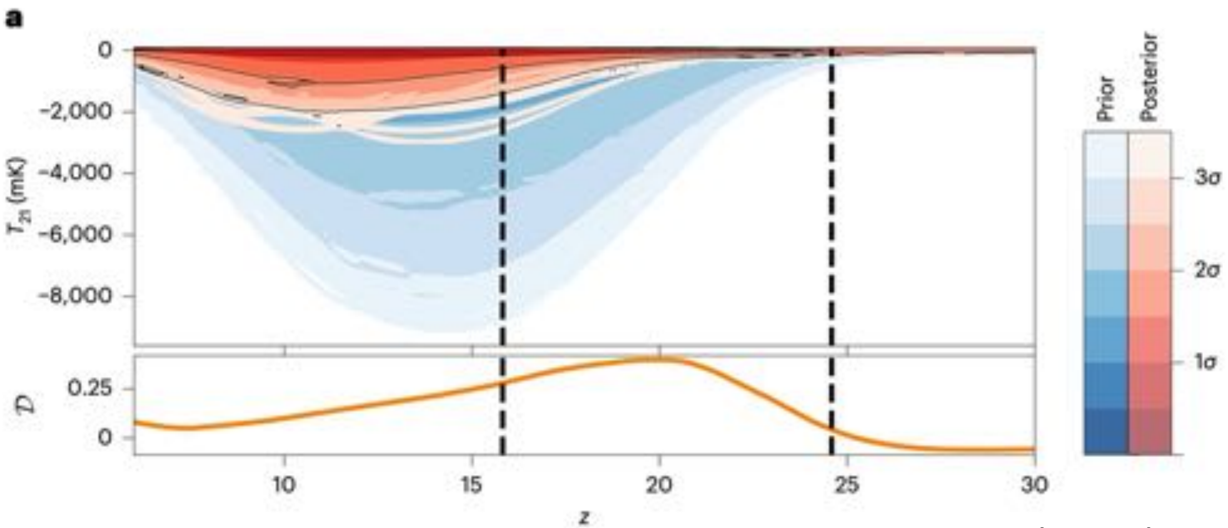
- The observe brightness of the 21 cm line is given by :

$$T_b = 27\, x_{HI}(1+\delta_b)\frac{T_s - T_{CMB}}{T_S}\left(\frac{1+z}{10}\right)^{\frac{1}{2}}\left(\frac{\partial_r v_r}{(1+z)H(z)}\right)^{-1} mK$$

# The first observational evidence of Cosmic Dawn




Bowman et al. (2018)




Bevins et al. (2022)

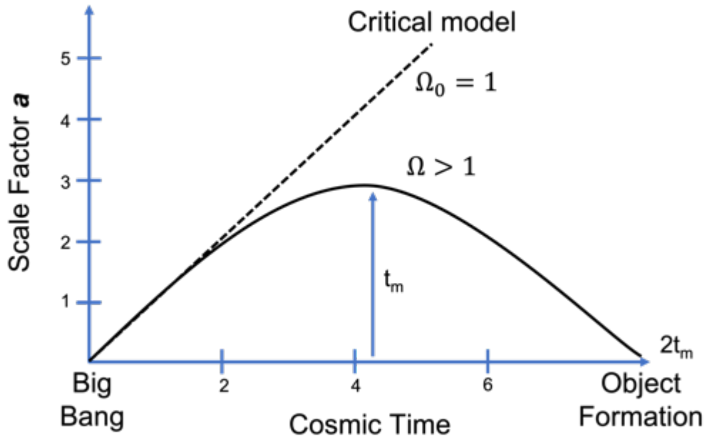
# Baryonic gas in dark matter halos

Overdensity in the baryonic gas catches up that in the dark matter :

$$\Delta_B = \Delta_D(1 - \frac{z}{z_0})$$

This suggests a simple picture for behaviour of baryonic gas :

- Baryonic gas should fall into pre-existing or growing dark-matter halos
- Typical infall velocities will be of order the free-fall speed ($\sim GM/R$) this greatly exceeds the sound speed in the baryonic gas.
- The gas therefore passes through a shock - a structure formation shock – and is heated




- The matter forms a virially supported structure of radius :

$$R_v = \frac{1}{2} R_m$$

- The mean matter density in the halo is given by :

$$\rho_v \sim 200\,\bar{\rho}(z_f) \sim 200\,\rho_0(1+z_f)^3$$

where $\rho_0$ is the total matter density in the Universe at the current epoch.

# Baryonic gas in dark matter halos

We now assume that matter has a density structure given by the singular isothermal sphere :

- "pressure support" for the dark matter we take to be due to the random motions of the non-interacting dark-matter particles
- For baryons we have normal pressure support.

The baryonic gas is in hydrostatic equilibrium in this potential well

The halo is characterised by its total mass – the virial mass $M_v$ :

- Mass of dark matter : $M_D = \frac{\Omega_D}{\Omega_M} M_v$
- Mass of baryonic gas : $M_B = \frac{\Omega_B}{\Omega_M} M_v$
- The virial radius of the halo is given by :

$$R_v = \left(\frac{3M_v}{4\pi\rho_v}\right)^{1/3} = \left(\frac{3M_v}{4\pi 200\rho_0}\right)^{\frac{1}{3}} (1+z)^{-1}$$

$$\rho_v \sim 200\,\bar{\rho}(z_f) \sim 200\,\bar{\rho}(z=0)(1+z_f)^3$$

The equation of hydrostatic equilibrium for the baryonic gas is :

$$\frac{1}{\rho_g}\frac{dP}{dr} = -\frac{GM(r)}{r^2}$$

For a singular isothermal sphere :

$$\rho = \frac{a^2}{2\pi G r^2} \text{ and } M = \frac{2a^2 r_0}{G}$$

Then :

$$\rho(r) = \frac{M_v}{4\pi R_v r^2} = \frac{\rho_v R_v^2}{3r^2}$$

Integrating :

$$M(r) = \int_0^r \rho(r')4\pi r'^2 dr' = M_v \frac{r}{R_v}$$

# Baryonic gas in dark matter halos

The equation of state of the baryonic gas is :

$$P = \frac{\rho_g k_B T}{\mu}$$

$$\frac{1}{\rho_g}\frac{dP}{dr} = -\frac{GM(r)}{r^2}$$

$$\rho(r) = \frac{\rho_v R_v^2}{3r^2}$$

$$M(r) = M_v \frac{r}{R_v}$$

Therefore :

$$kT_v = \frac{GM_v\mu}{2R_v} \propto M_v^{\frac{2}{3}}(1+z_f)$$

and

$$T_v \approx 3.6\times10^5 \left(\frac{M_v}{10^{12}M_\odot}\right)^{\frac{2}{3}} (1+z_f)\ \mathrm{K}$$

The growth of halo is done via hierarchical structure formation.

The cooling of baryonic gas is dominated by line emission (see chapter 2). In the low density limit the emissivity is given by :

$$\epsilon = n^2\Lambda(T) \approx n^2 \sum_i \frac{g_{u,i}}{g_{l,i}} C_{ul,i} h\nu_i e_i^{-h\nu/k_B T}$$

The collisional de-excitation rates have a temperature dependence in $T^{-1/2}$, then :

$$\epsilon \approx n^2\Lambda_0 \left(\frac{T}{T_0}\right)^{-1/2}$$

# Baryonic gas in dark matter halos

The cooling time for the gas depends on the radius, and is given by :

$$t_c(r) = \frac{\frac{\frac{3}{2}\rho_g(r)k_B T_v}{\mu}}{n^2\Lambda(T)} = \frac{3\mu k_B}{2\Lambda_0 T_0^{1/2}}\frac{T_v^{\frac{3}{2}}}{\rho_g} = A\frac{T_v^{-3/2}}{\rho_g}$$

We find the characteristic cooling time for the halo by averaging over all the particles in the halo :

$$\bar{t}_c = \frac{1}{M_v}\int_0^{R_v} t_c(r)\rho_g 4\pi r^2 dr = \frac{A}{M_v}\int_0^{R_v}\frac{T_v^{\frac{3}{2}}}{\rho_g}\rho_g 4\pi r^2 dr$$

$$= \frac{A}{M_v}T_v^{3/2}\frac{4}{3}\pi R_v^3 = A\frac{T_v^{\frac{3}{2}}}{\rho_v}$$

$$\propto \frac{\left(M_v^{2/3}(1+z_f)\right)^{3/2}}{\left(1+z_f\right)^3} \propto M_v\left(1+z_f\right)^{-3/2}$$

# Baryonic gas in dark matter halos

Inserting numbers gives :

$$\frac{\bar{t}_c}{yr} = 6.4\times10^{10}\left(\frac{M_v}{10^{12}M_\odot}\right)\left(1+z_f\right)^{-3/2}$$

**Cooling is therefore more efficient in low-mass halos and those which form at high-redshift**

If $t_c$ is much less than $t_u$ we expect all the gas to cool → predicts many small halos where all the gas has been converted to stars

$t_c$ and $t_u$ have the same redshift dependence therefore at all epochs the low-mass objects cool more efficiently

The Age of the Universe in a Einstein-de-Sitter Universe is given by :

$$t_u = 1.2\times10^{10}\left(1+z_f\right)^{-3/2}\ \text{yr}$$

Gas density depends on radius, there is a radius inside which the cooling time is less than the Hubble time : this is called **the cooling radius**

Outside of this radius, we do not expect gas cooling to be efficient : the cooling radius moves out with time.

# Baryonic gas in dark matter halos

**STAR FORMATION**

Star formation happens in the cool gas which accumulates at the center of the potential well.

Continuous input to the reservoir of cold gas as the warm gas at the virial temperature in the halo cools at a rate :

$$M_{c,\dot{a}cc} \approx \int 4\pi r^2 \frac{f_g^2 \rho_g^2 \Lambda(T)}{\frac{3}{2}\mu k_B T_v} dr$$

$f_g$ fraction of gas remaining hot at any time at a given radius.

Accreting gas settles into a disc (conservation of angular momentum).

Disc forms stars, when its surface density is :

$$\sigma > \frac{\kappa a_0}{\pi G} \text{ i.e. } Q<1$$

# The galaxy luminosity function and galaxy populations

**The Schechter function (Galaxy luminosity function)**

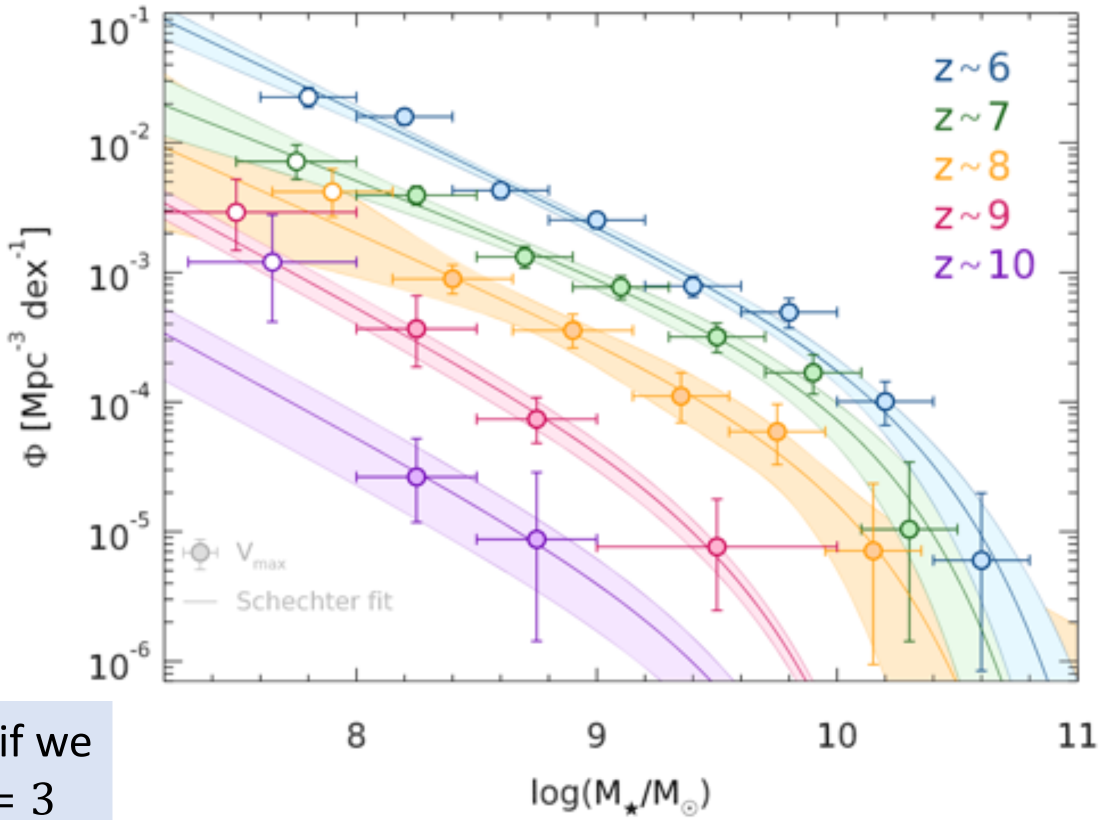
$$\Phi\left(\frac{L}{L^*}\right) d\left(\frac{L}{L^*}\right) = \Phi^*\left(\frac{L}{L^*}\right)^{\alpha} \exp\left(-\frac{L}{L^*}\right) d\left(\frac{L}{L^*}\right)$$

$\alpha \sim -1; -2$

**The Press-Schechter mass function**

$$N(M) = \frac{1}{2\sqrt{\pi}}\left(1+\frac{n}{3}\right)\frac{\overline{\rho}}{M^2}\left(\frac{M}{M^*}\right)^{(3+n)/6} \exp\left[-\left(\frac{M}{M^*}\right)^{\frac{3+n}{3}}\right]$$

By comparing the LF and the MF we clearly see that, if we assume a constant mass-to-light ratio, we require $n = 3$
However, the Harrison-Zeldovich spectrum has $n = 1$




Stefanon et al. (2021)

# The galaxy luminosity function and galaxy populations

The problem is even worst :

- Cooling is very efficient in the low mass halos

$$\frac{\bar{t}_c}{yr} = 6.4\times10^{10}\left(\frac{M_v}{10^{12}M_\odot}\right)\left(1+z_f\right)^{-3/2}$$

For halos not much less massive than that of the Milky Way the cooling time is less than the Hubble time.

↓

Low mass halos should have processed all of the gas into stars and have done so at early epochs

↓

Steepen the luminosity function even further !

# The galaxy luminosity function and galaxy populations

Also, high mass halos should still be gas rich and still actively forming stars in all cases since the cooling time is long.

What we observe :

- Massive elliptical galaxies in the local universe with little gas and evolved stellar population
- Many small irregular galaxies which are very gas rich and still forming stars – we do observe dwarf ellipticals but these do not dominate

It seems that our basic model has some fundamental problem…or missing some fundamental processes.

# The galaxy luminosity function and galaxy populations

BUT there also some important **successes** of this model :

- The form of the luminosity function is correct

- The formation redshifts for objects of different mass are correct

- The cooling argument suggests a change in behaviour in mass of order the mass of the Milky Way :
  - This is approximately where the break in the luminosity function occurs.
  - The star formation in higher-mass objects is suppressed due to longer cooling times.

- Very massive halos form late and will have little cooling-observed clusters have very hot extended gas in a single halo.

# The galaxy luminosity function and galaxy populations

However, the following fundamental problems remain :

- Drastically different slope between halo mass function and galaxy mass function
- Massive elliptical galaxies are expected to be currently accreting gas and forming stars while they are passive and gas poor (and with old stellar population)
- Low mass galaxies are expected to have processed most of their gas and now be passive and gas poor, while they are gas rich and star forming

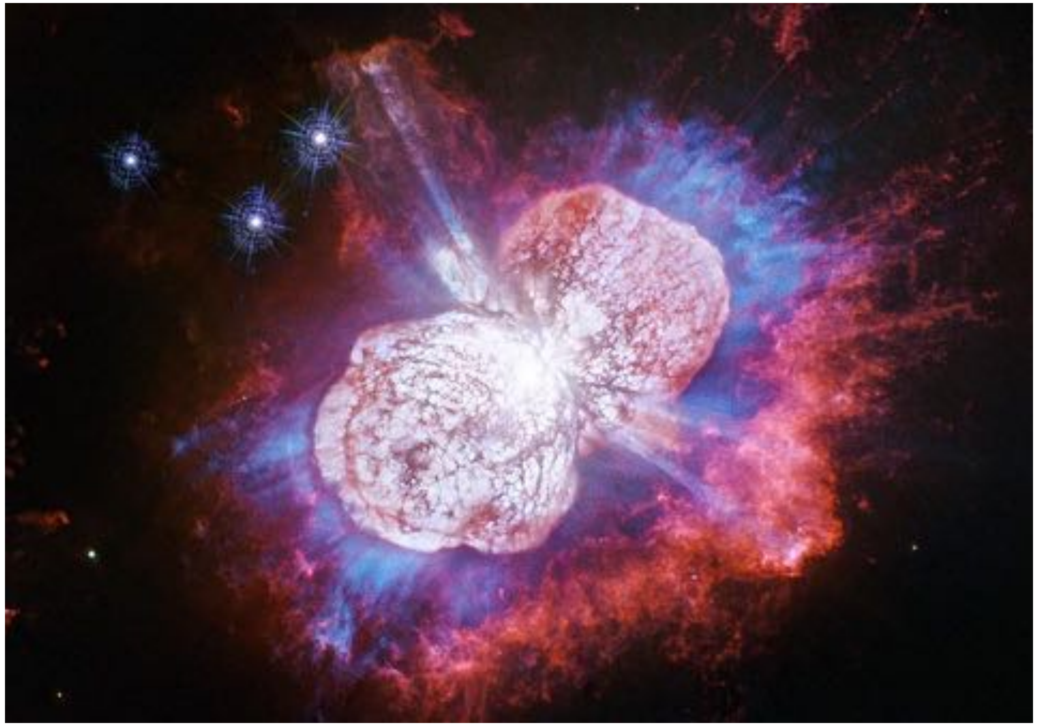
One key to solve these problems is : **Feedback** !

# Feedback

Including supernovae feedback in our model helps in two ways :

- Energy input heats the gas in the disc suppressing star formation
- Supernova-driven winds eject cold gas from the disc into the halo : if the gas is expelled beyond the cooling radius the feedback can halt star formation

A second process is also effective in suppressing star formation in the lowest mass halos : the reionisation of the gas from photons forming the UV background produced by star formation and AGN activity



# Feedback

More sophisticated models require numerical solution to follow the hot and cold gas

The process of halo growth is taken from numerical N-body models. In fact the basic build-up of the halos follows well the predictions of Press-Schechter, but the numerical approach includes the sudden increases in virial mass and gas which occurs as halos merge in the hierarchical structure formation scenario

Halo structure is modelled much as we have described

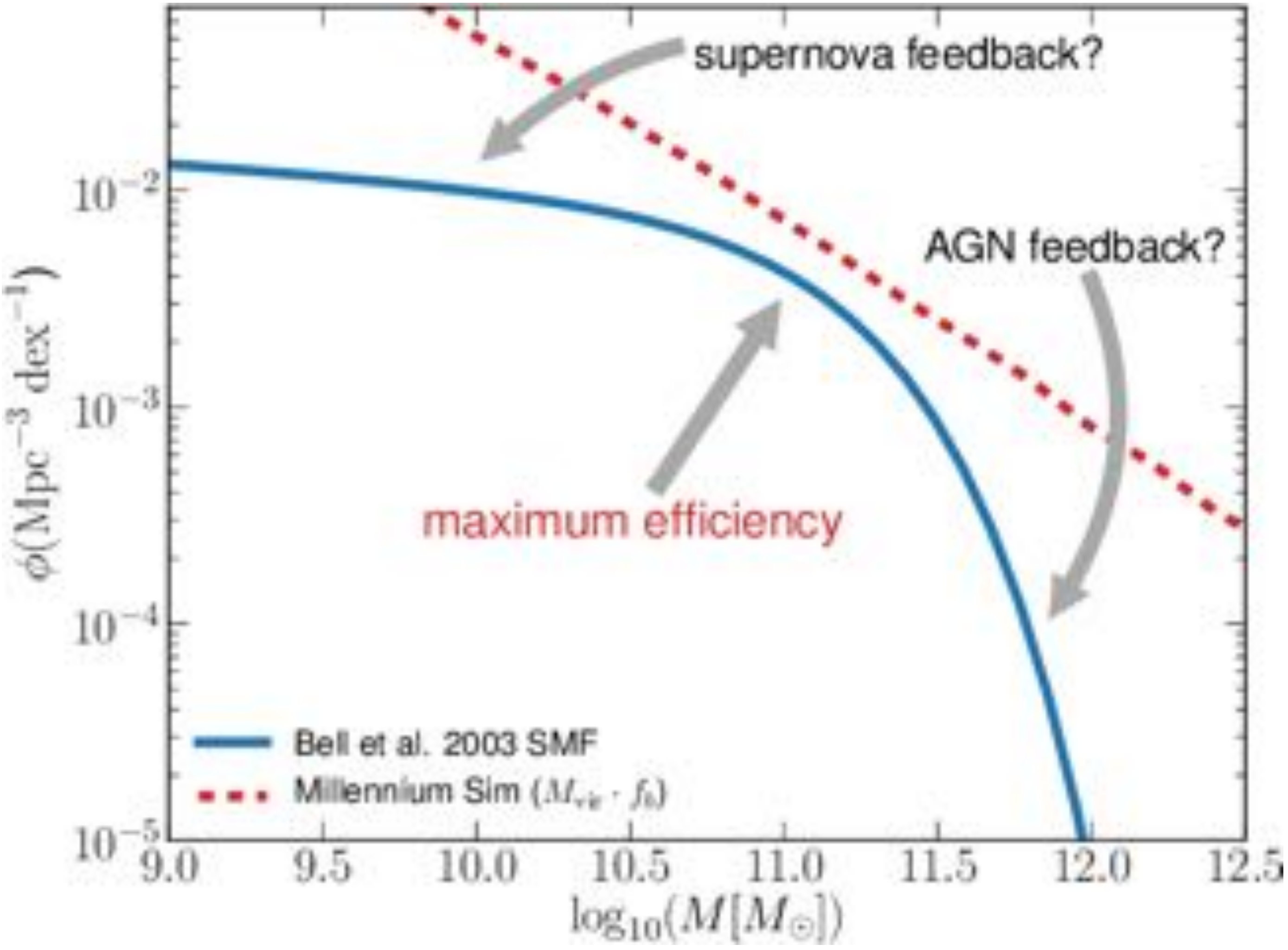
Cooling and star formation are modelled as we have described

Sne feedback is included by simply heating and ejecting the cold gas into the halo

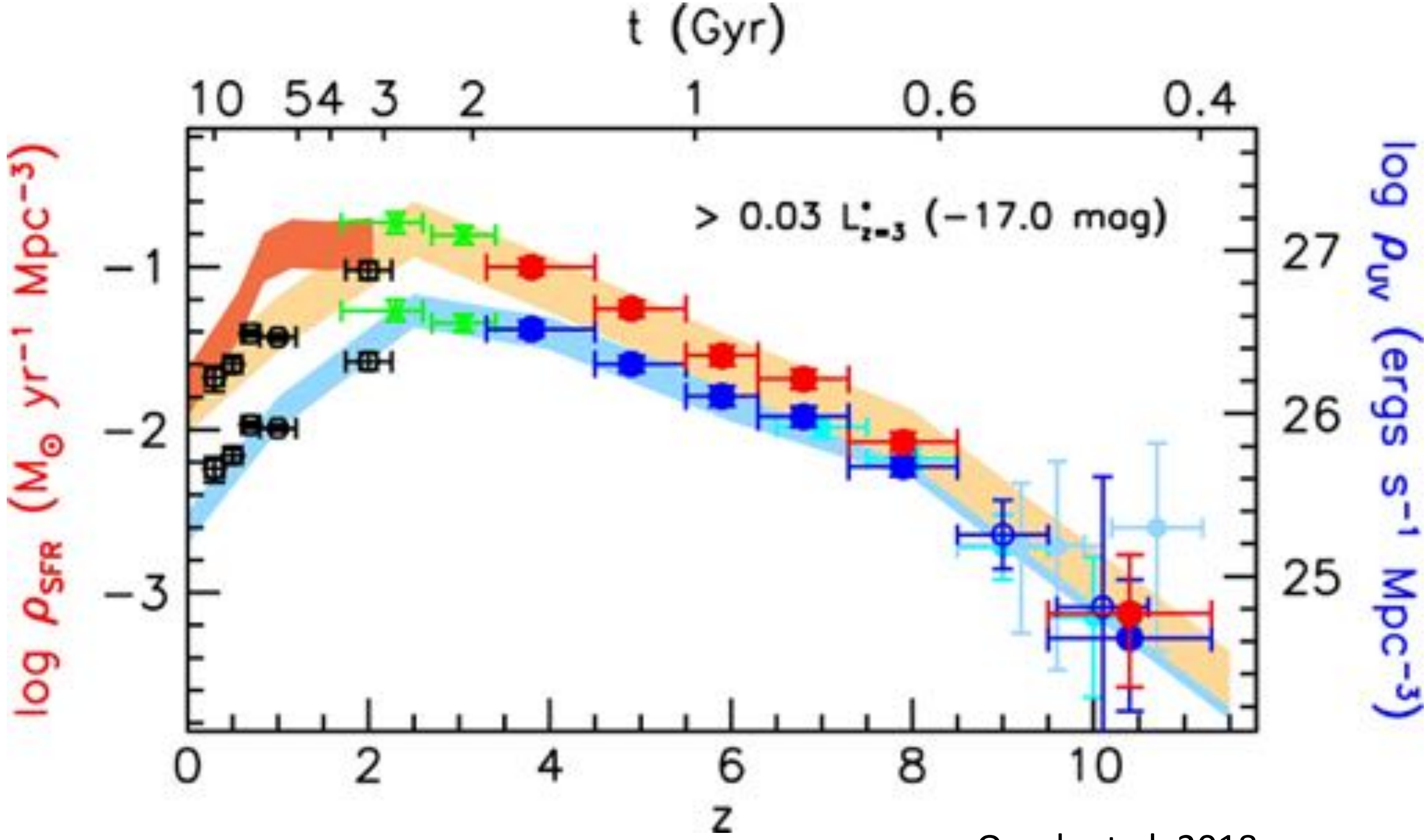
Reionisation feedback is also included

Recently, feedback from AGN has also been included as an additional heating source.

# Feedback


supernova feedback?
AGN feedback?
maximum efficiency
Bell et al. 2003 SMF
Millennium Sim
$\phi(\mathrm{Mpc}^{-3}\,\mathrm{dex}^{-1})$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
9.0
9.5
10.0
10.5
11.0
11.5
12.0
12.5
$\log_{10}(M[M_\odot])$

# Evolution of the Star-Formation Rate Density



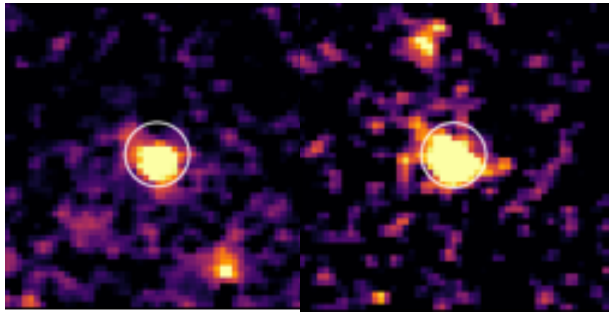

Oesch et al. 2018

# The first results from the JWST

z=12.22±0.15

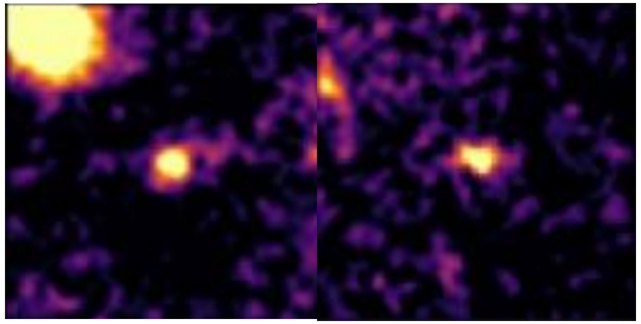
Castellano et al. (2022)
Naidu et al. (2022)

z=11.42 z=10.94



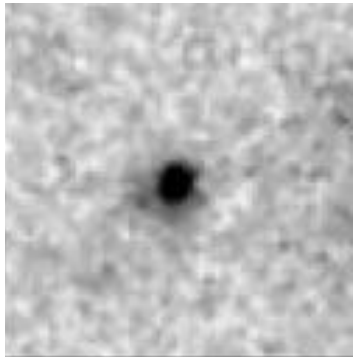
Adams et al. (2022)

z=15.92±0.17 z=15.2±0.21



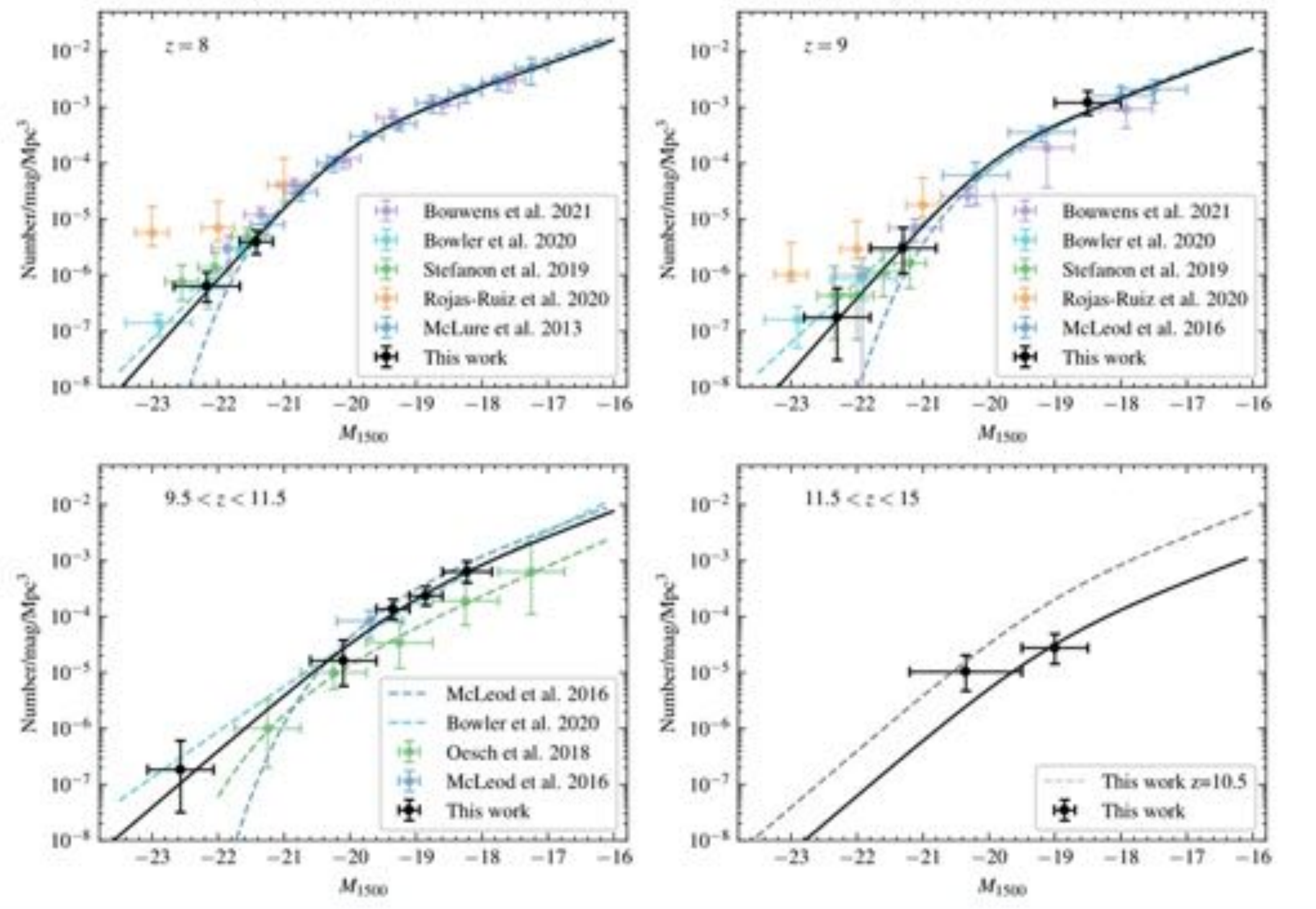
Atek et al. (2022)

z=16.39±0.27



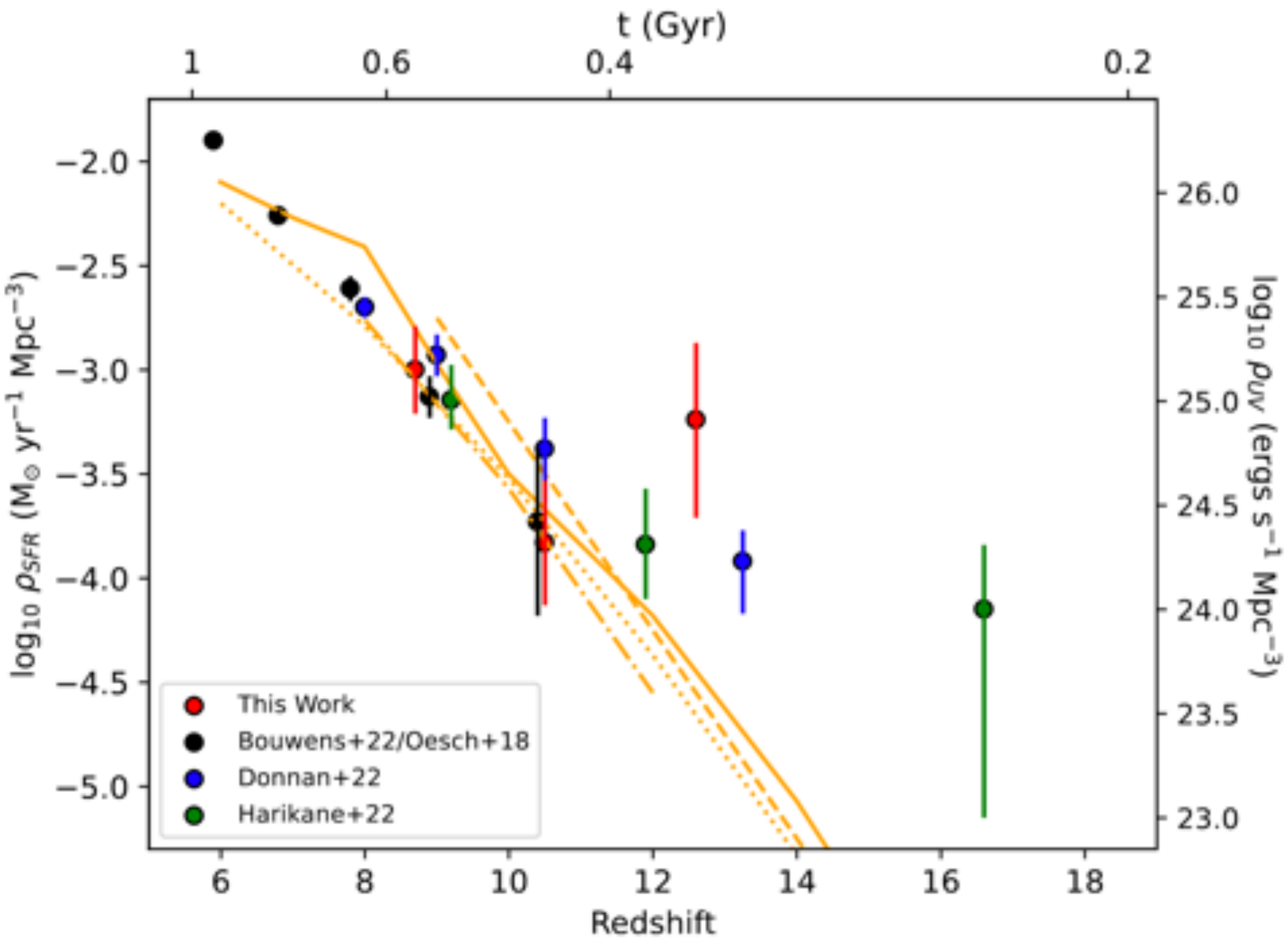
Donnan et al. (2022)

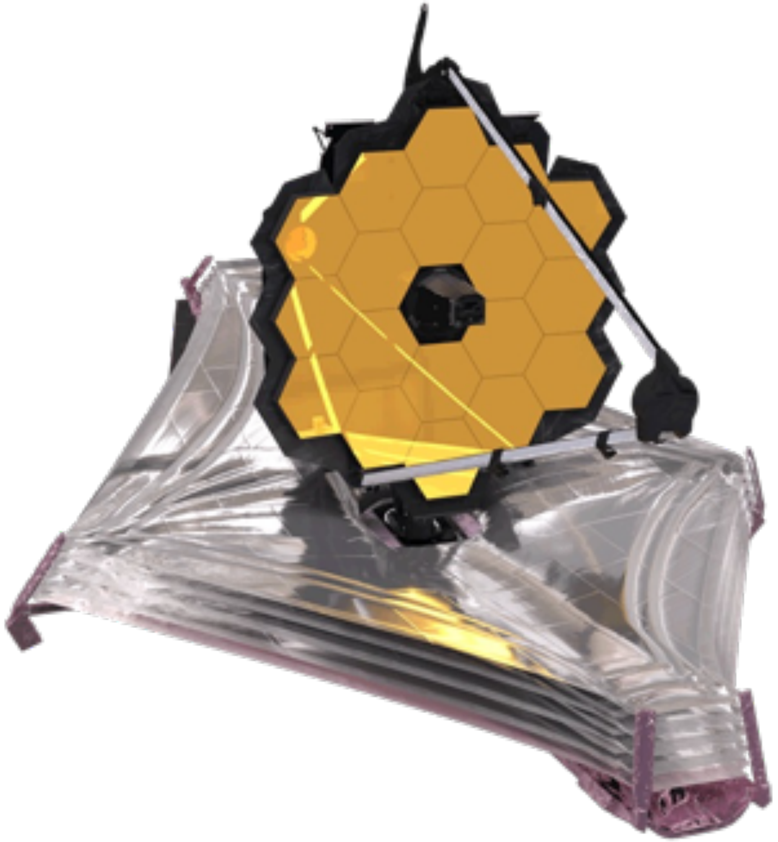
# The first results from the JWST




Donnan et al. 2023

# Evolution of the Star-Formation Rate Density

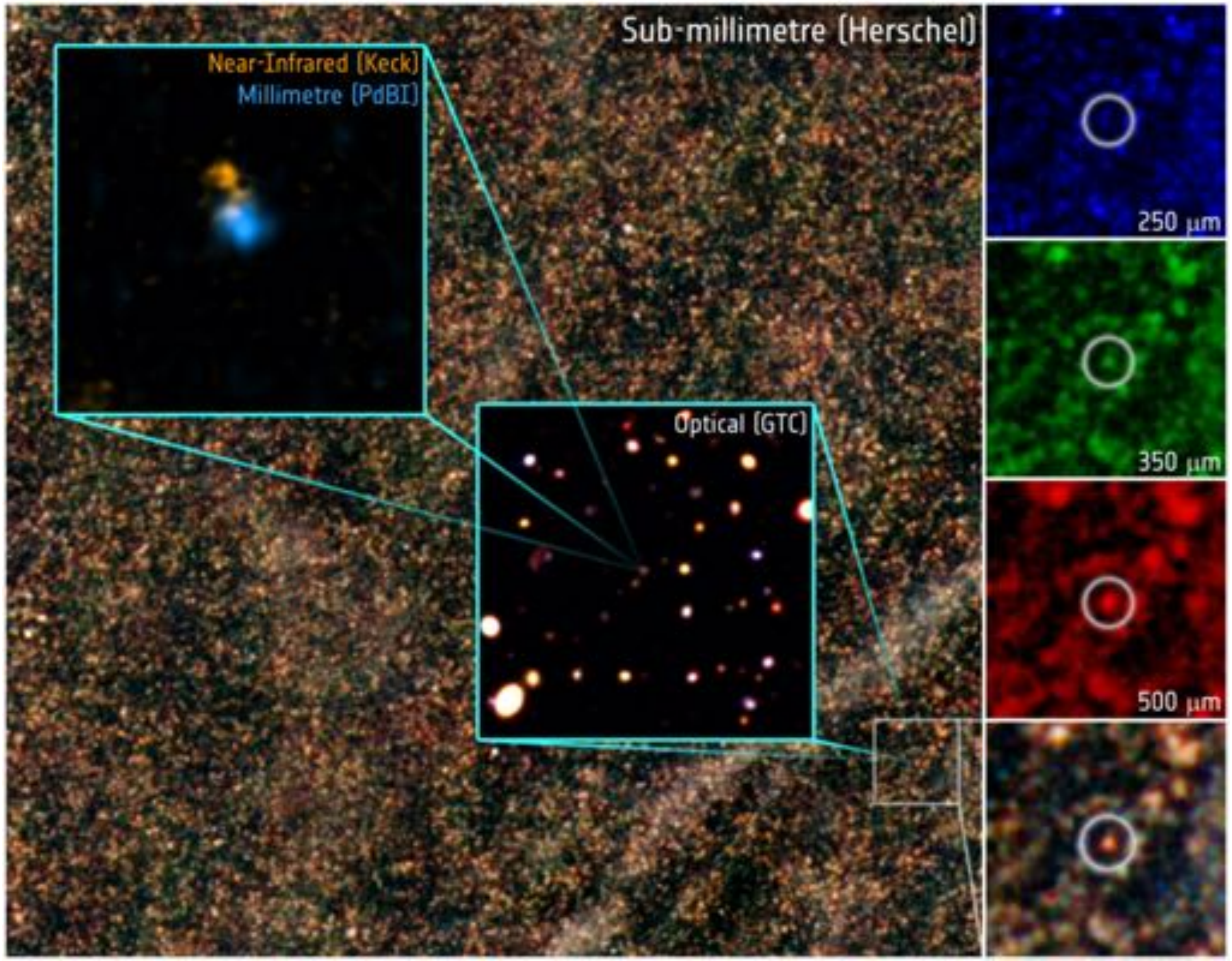


Bouwens et al. 2023


A new era in extragalactic astronomy: early results from the James Webb Space Telescope
20–24 MARCH 2023
KAVLI INSTITUTE FOR COSMOLOGY, CAMBRIDGE, UNITED KINGDOM
KICC
UNIVERSITY OF CAMBRIDGE

# Comments and Conclusions




Riechers et al. (2013)

In this course we have discussed our current understanding on structure formation in the Universe, mostly based on advances made over the past two decades.

Our model has several successes and leads to the emergence of a consistent picture. However several concerns arise with these models.

New telescopes (such as the E-ELT, the JWST, EUCLID, etc…) whose main goals will be to address all these questions will be commissioned within the next decades.

# Formation of Structure in the Universe

Lent Term 2023

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