UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics — NST3PHY
MASTER OF ADVANCED STUDY IN PHYSICS — MAPY

Thursday 28 April 11:30 to 13:00

MINOR TOPICS
Paper 2/FSU (Formation of Structure in the Universe)

*Answer **two** questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper contains 5 sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS

SPECIAL REQUIREMENTS
Mathematical Formulae Handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 One of the main challenges of modern astrophysics is to determine when the first galaxies formed in the early Universe (so-called *Cosmic Dawn*). Although this epoch is not observable yet with current telescopes, one may obtain relatively good observational constraints by measuring the ages of the most distant galaxies currently known. In the following, we will determine the ages of distant galaxies based on their dust content. [*You may assume that stellar masses are ranging from $M_\star$=0.1$M_\odot$ to $M_\star$=250 $M_\odot$*]

(a) Write brief notes on the physical properties of the first generation of stars and galaxies. Discuss in particular the Jeans Masses of the first stars and galaxies; the stellar masses, Spectral Class and lifetime of the first generation of stars, and the evolution and luminosity distribution of the first generation of galaxies. [5]

(b) Using a Salpeter Initial Mass Function (IMF) demonstrate that the fraction of newly born stars which will end their lives in type II SNe is ~ 0.3%. [4]

(c) In a similar manner, demonstrate that the fraction, f_M^{SNe}, of a galaxy's stellar mass contained in stars with $M_\star$>8$M_\odot$ is ~16%. [4]

(d) Assuming that the star-formation-rate (SFR) over the entire lifetime of the galaxy is constant, that dust is only produced in type II SNe, and that each SNe produces a constant amount of dust, M_{dust}^{SNe}, demonstrate that the age of a galaxy can be obtained from the total dust mass, M_{dust}^{tot}, as : [4]

$$\text{Age} = \frac{\langle M_\star^{SNe} \rangle \times M_{dust}^{tot}}{M_{dust}^{SNe} \times f_M^{SNe} \times \text{ SFR}},$$

where $\langle M_\star^{SNe} \rangle$ is the average mass of stars ending their lives as type II SNe.

(e) Recent observations suggest that each type II SNe produces 0.5$M_\odot$ of dust. Determine the age of the galaxies in Table 1, and use the information in Table 2 to estimate their formation redshift. [*You may assume that the average mass of stars ending their lives in type II SNe is ~8$M_\odot$*] [3]

Galaxy	M_{dust} [$M_\odot$]	SFR [$M_\odot$/yr]	$M_\star$ [$M_\odot$]	z_{obs}	$Age_{uni}(z_{obs})$ [Gyr]
A2744_YD4	6×10^6	20	2×10^9	8.38	0.589
MACS0416_Y1	4×10^6	60	3×10^8	8.31	0.596

Table 1: Dusty z ≥7.5 galaxies detected with ALMA. The last column gives the age of the Universe at the redshift at which the galaxies are observed.

	Age of the Universe [Myr]							
	520	530	540	550	560	570	580	590
Redshift	9.20	9.07	8.94	8.82	8.70	8.59	8.48	8.37

Table 2: Redshift as a function of the age of the Universe assuming H_0=70 km/s and Ω_Λ=0.7

2 The formation of the very first stars (popIII stars) in the early Universe is a highly active topic of cosmology. The successful launch of the *James Webb* Space Telescope in December 2021 has opened a new window to observe directly the birth of the first generation of stars. In the following, we will investigate whether the JWST has the capability to observe the first stars. [*In the following, we consider an isothermal spherical cloud of gas*]

(a) From the dispersion relation, $k^2 a_T^2 - \omega^2 = 4\pi G \rho_0$, show that the critical mass of a gas cloud above which gravity dominates is given by : [6]

$$M_J = \frac{\pi^{5/2}}{3} \frac{a_T^3}{\rho_0^{1/2} G^{3/2}}, \qquad (*)$$

where $a_T = \sqrt{\frac{k_B T}{\mu}}$, μ is the mass of a hydrogen atom, k_B the Boltzmann constant and G the gravitational constant.

(b) Simulations of the early Universe show that the first galaxies formed from gas clouds at a temperature T =200K and with a density of ρ_0=1.7×10^{-17} kg/m^{-3}. From equation (*), estimate the Jeans Mass of a popIII star and explain why this value is different from what is expected for stars currently forming in the Milky Way. [*$1 M_\odot \sim 2\times10^{30}$kg*] [3]

(c) From the expression of the Planck Function : [6]

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}(e^{hc/\lambda k_B T} - 1)^{-1},$$

where h is the Planck constant and c the speed of light, demonstrate that for a black body emitter, the wavelength at which the (specific) intensity is maximised is given by :

$$\lambda_{peak} = \frac{\alpha}{T},$$

with $\alpha \approx 2.9\times10^{-3}$ m K. [*The solution of the equation* $5(e^x - 1) = xe^x$ *is x=4.9654*]

(d) Theoretical studies demonstrate that the surface temperature of the first generation of stars is T ~50 000 K. Assuming that popIII stars are perfect thermal emitters formed 100 Myr after the Big-Bang (redshift z=30), determine at which wavelength their emission is maximum, and at which wavelength the peak wavelength in the emission spectrum of these stars will be received on Earth. [2]

(e) The first stars did not form in isolation, but rather in small galaxies containing ~10^5 stars. From Figure 1 showing the sensitivity of the NIRCam camera on-board the *James Webb* Space Telescope, estimate the minimum luminosity of the first galaxies emitting light at z ~30 (luminosity distance : D_L=1.08×10^{30}cm) which can be probed in 2.8hrs by NIRCam [1 Jy = 10^{-23}erg/s/cm^2/Hz]. The luminosity function of very high-redshift galaxies shows that the main population has a luminosity ranging between 10^{29} and 10^{30} erg/s/Hz. What do you conclude about the capabilities of the JWST to observe the first generation of stars ? [3]

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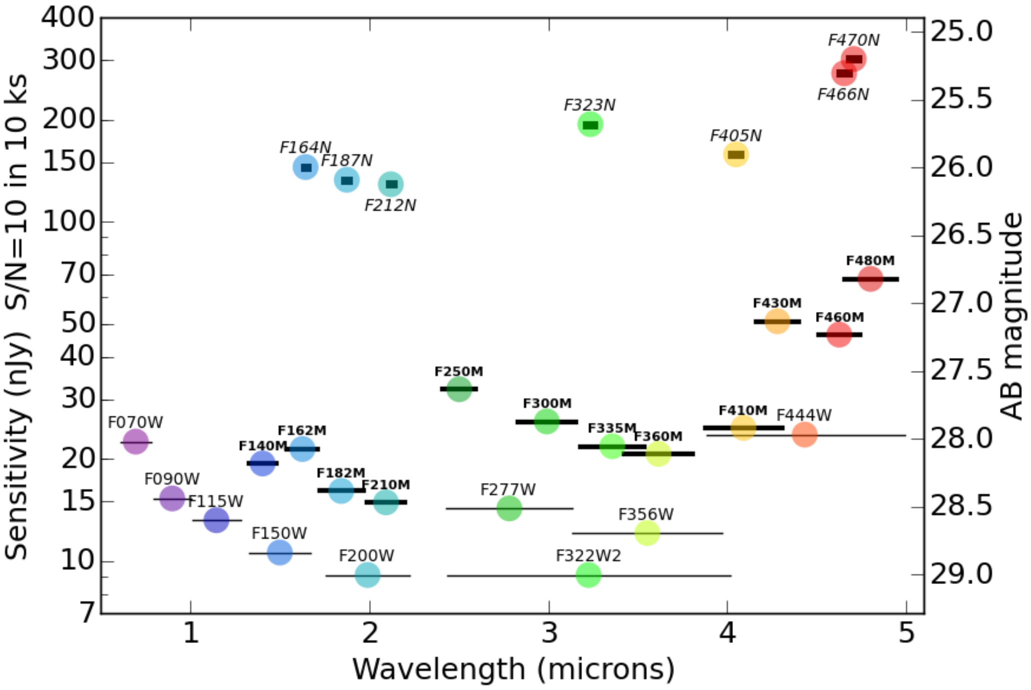

Figure 1: Sensitivity of the NIRCam/JWST filters : the fluxes on the left are for sources with a signal-to-noise ratio of 10 in 10ks exposure. Each filter is named using the following convention : F_ _ _W/M/N. Source : NASA/ESA

3 In 2019, the Event Horizon Telescope imaged the central black hole of the galaxy M87, a supergiant elliptical galaxy located 55 million light-years from the Milky Way with a total mass of M_{tot}= 2.4×10^{12}M$_\odot$. The mass of the black hole is estimated to be M_{BH}=6.5×10^9 M$_\odot$. In the following we will study the host galaxy and the central black hole.

(a) M87 can be viewed as a system with a gas mass M_{gas} and a stellar mass $M_\star$, where gas is converted into stars at a rate $\Psi=\alpha M_{gas}$, where α is constant, and is subject to a constant gas inflow rate of Φ. The mass of the black hole follows the scaling relation $M_{BH} = \beta M_\star$ which produces a galactic wind whose outflow rate is proportional to its luminosity in the form $\dot{M}_{outflow}=\lambda L_{AGN}$ (where β and λ are constants). Neglecting the fraction of gas returned to the ISM by supernovae and the fraction of gas accreted onto the black hole, and assuming that L_{AGN} is constant, determine that the gas mass evolves as : [7]

$$M_{gas}(t) = M_{gas}(t = 0)e^{-\alpha t} + \frac{1}{\alpha}(\Phi - \lambda L_{AGN})(1 - e^{-\alpha t}).$$

(b) Recent observations of stars within M87 allow us to obtain a robust estimate of its age as 13.24 Gyr. Assuming that, since its formation, the central black hole has been accreting mass at the Eddington rate with a radiative efficiency of ϵ =1.0, find the minimum mass of the black hole's progenitor. What can you conclude on the black hole accretion ?[*The Eddington luminosity is given by :* [5]

$$L_{Edd} = \frac{4\pi c G m_p}{\sigma_e} M_{BH},$$

where σ_e is the electron cross section equal to 6.65×10^{-29}m^2]

(c) Archaeoastronomy studies of M87 (total mass M_{87}) show that it merged with another galaxy (total mass m) 8 billion years ago. Demonstrate that the relaxation time scale for dynamical friction to act is given by : [7]

$$t_r = \frac{v^3}{4\pi G^2 M_{87} \rho \ln \Lambda},$$

where $\Lambda = \frac{b_{max}}{b_{min}}$ and $\rho = mn$ is the density of the interacting galaxy.

(d) Estimate the relaxation time for the merging of M87 with a smaller galaxy, assuming a velocity v ~200km/s and a density of the merging galaxy of $\rho \sim 10^8$M$_\odot$ kpc^{-3}, and ln Λ ~1. [1]

END OF PAPER