# 5. From gas cloud to collapsed object

That is one of the things that has come out of the discovery of pulsars - more knowledge about the space between the stars.

— Dame Susan Jocelyn Bell Burnell

In previous chapter, we have considered the equilibrium and then collapse of an isolated gas cloud. We now apply this physics to the formation of a central collapsed object forming at the centre of the cloud. Although we will use the formation of star, or more strictly a protostar as our example in this chapter, many of the ideas will be used again when we consider the formation of other collapsed objects in particular galaxies.

## 5.1 Basic physics of object formation

If the cooling time in the gas is much shorter than the collapse time ($\tau_c < t$), the system will evolve approximately isothermally. The shortest collapse time is the free-fall time, therefore we can say that if the cooling time is shorter than the free-fall time ($\tau_c < t_{ff}$), then the cloud will collapse approximately isothermally. From the way the Jeans mass depends on the temperature $T$ and density $\rho$ ($M_J \propto T^{3/2}\rho^{-1/2}$) we can see that the Jeans Mass decreases. This results in smaller regions of the cloud having the ability to collapse in a process called fragmentation, which we will discuss in the next chapter.

The increasing density has another consequence however :

- The optical depth is given by $\tau = \alpha R = \rho\kappa R$ (eq. 2.28). Assuming that the mass in the collapsing object is constant : $R \propto \rho^{-1/3}$ and hence $\tau \propto \rho^{2/3}$.

- At some point, the collapsed object, or as we will now call it the core, becomes optically thick ($\tau > 1$), cooling is then very inefficient and the core ceases to cool.
- It will in general continue to collapse as more mass is added due to accretion from the surrounding cloud because of the inside-out collapse. The temperature of the core now rises as the collapse proceeds adiabatically.

## 5.2 Evolution of the first core

From the discussion in the previous section, we see that the collapse of an isolated gas cloud results in what we call the first core, which is when the initial collapse ceases to be isothermal. We can apply the virial theorem to this first core, which is mainly composed of molecular hydrogen. Taking the gravitational potential energy to be $\sim -\frac{3}{5}\frac{GM^2}{R}$ and neglecting external pressure and magnetic fields in comparison to the thermal and gravitational terms we can estimate the temperature of the core:

$$T \approx \frac{\mu}{5k_B}\frac{GM}{R} \approx 850 \left(\frac{M}{5\times10^{-2}M_\odot}\right)\left(\frac{R}{5\,\mathrm{AU}}\right)^{-1} K \tag{5.1}$$

The accretion rate onto this core is, as we previously calculated:

$$\dot{M} \approx 2\times10^{-6}\left(\frac{T}{10^6 K}\right)^{3/2} M_\odot\,\mathrm{yr}^{-1} \tag{5.2}$$

Further mass addition, together with decreasing R, means that the temperature soon exceeds 200 K, and collisional dissociation of $H_2$ begins (although $k_BT$ is much less than the dissociation potential). This phase change is crucial ; further mass addition happens with a very much slower rise in temperature. The central region of atomic gas has a significant density gradient and eventually undergoes collapse to form a very dense central core (c.f. instability of an isothermal sphere with large density contrast).

## 5.3 Structure around the protostar

The collapse of the first core gives a protostar of mass $\sim 0.1M_\odot$ and radius of several $R_\odot$. From the virial theorem this gives $T$ $\sim 10^5$K, the density is now $\sim 10^{-2}$ g cm$^{-3}$ and this really can be considered a protostar. We will now consider the environment of this protostar.

### 5.3.1 Accretion luminosity

Accretion occurs onto the protostar via the inside-out collapse of the cloud. We start by considering the energetics of the protostar in more detail. Since the protostar forms from cold, low density gas, we can take the initial thermal and mechanical energy of the gas, then energy balance requires :

$$U_{gr} + U_{th} + E_p + L_r t = 0 \tag{5.3}$$

where $U_{gr}$ is the gravitional potential energy of the protostar ($U_{gr} \approx -GM_\star^2/R_\star$) ; its internal energy from the virial theorem is $U_{th} = -U_{gr}/2$; $E_p$ is the enery necessary to change the phase of the gas (i.e. dissociate and ionise the original molecular gas), and $L_r$ is the mean energy radiated over the formation time of the protostar $t$.
For an initial hydrogen fraction $X$ and helium fraction $Y$, we have :

$$E_p = \frac{XM_\star}{m_H}\left[\frac{E_d(H)}{2} + E_i(H)\right] + \frac{YM_\star E_i(He)}{4m_H} \tag{5.4}$$

where $E_d$(H) is the dissociation energy of $H_2$ ($E_d$(H) =4.2 eV), $E_i$(H) =13.6 eV and $E_i$(He)=75 eV are the total ionisation potential of H and He respectively.
If $L_r$=0, the protostar would have a radius of :

$$R_{max} = \frac{GM_\star^2}{2E_p} = 60\left(\frac{M_\star}{M_\odot}\right) R_\odot \tag{5.5}$$

which is much larger than the observed size of T Tauri stars which are the immediate descendants of the protostars we are considering. This implies that $L_r \times t$ is comparable to the first term, i.e. most of the excess energy is radiated away ; $L_r$ must be close to the accretion luminosity :

$$L_r \approx L_{acc} = \frac{GM_\star \dot{M}}{R_\star} = 61\left(\frac{\dot{M}}{5\times 10^{-5} M_\odot \mathrm{yr}^{-1}}\right)\left(\frac{M_\star}{M_\odot}\right)\left(\frac{R_\star}{5R_\odot}\right) \tag{5.6}$$

### 5.3.2 Accretion shock and dust envelope

The accreting gas is infalling at close to the free-fall velocity :

$$v_{ff} = \left(\frac{2GM_\star}{R_\star}\right)^{1/2} = 280\left(\frac{M_\star}{M_\odot}\right)^{1/2}\left(\frac{R_\star}{5R_\odot}\right)^{-1/2} \mathrm{km\ s^{-1}} \tag{5.7}$$

This greatly exceeds the sounds speed in the gas and a strong shock forms with $v_s \approx v_{ff}$ giving a post-shock temperature of order $10^6$ K ; at this temperature there is significant UV and soft X-ray production. This whole region is optically thick and radiates with an effective temperature such that :

$$L_{acc} \approx 4\pi R_\star^2 \sigma_B T_{eff}^4 \tag{5.8}$$

or

$$T \approx \left(\frac{GM_\star \dot{M}}{4\pi\sigma_B R_\star^3}\right)^{1/4} = 7300 \left(\frac{\dot{M}}{5\times 10^{-5} M_\odot yr^{-1}}\right)^{1/4} \left(\frac{M_\star}{M_\odot}\right)^{1/4} \left(\frac{R_\star}{5R_\odot}\right)^{-3/4} K \tag{5.9}$$

Surrounding the star, the gaseous envelope also contains dust. The UV flux produced at the stellar surface is able to vaporize dust grains within a region called the *opacity gap* out to a radius known as the dust destruction front.
Outside of this radius the dust absorbs the radiation and re-radiates. In the dusty layer, the dust temperature must drop until the layer becomes optically thin to the re-emission of infrared radiation from the dust. This occurs at a radius $R_{phot}$ such that :

$$\rho \kappa R_{phot} = 1 \tag{5.10}$$

and

$$L_{acc} = 4\pi R_{phot}^2 \sigma_B T_{phot}^4 \tag{5.11}$$

## 5.4 Evolution of the protostar

We now turn our attention to the structure and evolution of the protostar itself, also referred as *stellar evolution*. This could be the topic of an entire course, and we will only discuss it briefly in the following. Few analytical calculations are possible, but we must rely largely on the results of numerical simulations to guide our understanding of the physics.

### 5.4.1 Protostellar structure during accretion

The structure of the protostar will be governed by the equations of hydrostatic equilibrium plus equations describing the thermal structure of the protostar - these are just the equations of stellar structure you met last year, although the boundary conditions are now different as discussed later.
The first equations are identical to those we met in our analysis of hydrostatic equilibrium of a cloud except that everything is now a function also of time :

$$\frac{\partial P}{\partial r} = -\frac{GM\rho}{r^2} \tag{5.12}$$

$$\frac{\partial M}{\partial r} = 4\pi r^2 \rho \tag{5.13}$$

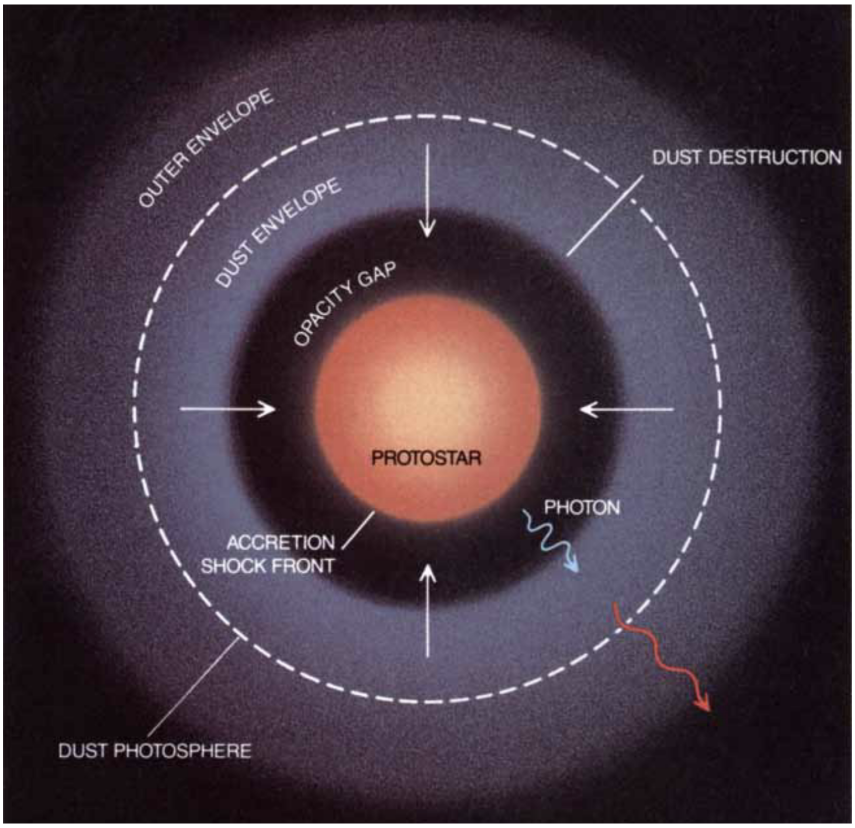


**Figure 5.1** : The environment of a protostar. From Stahler 1991

we again assume an ideal equation of state :

$$P = \frac{\rho k T}{\mu} \tag{5.14}$$

We then apply the first law of thermodynamics to obtain an equation for the heat production :

$$\rho T \frac{\partial S}{\partial t} = \rho \epsilon - \nabla \cdot \vec{F} \tag{5.15}$$

where $F$ is the radiative flux defined as :

$$F = \frac{L}{4\pi r^2} \tag{5.16}$$

hence :

$$\frac{\partial L}{\partial r} = 4\pi r^2 \rho \epsilon - 4\pi r^2 T \frac{\partial S}{\partial t} \tag{5.17}$$

This differs from the equation you had last year by the inclusion of the time derivative of the entropy. If the heat is transported by radiative diffusion we also have:

$$\frac{L}{4\pi r^2} = -\frac{4acT^3}{3\rho\kappa}\frac{\partial T}{\partial r} \tag{5.18}$$

These equations must be combined with a set of boundary conditions. In outline these are:

- The mass and luminosity go to zero as $r \to 0$
- The surface luminosity of the protostar itself is given by $L_\star - L_{acc}$
- The surface pressure of the protostar must balance the momentum flux, or *ram pressure* of the infalling gas which is $\sim \rho v_{ff}^2$ giving :

$$P(r_0) = \frac{\dot{M}}{4\pi}\left(\frac{2GM_\star}{R_\star^5}\right)^{1/2} \tag{5.19}$$

  The results of a numerical integration give for the accretion rate $\dot{M} = 1 \times 10^{-5} M_\odot \mathrm{yr}^{-1}$. This is shown in Figure 5.2.

- As parcels of gas fall onto the protostar they add an extra layer of material with both mass and entropy
- As the gas falls onto the star the energy density of $\sim -GM_\star\rho/R$ is thermalised at the shock and the entropy of this gas is just proportional to $\sim -GM_\star/R_\star$

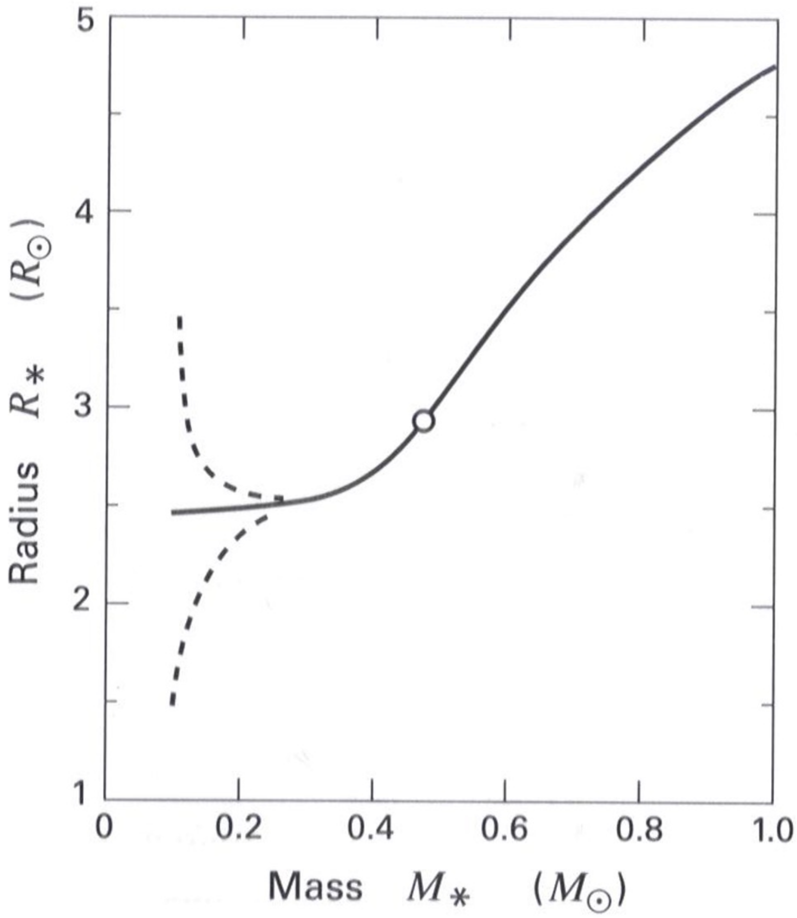


**Figure 5.2** : Mass-radius relation. The three curves have different initial values of $R_\star$ for a starting mass of $0.1M_\odot$

- if initially $R_\star$ is large, the entropy of the gas added to the protostar is low and the protostar shrinks under gravity
- The converse is true if $R_\star$ is initially small
- The protostar is characterised by its entropy profile $S(r)$
- The results show that $M_\star/R_\star$ is an increasing function in the early stages which gives an increasing entropy distribution with radius.

### 5.4.2 Onset of deuterium burning and convection

#### Convective stability

Consider a fluid element which moves a small distance through the atmosphere $\Delta r$ so that it remains in pressure balanced with the surrounding gas :

- The element expands adiabatically to a lower density
- For stability, the density of this element must be greater than the density of the surrounding gas
- If the entropy is increasing with radius the element is of lower entropy than the surrounding
- for an ideal gas :
$$S = c_v \log(p/\rho^\gamma) \tag{5.20}$$
where $\gamma = \frac{c_p}{c_v}$ is the specific heat ratio of a gas.
hence $\rho$ increases as the entropy decreases. The atmosphere is convectively stable therefore if :
$$\frac{\partial s}{\partial r} > 0 \tag{5.21}$$

#### Deuterium burning

At a temperature of about $10^6$ K the first nuclear fuel to ignite is deuterium :

$$^2\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma \tag{5.22}$$

which releases 5.5 MeV. The heating rate due to this process is very dependent on temperature with $\epsilon_D \propto T^{11.8}$ ! The protostar is unable to effectively transport the large luminosity produced in the core via radiative transport, the core heats up, reversing the entropy gradient and the protostar becomes convective.
We can calculate the maximum energy flux which can be carried by a pure radiative

flux. This occurs when the entropy is constant. From the equations of protostellar structure :

$$L_{crit} = 4\pi r^2 \frac{4acT^3}{3\rho\kappa}\left(\frac{\partial T}{\partial r}\right)_s = -\frac{GM16\pi acT^3}{3\rho\kappa}\left(\frac{\partial T}{\partial p}\right)_s \tag{5.23}$$

- Although the amount of deuterium is small, convection helps to bring new fuel to the core from the accreting gas
- This deuterium burning phase acts as a thermostat - the *deuterium thermostat*
- Any rise in $M_\star/R_\star$ increases the stellar entropy which, via convection, increases $T_c$ ; this leads to a substantial increase in $\epsilon_D$ which inflates the star, reducing $M_\star/R_\star$.

### 5.4.3 Deuterium shell burning

As the protostar mass continues to grow via accretion the energy production from the deuterium burning remains approximately constant determined by the rate of supply of new fuel from the accreting gas.

- $L_{crit}$ however rises
- we can show that $L_{crit}$ scales as $M_\star^{11.2}R_\star^{-1/2}$; eventually $L_{crit} = L_D$ and radiative energy transport can again remove energy from the core. The results of more detailed calculations are shown on Figure 5.3
- Without convection new deuterium accreted onto the protostar accumulates in a shell
- Effectively the radiative transport acts as a barrier preventing deuterium reaching the core
- deuterium in the core is quickly depleted
- Eventually the temperature of this shell reaches $10^6$K and the shell ignites
- The hot outer shell leads to a substantial increase in the stellar radius (Figure 5.4)

### 5.4.4 Contraction and hydrogen burning

The final stage of protostellar evolution we shall follow is the contraction of the star. Without deuterium burning in the core, the self-gravity of the protostar drives

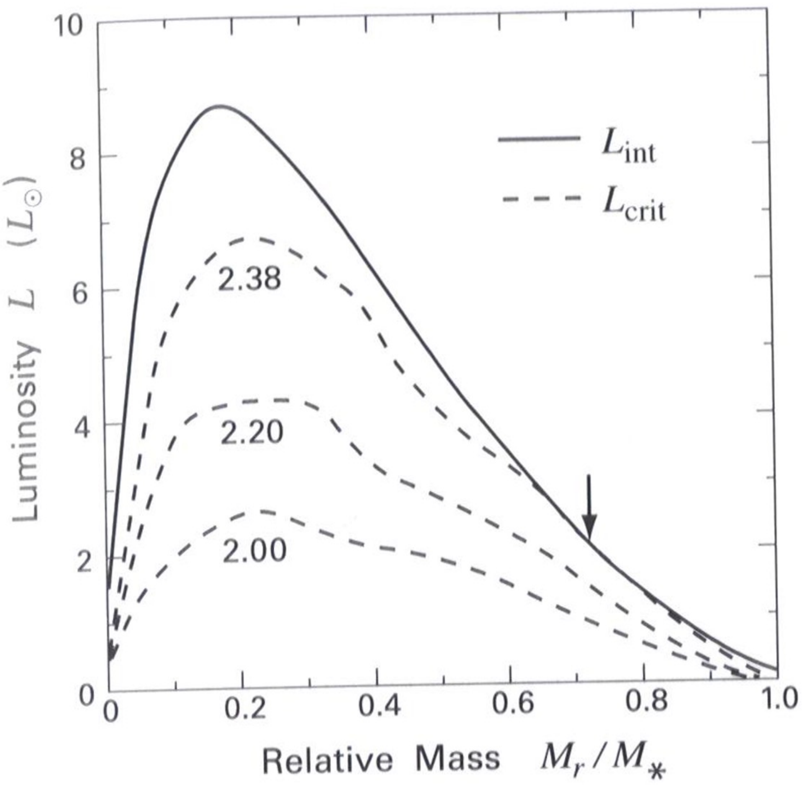


**Figure 5.3** : End of the convection phase for different stellar masses. The black arrow shows the radiative barrier

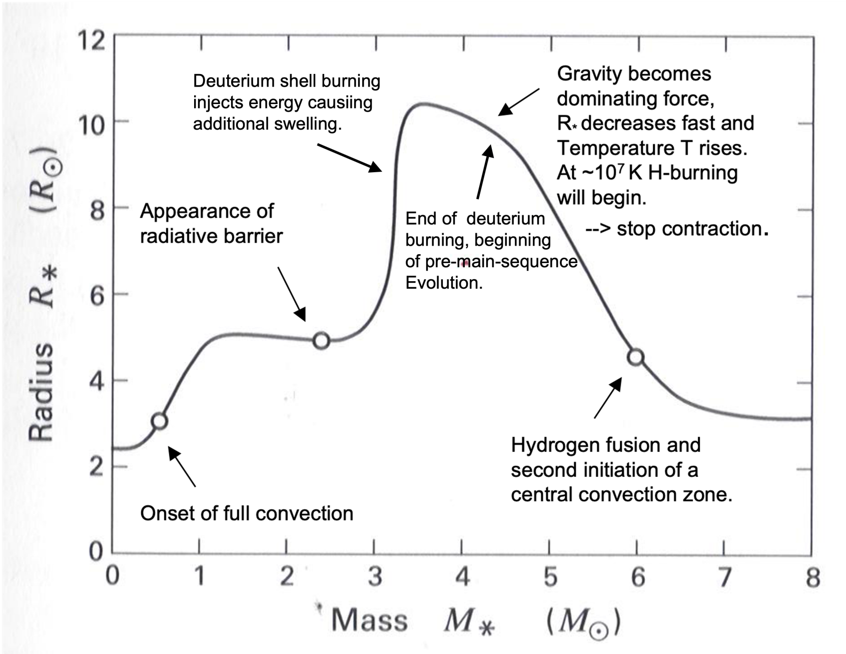


**Figure 5.4** : Radius mass relation accreting at $1 \times 10^{-5} M_{\odot} \mathrm{yr}^{-1}$. The circles mark the onset of full convection, the appearance of the radiative barrier and the onset of hydrogen burning.

the gravitational contraction of the star. The rate at which the star contracts is determined by the rate at which the star loses internal energy due to radiation. This is the *Kelvin-Helmholtz timescale:*

$$t_{KH} = \frac{GM_\star^2}{R_\star L_\star} = 3 \times 10^7 \left(\frac{M_\star}{M_\odot}\right)^2 \left(\frac{R_\star}{R_\odot}\right)^{-1} \left(\frac{L_\star}{L_\odot}\right)^{-1} \text{yr} \tag{5.24}$$

As the contraction proceeds the core temperature continues to rise until eventually $10^7$K, hydrogen burning commences and halts the contraction. Further temperature rise enables the CNO cycle. At this stage the protostar is regarded as a pre-main-sequence star.